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PRACTICAL PLANE AND SOLID GEOMETRY

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Elementary Science Manuals

PRACTICAL PLANE AND SOLID GEOMETRY

INCLUDING GRAPHIC ARITHMETIC

BY

I. H. MORRIS

AUTHOR OF 'GEOMETRICAL DRAWING'

LONDON
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PREFACE

In this text-book the Author has endeavoured to cover as fully as possible the needs, not only of candidates for the elementary examinations of the Science and Art Department, but also the wants of the general student of Geometry.

The special features which it is hoped will be found of service to both teachers and students are:—

The arrangement of the subject so that, as far as possible, similar Problems are grouped together; the arrangement of the Diagrams facing the text; the very large number of examples worked out; the numerous notes; and the copious number of Exercises which are appended. These Exercises are selected from the papers of the Science and Art Department, College of Preceptors, Oxford and Cambridge Locals, and various Military Colleges. They are carefully graduated, and, where necessary, hints for solution, and references to the problems upon which they depend, are given.

In the Solid Geometry an attempt has been made to make the subject as clear as possible by means of Diagrams and simple expedients.

The Author, who has had much practical experience in teaching the subject, would especially urge upon students the necessity of working out the examples, as more will be learnt by drawing carefully a number of well-graded problems, than by reading over many text-books.

I. H. M.

SHEFFIELD: *July*, 1890.

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SECTION I

PRACTICAL PLANE AND SOLID GEOMETRY

CHAPTER I

INTRODUCTION

In Practical Geometry we apply the principles of Theoretical Geometry to construction by the aid of instruments. The student who has a knowledge of Euclid will find it of considerable service in understanding the principles used, and also in remembering the methods of construction adopted.

It is of the highest importance that the problems should be worked with the greatest possible accuracy and neatness, and in a variety of positions, as problems frequently present fresh difficulties when the position of the points or lines is altered.

The *instruments* used need not be numerous, but should be of the best make and finish that the student can obtain, as inferior instruments frequently cause much trouble and vexation, and render the accuracy so indispensable in a geometrical drawing an impossibility. The following are absolutely essential :—

1. **A Drawing-board** with a perfectly level surface, and with its corners true right angles. Half-imperial is a very convenient size.

2. **A T square.**—This is used to draw lines parallel to the edge of the board. It is not advisable to use it for the purpose of drawing *vertical* as well as horizontal lines, as, if the board be not true, inaccuracies will be caused in the drawing. The *vertical* lines are best obtained by using the set square. It is advantageous to have the edge of the T square bound with hard wood, and bevelled. The blade should be screwed on to the head, as this arrangement allows the set squares to be used more freely.

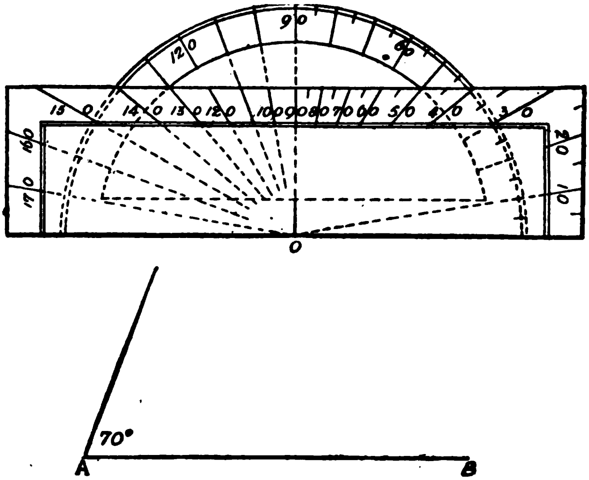
3. **Two set squares,** having angles of 45° and 60° respectively. These are used to obtain perpendiculars and parallels. Skilful manipulation of these useful instruments will enable the student to construct many simple figures by their aid alone.

4. **Pencils.**—These should be HH for the construction lines, and H for darkening the lines of the constructed figure. They are best

sharpened like the edge of a *chisel* for geometrical drawing, as the point lasts longer.

5. **Mathematical instruments.**—These should include:—**A compass** with movable pen and pencil legs (those with needle-points are preferable, as they do not make so large a hole in the paper); a pair of **dividers**, for measuring; a **mathematical pen**, for ruling lines in ink. In addition to these, a set of *spring bow compasses*, for describing small arcs and circles, are of great assistance. **Indian ink** must always be used with the instruments, as it does not corrode them; that sold in a liquid form is very convenient. After the pens have been used they should be carefully wiped, to prevent rust.

6. **A protractor.**—This is made either semi-circular or as a flat rule; the latter form is more convenient. The illustration shows how the flat protractor is made from the round one. The instrument is used as follows:—Suppose an angle of 70° be required at point A in the line A B. Place the mark O, which is always indicated on the protractor, on the point A, keeping the edge of the instrument on the line A B, and make a tick with the pencil at the number 70°; remove the protractor, and join the tick with the point A. An angle of 70° will then have been made with the line A B. The degrees are numbered from each end of the protractor, so that the angle of 70° may be made either to the left or right of the point A.



7. **Paper and pins.**—For ordinary pencil-work cartridge-paper is the most suitable; when the drawings have to be inked in, a better quality is desirable, such as Whatman's smooth papers. Pins with heads soldered on are the most convenient.

8. Parallels and perpendiculars.—These should be obtained by means of the T square and set squares. The following instruction is given on the examination papers of the Science and Art Department:—'Lines parallel or perpendicular to others may be drawn mechanically, without showing any construction. Lines may be bisected by trial.'

CHAPTER II

DEFINITIONS, TERMS USED, &c.

1. **A point** has no magnitude. It merely indicates *position*, and is marked either by a dot or as in Fig. 1.

LINES.

2. **A line** has *length* and *position*, but neither breadth nor thickness. It is indicated by the letters placed at its extremities, as A B, Fig. 2. Various methods of drawing lines are used in practice, as thick, thin, dotted, and chain lines. Fig. 2.

3. **A straight line** is the shortest distance between two given points. It is also called a *right* line.

4. **A curved line** is nowhere straight. There are endless varieties of curved lines. Fig. 3 is a *simple* curve. Fig. 4 is a *compound* curve.

5. **A horizontal line** is perfectly level, like the surface of still water. Fig. 5.

6. **A vertical line** is perfectly upright, like a plumb-line. Fig 6.

7. **An oblique line** is neither horizontal nor vertical. Fig. 7.

8. **Parallel lines** are the same distance apart, and cannot meet, however far they may be produced. Fig. 8.

ANGLES.

9. **An angle** is the inclination of two straight lines to each other, which meet together.

10. **A right angle.**—When two straight lines meet each other so that the adjacent angles are equal to each other, each of these angles is a right angle, and the lines are **perpendicular** to each other. Fig. 9.

11. **An obtuse angle** is greater than a right angle. Fig. 10.

12. **An acute angle** is less than a right angle. Fig. 11.

13. **The vertex** is the point where the two lines forming an angle meet, as at A, Fig. 11.

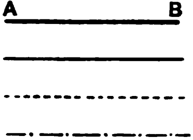
14. **Adjacent angles** have a common vertex and one common arm. In Fig. 12, the angle D A B is adjacent to the angle D A C.

15. **The complement** of an angle is the difference between it and a right angle. In Fig. 13, the angle C A B is the complement of the angle C A D, and C A D is the complement of C A B.

16. **The supplement** of an angle is the difference between it and two right angles. In Fig. 14, the angle C A B is the supplement of the angle C A D, and *vice versâ*.

Fig. 1.

Fig. 2.



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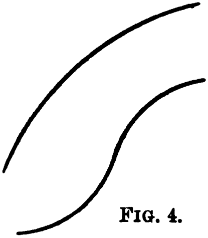
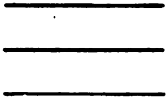


Fig. 4.

Fig. 5.



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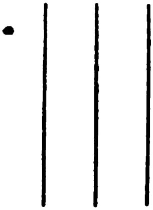

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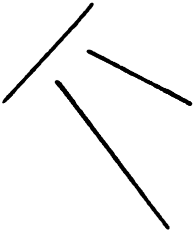

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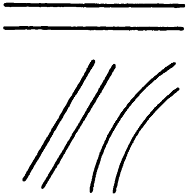

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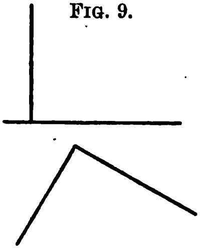

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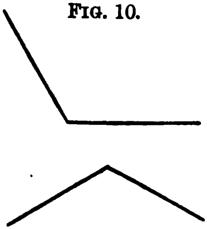

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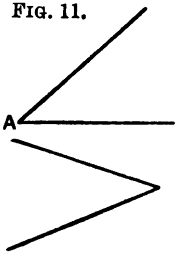

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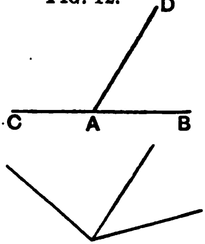

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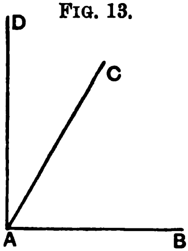

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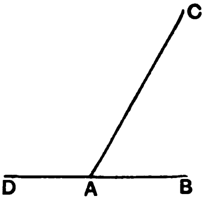


TRIANGLES.

17. **A triangle** is a figure contained by three straight lines. If it be supposed to stand upon one of its sides, that side is termed its **base**; the point where the other two sides meet is its **vertex** or **apex**, the angle at the vertex is the **vertical angle**, and the perpendicular from the apex to the base or base produced is the **altitude.** Thus, in Fig. 15, if A B be the base, then C is the vertex or apex, A C B the vertical angle, and C D the altitude.

Triangles are named either from the comparative lengths of their sides, or from the magnitudes of their angles.

1*st. With reference to their sides there are three kinds :—*

18. **An equilateral triangle** has three equal sides. Fig. 16.

19. **An isosceles triangle** has two of its sides equal. Fig. 17.

20. **A scalene triangle** has three unequal sides. Fig. 18.

2*nd. With reference to the angles there are also three kinds :—*

21. **A right-angled triangle** has one of its angles a right angle. The side opposite to the right angle is called the **hypotenuse.** In Fig. 19, A B is the hypotenuse.

22. **An obtuse-angled triangle** has one of its angles obtuse. Fig. 20.

23. **An acute-angled triangle** has three acute angles. Fig. 21.

Note.—All the angles of any triangle equal two right angles, or 180°; thus, if two of the angles of a triangle are respectively 60° and 50°, then the remaining angle must be 70°, to make up the 180°.

QUADRILATERALS.

24. **A quadrilateral** figure is bounded by four straight lines. It is also termed a **quadrangle,** from having four angles.

Note.—The four angles of any quadrilateral figure must always equal four right angles, or 360°.

25. **A parallelogram** is a quadrilateral figure in which the opposite sides are parallel. The straight line joining the opposite angles of the parallelogram is called the **diagonal.** In Fig. 22, A B is the diagonal. There are four parallelograms—viz., the square, the rectangle or oblong, the rhombus, and the rhomboid.

26. **A square** has all its sides equal and all its angles right angles. Fig. 22.

27. **A rectangle** or **oblong** has its opposite sides equal and all its angles right angles. Fig. 23.

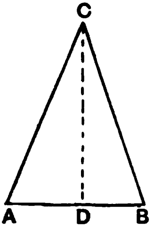
28. **A rhombus** has all its sides equal, but its angles are not right angles. Fig. 24.

29. **A rhomboid** has its opposite sides equal, but its angles are not right angles. Fig. 25.

30. **A trapezoid** has only two sides parallel. Fig. 26.

31. **A trapezium** has none of its sides parallel, but may have two of its sides equal. Fig. 27. When two of the sides are equal, the figure is sometimes called a **trapezion,** or **kite.**

FIG. 15.



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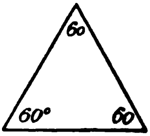

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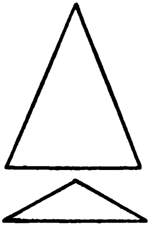

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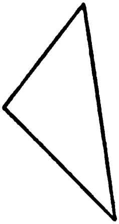

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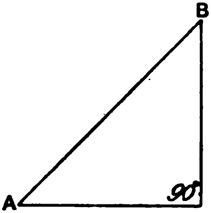

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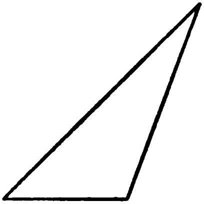

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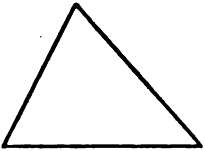

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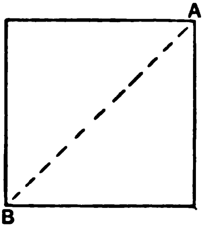

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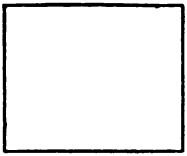

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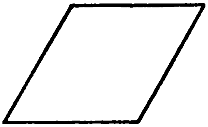

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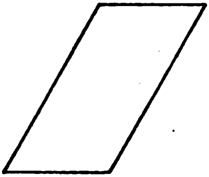

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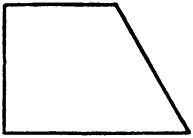

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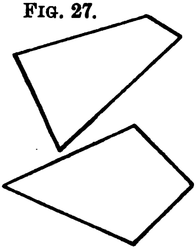


POLYGONS.

32. **A polygon** is a plane figure bounded by straight lines.

Note.—In some works a polygon is defined as a figure bounded by more than four straight lines.

If the sides of the figure are equal, it is termed a **regular** polygon. Fig. 28. If the sides are unequal, it is called an **irregular** polygon. Fig. 29.

Polygons are named according to the number of their sides:—

A pentagon has five sides.
A hexagon has six sides.
A heptagon has seven sides.
An octagon has eight sides.
A nonagon has nine sides.
A decagon has ten sides.
An undecagon has eleven sides.
A duodecagon has twelve sides.

THE CIRCLE.

33. **A circle** is a plane figure contained by one line, called the **circumference.** Every part of the circumference is equally distant from a point within the circle, which is called the **centre.** A, Fig. 30.

Note.—The circumference is sometimes spoken of as the circle.

34. **The diameter** of a circle is the straight line passing through its centre, and terminated at both ends by the circumference, as B C, Fig. 30. It divides the circle into two semicircles. If two diameters be drawn at right angles to each other, the circle is divided into **quadrants.** Fig. 30.

35. **The radius** of a circle is the distance from the centre to the circumference, as A B, Fig. 30.

36. **An arc** of a circle is the portion between any two points in the circumference, as A B, Fig. 31.

37. **A chord** is the straight line joining the ends of an arc, as the straight line C D, Fig. 31.

38. **A segment** is the part of a circle bounded by an arc and its chord. Fig. 32.

39. **A sector** is the part of a circle contained by two radii and the arc between them. Fig. 33.

40. **A tangent** is a straight line which touches a circle, but does not cut it when produced, as A B, Fig. 34.

41. **Concentric circles** have the same centre but different radii. Fig. 35.

Fig. 28.

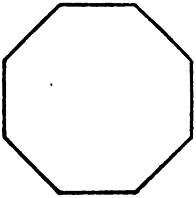


Fig. 29.

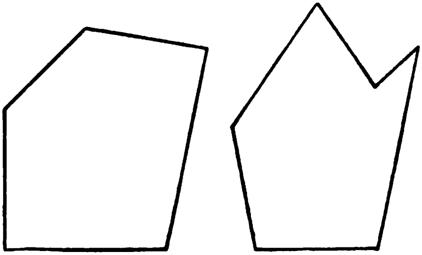
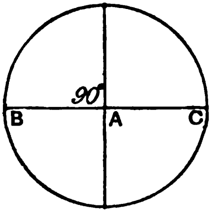


Fig. 30.



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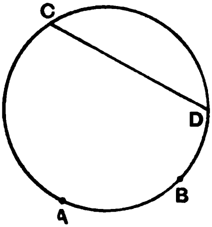
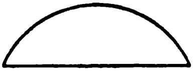


Fig. 32.



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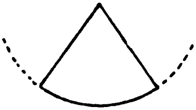


Fig. 34.

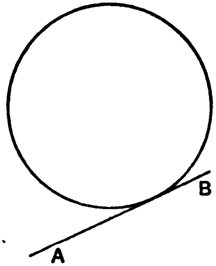
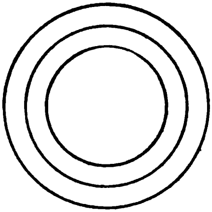


Fig. 35.



CHAPTER III

LINES AND ANGLES

PROBLEM 1.—To bisect a given line.

With centre A, and any distance greater than half the line, describe an arc. With centre B, and the same radius, intersect it in C and D. Draw the line C D. Then the line C D bisects the given line at right angles, or is perpendicular to it. (EUC. I. 10.)

The curved line E F may be bisected in a similar manner. By this problem a line may be divided into 4 equal parts by again bisecting each half as shown. If those parts be again bisected, the line would be divided into 8 equal parts; and in a similar manner into 16, &c.

PROBLEM 2.—To draw a perpendicular to a given line, from a given point in the line.

Let C be the given point. With C as centre, and any radius, set off equal distances C 1 and C 2. With 1 and 2 as centres, and any radius greater than half 1 2, describe arcs intersecting at D. Draw C D, the required perpendicular.

PROBLEM 3.—To draw a perpendicular when the given point is at or near the end of the line.

Let B be the given point. With centre B describe an arc. With the same radius cut this arc in points 2 and 3. From 2 and 3 describe arcs intersecting at C. Draw B C, the required perpendicular.

Note.—Keep the same radius for describing all the arcs.

PROBLEM 4.—The same. (Another method.)

Take any point C not in the given line A B. Join C B. With centre C and radius C B describe an arc cutting A B in D. Join C D, and produce until it cuts the arc in E. B E will be the required perpendicular. (EUC. III. 31.)

PROBLEM 5.—The same. (For large work when instruments are not available.)

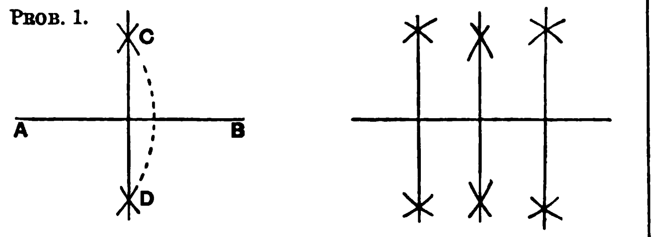
From B set off 3 equal divisions towards A to any convenient unit. With centre B and 4 of the same units describe an arc at C. With centre A and 5 units cut the arc at C. B C will be the required perpendicular. (EUC. I. 48.)

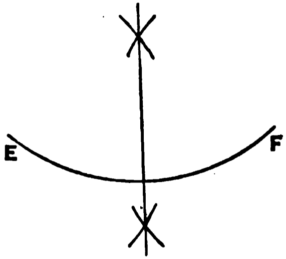
Note.—Any triangle in which the square on one side equals the sum of the squares on the other two sides is a right-angled triangle. $AC^2 = AB^2 + BC^2$, or $5^2 = 3^2 + 4^2$.

PROBLEM 6.—To draw a perpendicular to a line from a given point without it.

Let A be the given point. With centre A describe an arc cutting the given line B C in points 1 and 2. With centres 1 and 2 describe arcs intersecting at D. Draw A D, the required perpendicular. (EUC. I. 12.)

PROB. 1.





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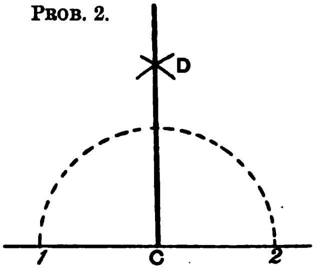

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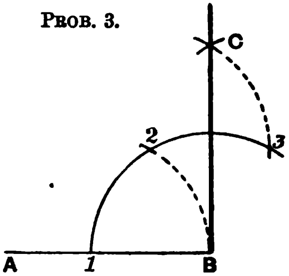

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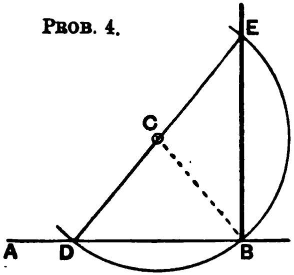

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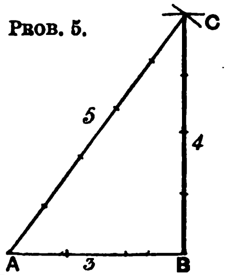

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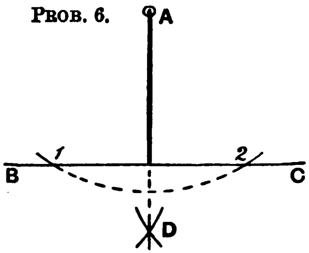

PROBLEM 7.—To draw a perpendicular when the given point is nearly over the end of the given line.

Let A be the given point. With centre B and radius B A describe an arc. With centre C and radius C A describe an arc cutting the first arc in A and D. Draw A D, the required perpendicular.

Note.—Two positions of the point are shown.

PROBLEM 8.—The same. (Another method.)

Let A be the given point. Take any point D in the line B C. Join A D, and bisect it in E. With centre E describe the semicircle A F D, cutting B C in F. Draw the perpendicular A F. (Euc. III. 31.)

PROBLEM 9.—To draw a line parallel to a given line, at a given distance from it.

Let A B be the given line, and C D the given distance. Take any points 1 and 2, and with radius C D describe two arcs, E and F. Then the line touching the arcs will be the required parallel.

PROBLEM 10.—To draw a line parallel to a given line through a given point.

Let A be the given point, and B C the given line. With any point 1 as centre, and radius 1 A, describe the arc A 2. With A as centre and the same radius describe another arc. Cut off 1 D equal to A 2. Draw A D, the required parallel.

PROBLEM 11.—To bisect a given angle.

With centre A and any radius describe the arc B C. With centres B and C describe arcs intersecting in D. Then the line A D bisects the angle.

By bisecting each half again, the angle will be divided into 4 equal parts, as shown. In a similar manner, it may be divided into 8, 16, &c., equal parts.

PROBLEM 12.—To trisect a right angle.

Let A B C be the given right angle. With centre B and any radius describe the arc 1 2. With centres 1 and 2 and the same radius cut the arc in 3 and 4. The lines drawn from B through these points will trisect the right angle.

PROBLEM 13.—To make an angle equal to a given angle.

Let B A C be the given angle. Draw any line, D E. With centres A and D describe arcs B C and F G. Cut off F G equal to B C. (Euc. III. 27.)

PROBLEM 14.—Through a given point to draw a line meeting another line at an angle equal to a given angle.

Let A be the given point, B C the given line, and D the given angle. Through A draw a line parallel to B C. (*Prob.* 10.) At the point A make an angle 1 A 2 equal to the angle D. (*Prob.* 13.) Produce A 2 to E. Then the line A E meets B C, and makes the angle A E C equal to D. (Euc. I. 29.)

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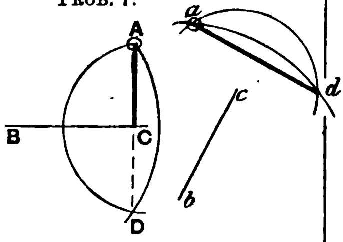

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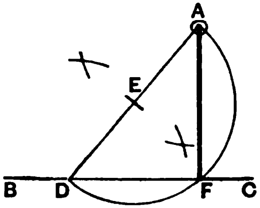

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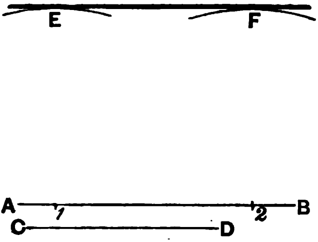

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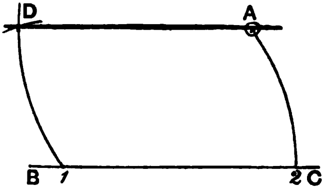

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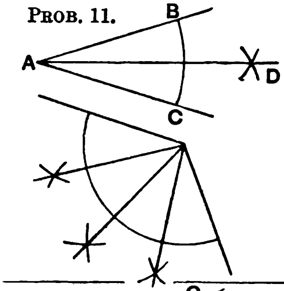

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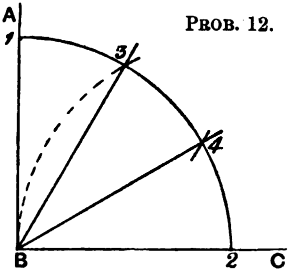

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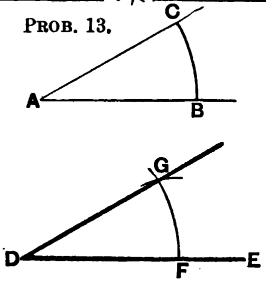

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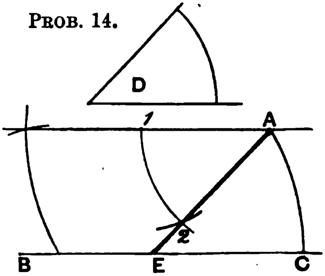


PROBLEM 15.—To bisect the angle made by two converging lines without producing them.

Let A B and C D be the two lines. Draw a line parallel to A B at any convenient distance, and at the same distance draw another line parallel to C D intersecting the first parallel at E. Bisect the angle thus obtained. Then E F bisects the angle at which A B and C D are inclined.

Notes.—1. The angle of inclination might be found by drawing one parallel only to meet the opposite line. 2. All circles required to touch the lines A B and C D would have their centres in E F.

PROBLEM 16.—From any given point to draw a line which should meet at the same point as two converging lines would meet if produced.

Let A be the given point, and B C, D E the two converging lines. Draw any line A 2. Mark a point 1 in the line B C not in the same straight line as A 2. Join A 1 and 2 1, forming the triangle A 2 1. Take any point 3 in D E. From point 3 draw 3 4 parallel to 2 1, and 3 F parallel to 2 A. From 4 draw 4 F parallel to 1 A, forming the triangle 3 4 F. The line A F passing through the corresponding angle of each triangle is the line required.

Notes.—1. Two positions of the point A are given, one between and one outside the lines. 2. Use the set square in obtaining the parallel lines in this and all complicated figures, as the describing of a number of arcs would cause confusion.

PROBLEM 17.—To find a point in a line equally distant from two given points without it.

Let A B be the given line, and C and D the given points. Join C D, and bisect by a line meeting A B in E. Then E is the required point, and E C equals E D.

Note.—From E a circle could be described passing through C and D.

PROBLEM 18.—From two given points without a straight line to draw two straight lines to meet the given line and make equal angles with it.

Let A and B be the two points, and C D the given line. Draw A E perpendicular to C D, by *Prob.* 7. Draw E B, cutting C D in F. Join A and F. Then A F and B F are the required lines.

PROBLEM 19.—Through a given point between two converging lines to draw a straight line which shall be terminated by the given lines and bisected in the given point.

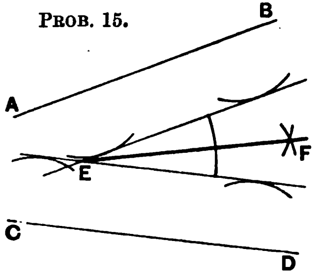
Let A be the given point, and C D, E F the given lines. Draw A B perpendicular to E F, and produce it, making A G equal to A B. From G draw G D parallel to E F. From D draw D E through the point A. Then D E is bisected in the point A. (Euc. I. 26.)

PROBLEM 20.—To draw a line from a given point which shall make equal angles with two converging lines.

Let A be the given point, and B C, B D the converging lines. Produce B D, and bisect the *supplemental* angle thus formed. From A draw A F parallel to the bisecting line, B E.

If the converging lines do not meet, then obtain the *supplemental* angle by drawing a line parallel to B D, as shown in the second figure. Bisect the angle thus formed, and proceed as in the previous figure.

PROB. 15.



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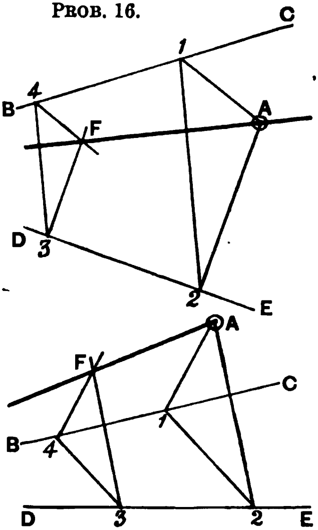
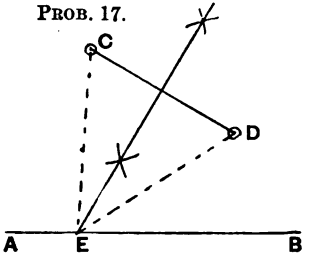


PROB. 17.



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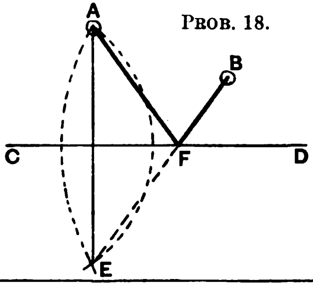

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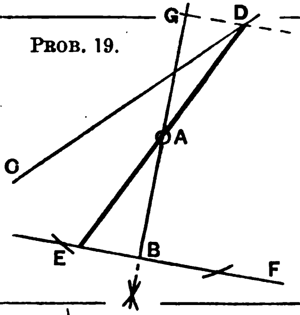
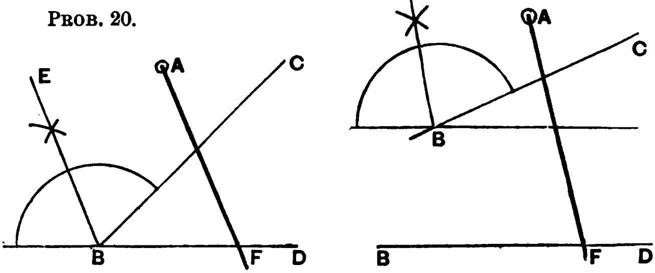


PROB. 20.



PROBLEM 21.—From a given point to draw a line so that the part intercepted between two given parallel lines shall be equal to a given distance.

Let A be the given point, B C and D E the two parallel lines, and F the given distance.

Take any point G in B C. With centre G and radius equal to F describe an arc cutting D E in H. Draw G H.

From A draw a line, A K, parallel to G H. Then the distance J K is equal to the given distance F.

The solution when the given point is between the two parallel lines is exactly similar. Both positions are shown.

PROBLEM 22.—To divide a line into any number of equal parts (say 5).

Let A B be the given line. Draw a line at any angle, and set off any convenient distance five times. Draw 5B, and from the points 4, 3, 2, 1, rule parallels to 5B with the set square. These parallels will divide the line as required.

PROBLEM 23.—The same. (Without using the set square.)

Draw A 5 at any angle to A B. At B make the angle A B 5 equal to the angle B A 5. (*Prob.* 13.) From A and B, on the lines A 5 and B 5, set off 5 equal distances. Join 5 B, 4 1, 3 2, 2 3, 1 4, and A 5, thus cutting the line A B into 5 equal parts.

PROBLEM 24.—To construct an angle containing a given number of degrees.

The circumference of a circle is supposed to be divided into 360 equal parts, called *degrees*. The radius of a circle may be set off exactly six times round the circumference; hence, if an arc be described, and a portion cut off equal to the radius of the arc, an angle containing 60° will be obtained. With a knowledge of this principle, a variety of angles may be constructed. Constructions for angles of 60°, 120°, 30°, 15°, 45°, and 75° are shown on the opposite page.

For 60°, describe an arc, and cut it with the same radius.

For 120°, describe an arc, and set off twice the radius.

For 30°, obtain an angle of 60°, and bisect it.

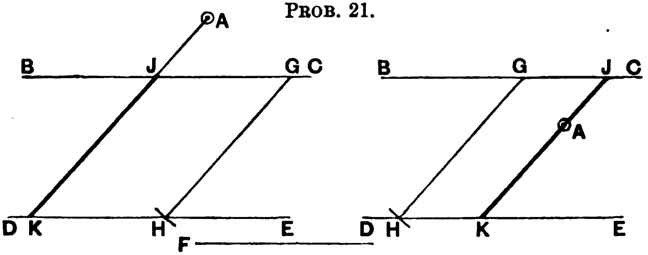
For 15°, obtain an angle of 30° as above, and bisect it.

For 45°, obtain an angle of 30°, and bisect the arc A B. (30° + 15° = 45°.)

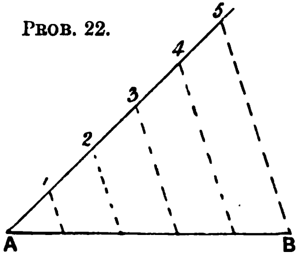
For 75°, draw a right angle, trisect it, and bisect the top division, A B.

Other angles also may be easily made. 150° = 120° + 30°. 105° = 90° + 15°. 135° = 90° + 45°. $22\frac{1}{2}°$ = half 45°. $67\frac{1}{2}° = 45° + 22\frac{1}{2}°$; &c.

PROB. 21.



PROB. 22.



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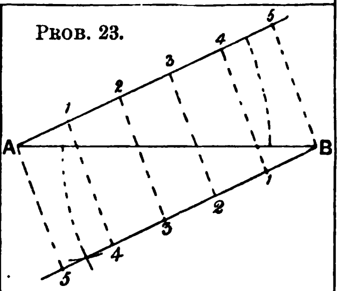

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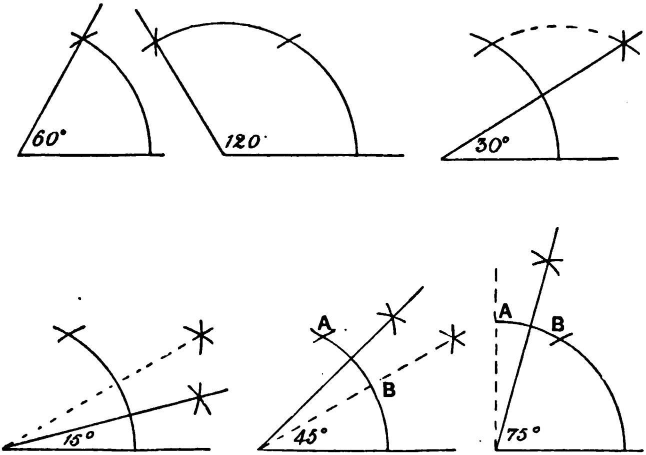


CHAPTER IV

PROPORTIONALS

If we compare two numbers with respect to the number of times one contains the other, a **ratio** is formed. Thus, as 9 : 3 is a ratio, and means the same as the fractional expression $\frac{9}{3}$, or that 9 contains 3 three times. If we take two equal ratios, we have a **proportion**; for example, $\frac{9}{3} = \frac{6}{2}$; or 9 : 3 as 6 : 2.

The first term of a ratio is called the **antecedent**; and the second term, the **consequent**.

If the numbers are in true proportion, the product of the end terms, or **extremes**, equals the product of the middle terms, or **means**.

'*If a straight line be drawn parallel to one side of a triangle, it cuts the other two sides, or those produced, proportionally.*' (Euc. vi. 2.)

PROBLEM 25.—To divide a line in the same manner as another given divided line.

Let **A B** be the line. It is to be divided similarly to **C D**.

Draw a line at any angle to **A B**, and set off **C 1**, **C 2**, **C 3**, and **C D** on it.

Join **D′ B**, and draw parallels from 3′, 2′, and 1′ to **D′ B**. Then **A B** will be divided similarly to **C D**.

PROBLEM 26.—To divide a line into three parts in the same proportion as the numbers 2, 3, and 4.

Let **A B** be the given line. From **A** draw a line at any angle. Set off 2 + 3 + 4 equal parts. Join 9 **B**, and from the points 5 and 2 rule parallels cutting **A B**, so that **A C : C D : D B** as 2 : 3 : 4.

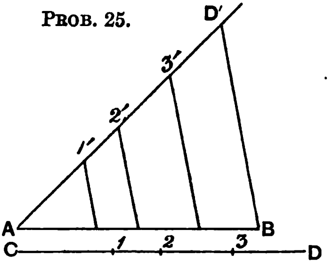
PROBLEM 27.—To find a fourth proportional to three given lines.

Let **A**, **B**, and **C** be the three given lines. Draw **D J** and **D K**, of indefinite length, and at any angle to each other. Set off **D E** equal to **A**, **D F** equal to **B**, and **E G** equal to **C**. Join **E F**. From **G** rule a parallel to **E F**, cutting off **F H**, the fourth proportional required. **D E : D F : : E G : F H.**

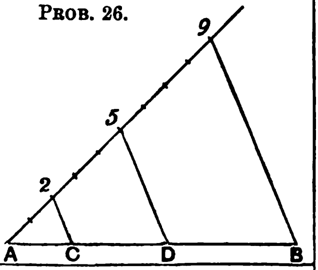
Notes.—1. This construction answers for all cases. It should be worked to scale, and the result verified by the student. If the three lines be 3, $2\frac{1}{2}$, and 2 inches long, then the fourth proportional will be found thus :—3 : $2\frac{1}{2}$ :: 2 : F H. But 3 F H = $2\frac{1}{2} \times 2$. Therefore, $\text{F H} = \frac{2\frac{1}{2} \times 2}{3} = 1\frac{2}{3}$ inches.

2. When the lines are long, it is sometimes more convenient to proceed as in **Problem 27a** :—Draw two lines at any angle as before, and set off D E equal to A, and D F equal to B. Make D G equal to C, and from G draw G H parallel to E F. Then D H will be the fourth proportional **less** than any of the given lines. The same method of construction is applicable when a fourth proportional **greater** than any of the given lines is required, but it is necessary to commence with the shortest line. See **Problem 27b**. Draw two lines at any angle, and set off D E equal to C, D F equal to B, and D G equal to A. Join E F, and from G draw G H parallel to E F. Then D H will be the fourth proportional **greater**.

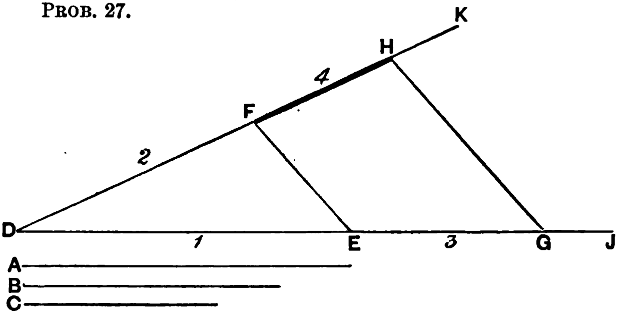
Prob. 25.



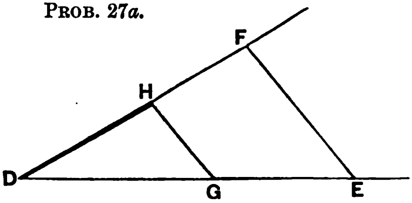
Prob. 26.



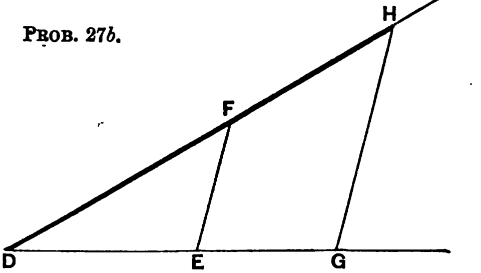
Prob. 27.



Prob. 27*a*.



Prob. 27*b*.



PROBLEM 28.—To find a third proportional to two given lines.

Let A and B be the given lines.

This problem is exactly the same as finding a fourth proportional to three magnitudes, the last two of which are equal. If we take numbers, for example, and require a third proportional to 8 and 4, the statement would read thus—8:4 as 4:2. Here 2 is manifestly the third proportional, because it bears the same relation to 4 that 4 does to 8.

In the case of the given lines, proceed in a similar manner, remembering that the line B is used as both the second and third term.

Draw two lines at any angle. Set off C D equal to A, and C E equal to B. Join D E. Now set off D F, also equal to B. Draw F G parallel to D E. E G is the required third proportional. A:B as B:E G.

Note.—The line B may be set off from C as in *Problem* 27*a*: the parallel *f g* cutting off the third proportional C *g*, which will be found to be of exactly the same length as E G.

PROBLEM 29.—To find a mean proportional to two given lines.

Let A B and C D be the given lines.

Produce A B, and set off B E equal to C D. Bisect A E, and describe a semicircle. At B draw B F perpendicular to A E. Then B F is the mean proportional required. A B:B F as B F:B E. (Euc. vi. 13.)

Note.—The square constructed on B F equals the rectangle constructed with sides equal to A B and C D.

PROBLEM 30.—To divide a line into an extreme and mean ratio— that is, so that one part shall be a mean proportional between the whole line and the other part.

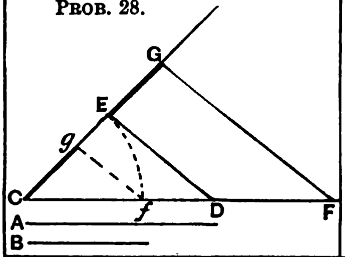
Let A B be the given line. Draw A C perpendicular to A B, and equal to half of it. Join B C. With centre C and radius C A cut off C D. With centre B and radius B D cut off B E. Then the line is divided at E so that A E:E B as E B:A B, or so that the rectangle contained by A E and A B equals the square on E B. (Euc. vi. 30 and ii. 11.)

PROBLEM 31.—To make a proportional copy of any irregular figure, such as a plan, map, or picture.

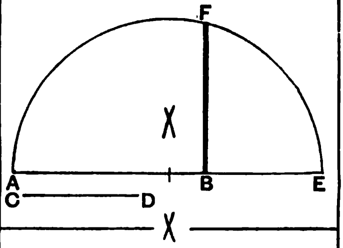
It is required to copy the given map so that the line A B shall be enlarged to the line X Y. Draw A C at right angles to A B. Set off equal spaces on A B and A C, and draw parallels forming a network of squares as shown. We now require the proportionate width of the new drawing—that is, the fourth proportional to the lines X Y, A B, and A C. Set off X Y at any angle to A B. Join A Y. Set off B C equal to the width A C on A B, and draw C Z parallel to A Y. Then B Z is the required width.

The parallel E F cuts off B F equal to the side of a square in the new drawing; or X Y and X Z may be divided into the same number of equal parts as A B and A C are divided into. Draw parallels. With the pencil draw the figure carefully, taking care that each portion of the figure occupies a corresponding square to the original For example, the point A is at the junction of vertical 3 and horizontal 5 on the original drawing, therefore it must occupy a similar position on the copy.

Prob. 28.

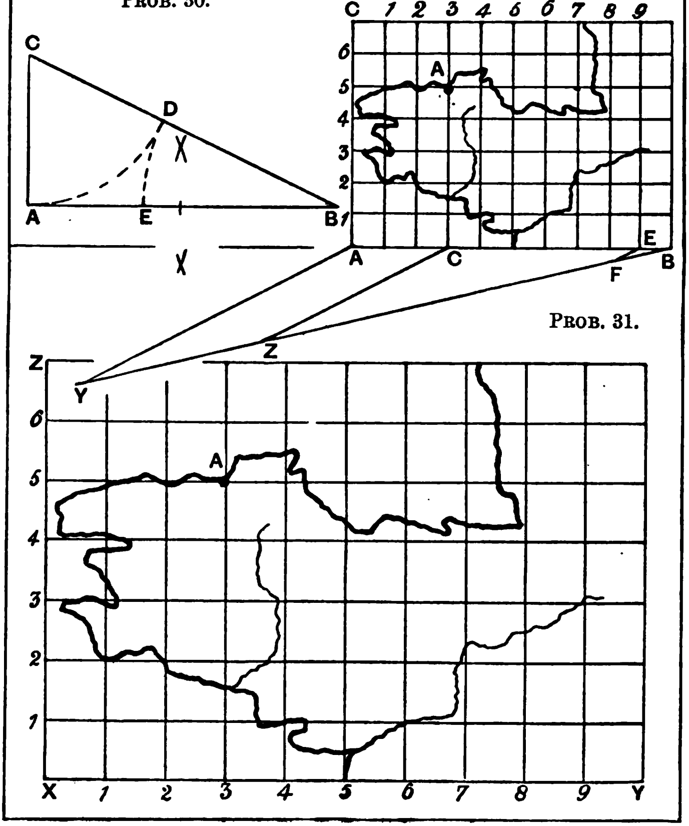


Prob. 29.



Prob. 30.

Prob. 31.



CHAPTER V

TRIANGLES

PROBLEM 32.—To construct an equilateral triangle on a given straight line.

Let A B be the given line. With centres A and B, and the line A B as radius, describe arcs intersecting at C. Join A C and B C. Then A B C is the required triangle. (EUC. I. 1.)

PROBLEM 33.—To construct an equilateral triangle, the altitude being given.

Let A B be the given altitude. At B draw C D at right angles to A B. With centre A and any radius describe an arc. On each side of A B construct an angle of 30° by cutting the arc in points 2 and 3 from centre 1, and bisecting 1 2 and 1 3 by the lines A C and A D. A D C is the required triangle.

PROBLEM 34.—To construct an isosceles triangle, the base and altitude being given.

Let A B be the base, and C the altitude. Bisect the base by the perpendicular, D E. Make D E equal to C. Draw A E, B E, forming the required triangle, A E B.

PROBLEM 35.—To construct an isosceles triangle, the base and one side being given.

Let A B be the given base, and C the given side. With centres A and B, and radius equal to C, describe arcs intersecting at D. Join D with A and B. A B D is the required triangle.

PROBLEM 36.—To construct an isosceles triangle, the base and vertical angle being given.

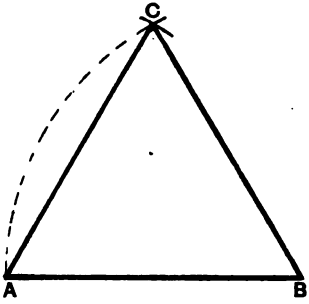
Let A B be the base, and C the given vertical angle. With centre C describe an arc cutting off equal distances C 1 and C 2. Join 1 2, forming an isosceles triangle, C 1 2. At A and B construct angles equal to the angle at 1. (*Prob.* 13.) A B D will be the required triangle.

Note.—A B D and 1 2 C are similar triangles. If the two base angles of the one equal the two base angles of the other, then the vertical angle D must equal the vertical angle C, because the three angles of any triangle must equal 180°. (EUC. I. 32.)

PROBLEM 37.—To construct an isosceles triangle, the altitude and the vertical angle being given.

Let A B be the altitude, and C the vertical angle. Draw D E perpendicular to A B. Bisect the angle C. At A construct angles B A E and B A D, each equal to half the angle C. D E A is the required triangle.

PROB. 32.



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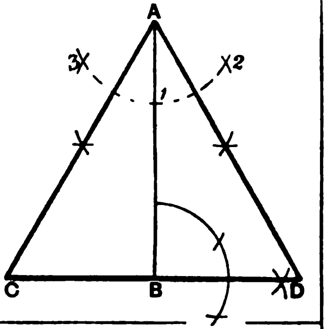

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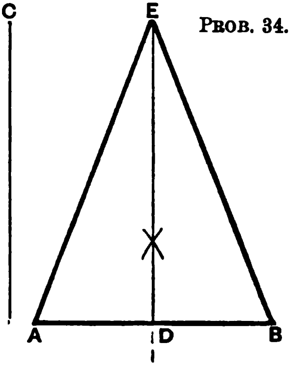

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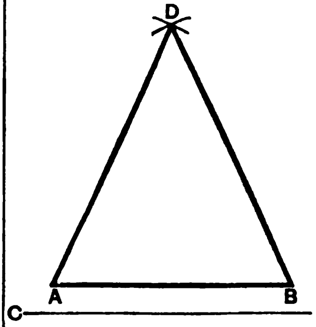

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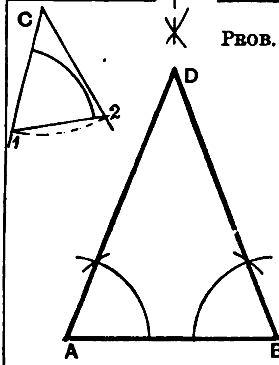

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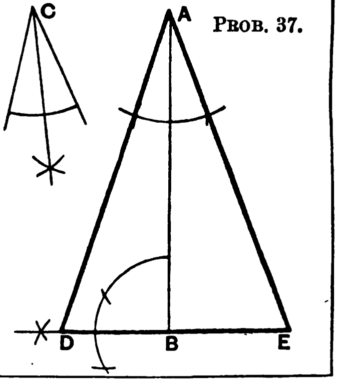


PROBLEM 38.—To construct an isosceles triangle, one of the equal sides and an angle at the base being given.

Let A B be the side, and C the given angle.

Draw any line D E. At D make an angle equal to C, and cut off D F equal to A B. With centre F and radius F D describe an arc cutting D E in G. Join F G. Then F D G is the required triangle.

PROBLEM 39.—To construct a right-angled triangle, the base and hypotenuse being given.

Let A B be the base, and C the hypotenuse.

At A erect a perpendicular. With centre B and radius equal to C cut the perpendicular in D. Join B D.

PROBLEM 40.—To construct a right-angled triangle, the hypotenuse and an acute angle being given.

Let A B be the hypotenuse, and C one of the acute angles.

Bisect A B in D. With centre D describe a semicircle on A B. At A construct an angle, B A E, equal to C. Join B E. Then B A E is the required triangle, the angle in a semicircle being a right angle. (Euc. III. 31.)

PROBLEM 41.—To construct a triangle, the three sides being given.

Let A B, C, and D be the lengths of the three given sides.

With centre A and radius equal to C describe an arc. With centre B and radius equal to D intersect the arc in E. Join E A and E B. Then A B E is the required triangle. (Euc. I. 22.)

PROBLEM 42.—To construct a triangle, the base and the two base angles being given.

Let A B be the base, and C and D the given angles.

At A and B make angles equal to the given angles C and D. Then A B E is the required triangle.

Note.—Take the same radius for all the arcs.

PROBLEM 43.—On a given base to construct a triangle similar to a given triangle.

Let A B be the given base, and C D E the given triangle.

Make the angles at A and B equal to those at C and D, as in *Prob.* 42.

PROBLEM 44.—To construct a triangle, the altitude and two sides being given.

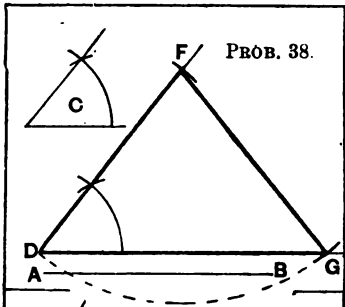
Let A B be the altitude, and C and D the lengths of the sides.

At B draw a base line at right angles to A B. With centre A and radius C cut the base in E. With centre A and radius D cut the base in F. Join A E, A F.

PROBLEM 45.—To construct a triangle, the altitude and base angles being given.

Let A B be the altitude, and C and D the base angles.

At A and B draw lines G H and E F perpendicular to A B. At A make the angle G A E, equal to C, and the angle F A H, equal to D. A E F is the required triangle.



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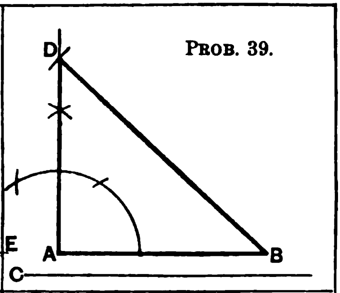
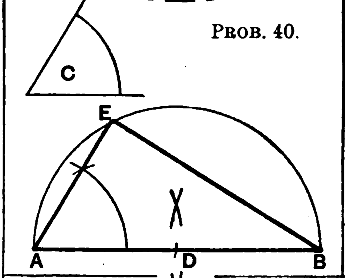


Prob. 39.



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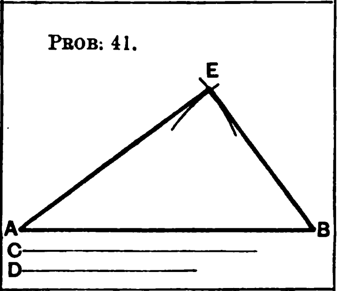

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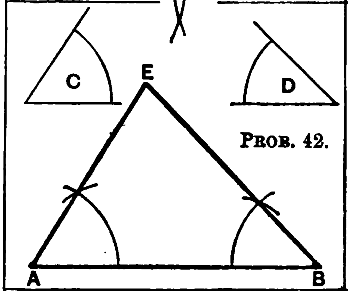

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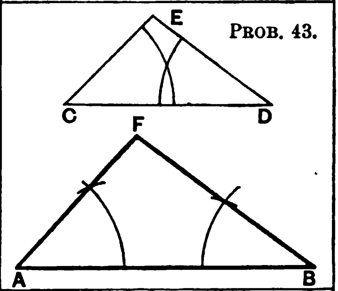

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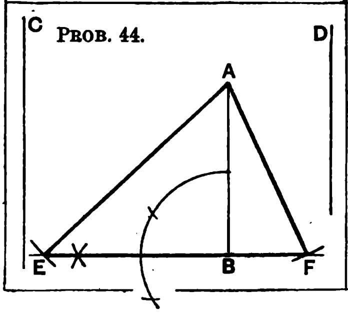
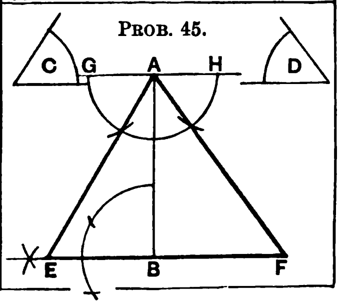


Prob. 44.



Prob. 45.

PROBLEM 46.—To construct a triangle, the base, altitude, and one side being given.

Let A B be the base, C the altitude, D one of the sides.

Draw E F parallel to A B, at a distance equal to C.

With centre A and radius D cut E F in G. A B G is the required triangle.

PROBLEM 47.—To construct a triangle, the base, altitude, and vertical angle being given.

Let A B be the base, C the altitude, and D the vertical angle.

At A construct an angle, B A F, equal to D. Draw A G at right angles to A F. Bisect A B by the perpendicular E G. With centre G and radius G A describe the segment of a circle on A B.

Draw H J parallel to A B, at a distance equal to C. Join H A and H B. Then A H B is the required triangle.

Notes.—1. Any triangle on A B whose vertex lies in the arc A H J B will have its vertical angle equal to D. (EUC. III. 33.) Hence there will be two triangles satisfying the conditions, A H B and A J B.

2. This problem offers another solution to Problem 36. If an isosceles triangle were required on base A B, with its vertical angle equal to D, it would only be necessary to join A and B with K. (See Prob. 47a.)

PROBLEM 48.—To construct a triangle, the base, one side, and the angle opposite the base being given.

Let A B be the given base, C the side, and D the angle.

At A construct an angle, B A F, equal to the angle D.

Draw A G perpendicular to A F. Bisect A B by the perpendicular E G. With centre G and radius G A describe the segment A H B. With centre A and radius equal to C cut the segment in H. A H B is the required triangle.

PROBLEM 49.—To construct a triangle, the base, the sum of the other two sides, and one of the base angles being given.

Let A B be the base, C the sum of the other two sides, and D one of the base angles. At A make an angle equal to D.

Cut off A E equal to C. Join B E, and bisect it by the perpendicular F G. Join G B. Then A G B is the required triangle.

Note.—The triangle B G E is isosceles, therefore G E = G B, and A G, G B = A E.

PROBLEM 50.—To construct a triangle, the base and the ratio of the angles being given.

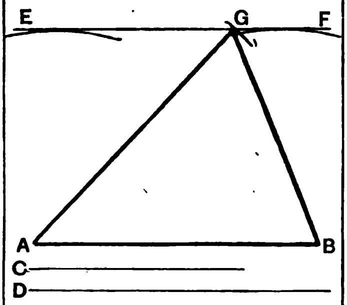
Let A B be the base, and let the angles be as 2 : 3 : 4.

Produce A B, and describe a semicircle. Divide this semicircle in 9 parts (2 + 3 + 4). Draw A 2 and A 5, giving the three angles of the triangle. At B construct an angle, A B C, equal to the angle 9 A 5, and produce A 2 to C, forming the required triangle, A B C.

Notes.—1. B C may be drawn parallel to A 5 to form the angle A B C.

2. The three angles, 9 A 5, 5 A 2, and 2 A B, are equal to two right angles, therefore they must equal the angles of the triangle A B C.

PROB. 46.



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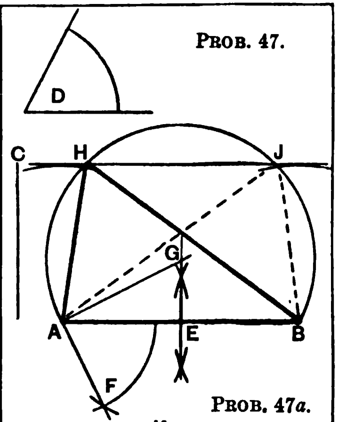

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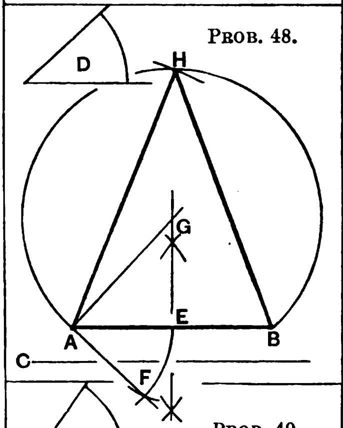

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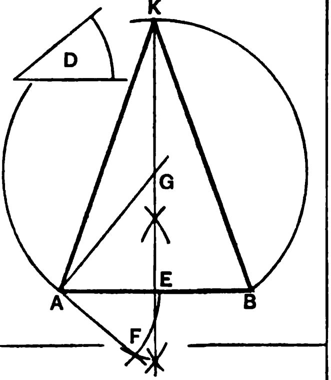

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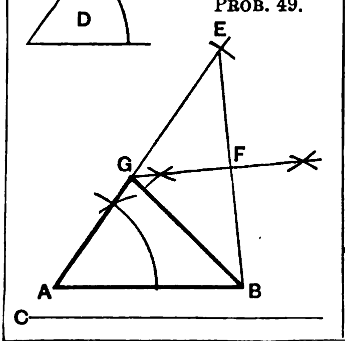

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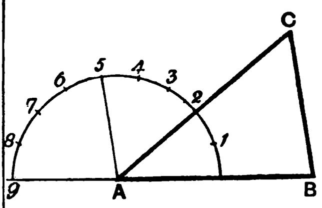


PROBLEM 51.—To construct a triangle, the perimeter and two angles being given. (The *perimeter* of any plane figure equals the sum of its sides.)

Let A B be the perimeter, and C and D the two angles.

On A B construct a triangle with its base angles equal to the angles C and D. Bisect the angles at A and B by lines meeting at F. From F draw F G parallel to A E, and F H parallel to E B, giving F G H, the required triangle.

Note.—A G F and B H F are isosceles triangles, therefore A G = G F, and B H = H F. The angle F G H = the angle E A B, and the angle F H G = the angle E B A. (Euc. I. 29.)

PROBLEM 52.—To construct a triangle, the perimeter and the proportion of the sides being given.

Let A B be the perimeter, and let the sides be as 3 : 4 : 5. Draw a line at any angle to A B, and set off 3 + 4 + 5 equal parts. Join B 12, and divide A B so that A C, C D, and D B shall equal 3, 4, and 5 parts respectively. With centre C and radius C A describe an arc, and with centre D and radius D B intersect the first arc in E. Join D E and C E, forming the required triangle, C D E.

PROBLEM 53.—To construct a triangle, the base, the ratio of the other two sides, and the angle opposite to the base being given.

Let A B be the given base, C the angle opposite to the base, and let the two sides be as 5 : 6.

Draw any line, D E. At D make an angle equal to C. On D E set off five, and on D F, six, equal parts. Join 5 and 6. On line 5 6 cut off 6 G equal to A B. Draw G H parallel to D 5. Then G H 6 is the required triangle, and is similar to the triangle 6 5 D.

Note.—If A B be longer than 5 6, then both 5 6 and 6 D must be produced as in **Problem 53a.**

CHAPTER VI

QUADRILATERALS

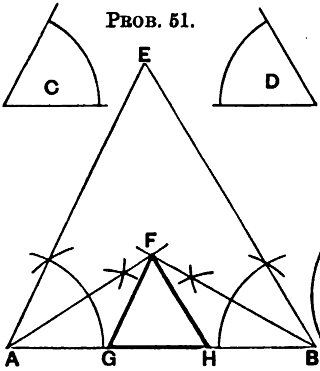
PROBLEM 54.—To construct a square, the side being given.

Let A B be the given side. At A erect a perpendicular, A C, and make it equal to A B. With centres C and B and radius A B describe arcs intersecting at D. Draw C D, B D.

PROBLEM 55.—To construct a square, the diagonal being given.

Let A B be the diagonal. Bisect A B by the perpendicular C D. With centre F and radius F A describe arcs cutting the perpendicular in C and D. Join A D, D B, B C, and C A.

PROB. 51.



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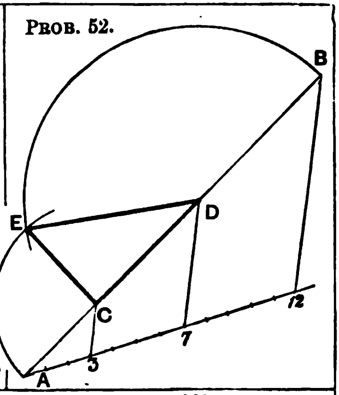

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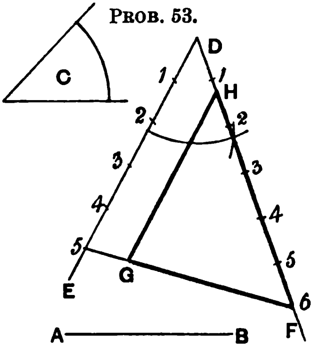

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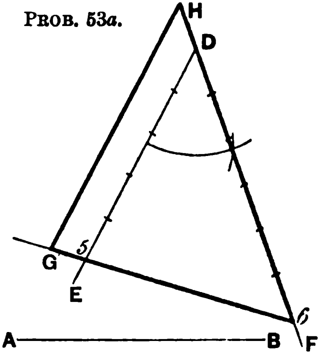

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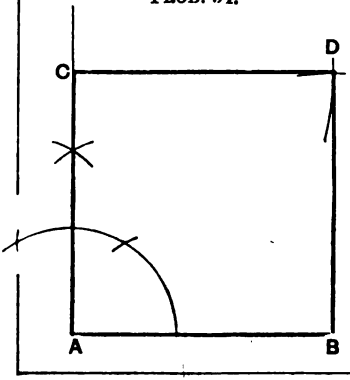

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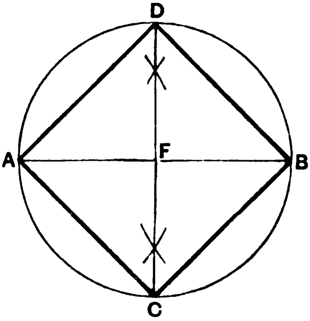


PROBLEM 56.—To construct a rectangle, the two sides being given.

Let A B and C D be the two sides. At A erect a perpendicular, A E, and make it equal to C D. With centre E and radius A B describe an arc, and with centre B and radius C D intersect the arc in F. Draw E F, F B.

PROBLEM 57.—To construct a rectangle, the diagonal and one side being given.

Let A B be the diagonal, and C D the side. Bisect A B in F.

With centre F and radius F B describe a circle. With radius C D and centres A and B cut the circle in G and H. Draw A G, G B, B H, H A.

Note.—The angle in a semicircle is a right angle. (Euc. III. 31.)

PROBLEM 58.—To construct a rhombus, the side and one of the angles being given.

Let A B be the given side, and C the given angle.

At A construct an angle equal to C, and make A D equal to A B.

With centres D and B and radius A B describe arcs intersecting at E. Draw D E, B E.

PROBLEM 59.—To construct a rhombus, the diagonal and side being given.

Let A B be the diagonal, and C the side. With radius C and centres A and B describe arcs intersecting at D and E. Draw A D, D B, B E, E A.

PROBLEM 60.—To construct a rhomboid, the two sides and one of the angles being given.

Let A B and C be the two sides, and D the given angle.

At A construct an angle equal to D, and make A E equal to C.

With centre E and radius A B describe an arc, and with centre B and radius C intersect the arc at F. Draw E F, B F.

PROBLEM 61.—To construct a rhomboid, the diagonal and the two sides being given.

Let A B be the diagonal, and C and D the two sides.

With radius C and centres A and B describe two arcs.

With radius D and the same centres intersect the arcs in E and F. Draw A E, E B, B F, F A.

PROBLEM 62.—To construct a trapezium, the diagonal and two pairs of equal sides being given.

Let A B be the diagonal, and C and D the sides.

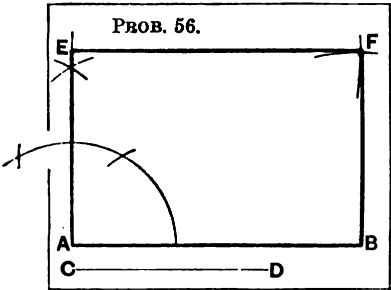
With centre B and radius C describe an arc. With centre A and radius D intersect this arc in E and F. Draw A F, F B, B E, and E A.

PROBLEM 63.—To construct a trapezium equal to another given trapezium.

Let A B C D be the given trapezium. Draw *a b* equal to A B.

With centres *a* and *b*, and radii equal to A C and B C, describe arcs intersecting in *c*. With centres *b* and *c*, and radii equal to C D and B D, describe arcs intersecting in *d*. Draw *a c*, *c d*, and *b d*.

Note.—The same principle of cutting the figure into triangles may be applied to figures with any number of sides.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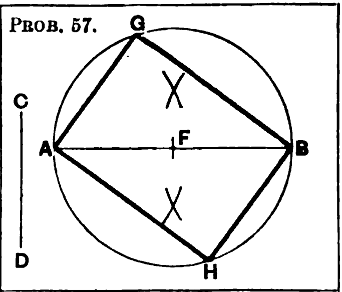
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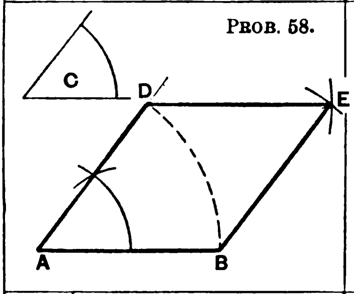
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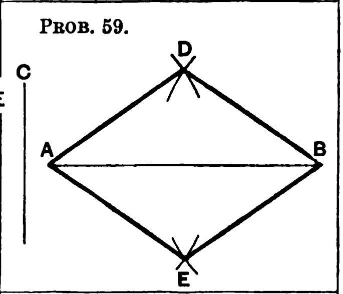
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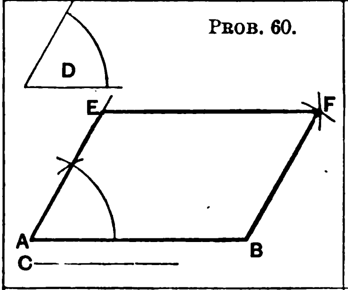
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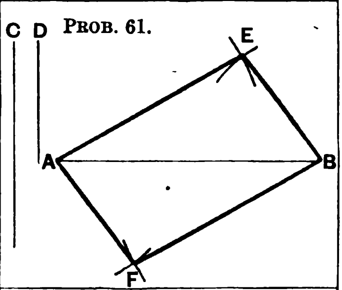
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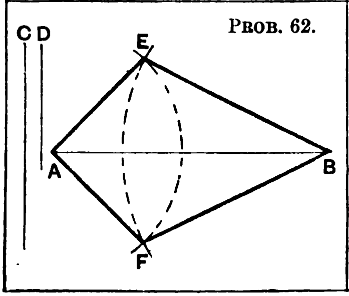
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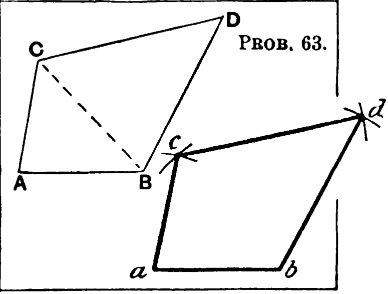

PROB. 63.

EXERCISES

Note.—The exercises taken from the examination papers of the Science and Art Department in Subject I. are marked with the letters *Sc.*, and the year when set. Those from the various Art examinations are marked *Art.* They should be worked in connection with the chapter to which they relate.

CHAPTER III

1. Draw a line 3" long; mark a point 1⅞" below it, and from this point draw a perpendicular to the line.

2. Draw a line, A B, 2·5" long. From a point C in it, ·5" from A, draw a line making 45° with C B; and at B draw a line at 75° with B C to meet it.

3. Draw a line 3·5" long. Divide it into 7 equal parts, and on $\frac{2}{7}$ of the line describe a semicircle.

4. Draw a line 8" long, and erect a perpendicular from a point ½" from one end, and without the line, without producing it.

5. From a point A draw two lines, A B, A C, making with each other an angle of 75°; bisect this angle.

6. Set off with your protractor an angle of 57°, and divide it into 4 equal parts.

7. At the extremities of a line 3" long erect perpendiculars 2" and 2½" long. From the upper ends of these perpendiculars draw two equal lines meeting each other in the first line. (*Prob.* 17.)

8. From A and B, two points 2" apart, draw two right lines to meet at an angle of 70°. (*Art.*)

9. Through a point C, 1½" from a line A B, draw two right lines, one parallel and the other perpendicular to A B. (*Art.*)

10. Draw two lines intersecting at an angle of 52° and between them place a line 2·25" long, making 58° with one of them. (*Sc.* 1884.)

Note.—Draw any line 2·25" long, and making 58° with one line, and through its extremity draw a parallel to the one line until it meets the other. A line from the point of intersection will be the line required.

11. Draw a line 8·5" long, and at one extremity erect a perpendicular 1·75" long. From the top of this perpendicular draw a line to make an angle of 30° with the given line. (*Sc.* 1871.) (*Prob.* 14.)

12. Draw any two parallel lines, A and B, 1½" apart. Mark a point P, ¾" above A. Through P draw a line cutting the given lines in points 1⅞" apart. (*Sc.* 1888.) (*Prob.* 21.)

13. A B C is an obtuse angle, and X is a point within it. Determine X's position when A B = 2·83", B C = 3·15", A B C = 160°, A B X = 65°, B C X = 87°. State the lengths of A X and B X. (*Sc.* 1872.)

CHAPTER IV

1. Find a fourth proportional to three lines whose lengths are 2¼", 3⅜", $1\frac{3}{10}$" respectively.

2. Find a mean proportional between two lines, 2" and 1" long, and figure its length on the line.

3. Draw a line 8·75" long, and divide it in the proportion of the numbers 2, 7, 3, 6. Figure the parts.

4. Find a third proportional to two lines, 1½" and 1¾" long.

5. Find a line which shall have the same ratio to a line 1¾" long that 2½" has to 1½".

Note.—2½" : 1½" :: 1¾" : required line.

6. Draw a line 2" long, and divide it into 4 unequal parts. Draw another line two-thirds of the length of the first line, and divide it proportionally to it. (*Prob.* 25.)

7. From the extremity A, of a line A B, obtain the ½, ⅓, and ⅕ of the line.

8. Divide a line 3·25" long into 4 parts, A, B, C, D, so that B is double of A, C three times A, and D four times A.

9. Produce a line, **A B**, 3″ long, to a point **P**, so that **B P : A B** as 3 : 5. (*Sc.*, 1872.)

Note.—Draw a line at any angle, and set off 5 + 3 parts. Join 5 B, and from 3 draw 3 P parallel to 5 B.

10. Find a point, P, in a line **A B** (2″ long), produced, so that **A P : A B** as 7 : 4. (*Sc.*, 1870.)

Note.—Draw a line at any angle to **A B**, and set off 7 parts. Join 4 B. From 7 draw 7 P parallel to 4 B.

11. Divide a line, **A B**, 3·5″ long, in a point **P**, so that the rectangle contained by **A B**, **A P** may be equal to the square on **B P**. (*Sc.*, 1870.) (*Prob.* 30.)

12. Find a mean proportional to two lines, 2·5″ and 1·5″ long respectively. State any Problem that you know of which this is the solution. (*Sc.*, 1871.) (Euc. II. 14.)

CHAPTER V

1. Construct a triangle with sides **AB** = 3″, **A C** = $2\frac{3}{8}$″, **B C** = $1\frac{3}{10}$″, and construct an angle equal to **B A C**.

2. Two lines, A and **B**, 4″ long, contain an angle of 45°. From a point C in one line draw a line to make equal angles with the two converging lines.

3. Upon a base of $2\frac{1}{2}$″ construct a triangle having two of its angles 75° and 45° respectively, and then construct a similar triangle on a base of 2″. (*Art.*)

4. An isosceles triangle has a base of $1\frac{1}{2}$″ and a vertical angle of 42°. Construct it by using *Problem* 47*a*.

5. Describe a segment of a circle which shall contain an angle equal to the angle of an equilateral triangle. (*Prob.* 47.)

6. Construct an isosceles triangle with its equal sides $2\frac{1}{2}$″ long and the included angle 30°. On the same base describe another isosceles triangle with its vertical angle double that of the first triangle. (*Art.*)

Note.—Bisect the sides, and find the centre for the circumscribing circle. The angle at the centre of a circle is double the angle at the circumference, standing upon the same base. (Euc. III. 20.)

7. Draw a triangle, two of whose sides are 2·5″ and 3″ respectively, the angle opposite the shorter side being 40°. (*Sc.*, 1876.) (*Prob.* 38.)

8. Draw a triangle having its vertical angle 30°, the base 1·7″, and the sides as 4 : 5. (*Prob.* 53.)

9. Construct a triangle of which the sides are as 1, 1·5, 2, the perimeter being 4″. (*Prob.* 52.)

10. Construct a triangle on a base of 2″, altitude 1·75″, and the angle opposite the base 42°. (*Sc.*, 1871.) (*Prob.* 47.)

11. Construct a triangle whose sides, *a b*, *b c*, *c a*, are $3\frac{1}{4}$″, $2\frac{3}{8}$″, and 2″ respectively. On *a c* construct a second triangle, *a d c*, whose vertical angle, *a d c*, is equal to the angle *a b c*, and the side *a d* $1\frac{5}{8}$″. (N.B.—The angles upon the same base and in the same segment of a circle are equal.) (*Sc.*, 1887.)

Note.—At *a* draw a line making with *a c* an angle equal to *a b c*, and use *Problem* 48.

CHAPTER VI

1. Construct a square of 4″ sides, bisect the sides, and join the adjacent points of bisection, thus obtaining a second square; bisect the sides of this square, and obtain a third square. Continue the process until five squares have been drawn. (*Sc.*, 1879.)

2. Construct a rhomboid, one diagonal being 2″, and the adjacent sides $1\frac{1}{2}$″ and 1″ respectively. (*Prob.* 61.)

3. Construct a rhombus having an angle of 65° and a base of 3″. Measure its two diagonals accurately, and write down their lengths.

4. The adjacent sides of a trapezium are 2·3″ and 1·8″ long respectively. The included angle is 60°. The other sides are $2\frac{3}{4}$″ and 3″. Construct the figure, and give the length of the longest diagonal.

5. Upon a line $1\frac{7}{8}$″ long describe a square, and divide it by parallel lines, alternately thin and dotted, into 5 equal rectangles.

6. The diagonals of a parallelogram 2·4″ and 4·2″ long contain an angle of 61°. Construct the parallelogram. (*Sc.*, 1883.)

CHAPTER VII

THE CIRCLE AND TANGENTS

PROBLEM 64.—To find the centre of a circle.

Draw any two chords, A B, B C. Bisect them by lines at right angles. The point D, where the bisecting lines intersect, is the centre. (Euc. III. 1, Cor.)

PROBLEM 65.—To describe a circle passing through three given points not in the same straight line.

Let A, B, and C be the three points.

Join A B, B C. Bisect both lines as above. From D, the point where the bisecting lines meet, describe the circle.

PROBLEM 66.—To describe a circle about a triangle.

Bisect two sides, and proceed as in Problem 64.

PROBLEM 67.—To describe an arc equal to a given arc, and having the same radius.

Draw any two chords, A B and C D, in the given arc. Bisect them, and find the centre E, from which the arc was described. With centre O and radius E F describe the arc G H, and make it equal in length to the given arc.

PROBLEM 68.—To draw a tangent to a circle through a given point in the circumference.

Let A be the given point. Find the centre, B. Join B A and produce, making A C equal to A B. With centres B and C describe arcs intersecting at D. Draw D A, the required tangent.

Notes.—1. This method of drawing the perpendicular is preferable to the other methods when there is room to produce the radius, as it is likely to be more accurate.

2. A tangent is always at right angles to the radius.

PROBLEM 69.—To draw a tangent to a circle from a given point without it.

Let A be the given point. Find the centre, B, and join B A. Bisect B A, and describe a semicircle cutting the given circle in C. Draw A C, the required tangent.

Note.—If the whole circle on A B were described, another tangent might be obtained.

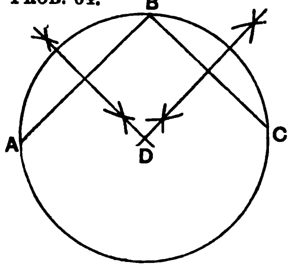
PROBLEM 70.—To draw a tangent to an arc from a point in it, when the centre is not accessible.

Let A be the given point.

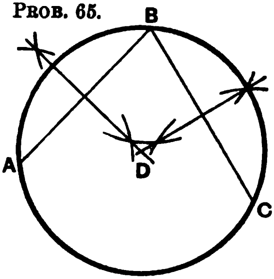
With centre A describe a circle. With centres B and C describe arcs intersecting in E and F. Draw E F. At A draw the tangent at right angles to E F, using H and G as centres for the intersecting arcs.

Another solution is shown in **Problem** 70*a*. Draw a chord, A B, from the given point A. Bisect this chord. Join C A. Make the angle C A D equal to C A B. Then D E is the required tangent, the angle D A C being equal to the angle A B C. (Euc. III. 32.)

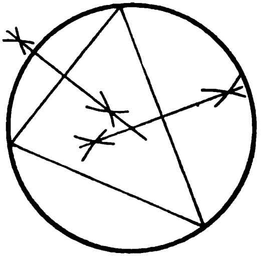
PROB. 64.



PROB. 65.



PROB. 66.



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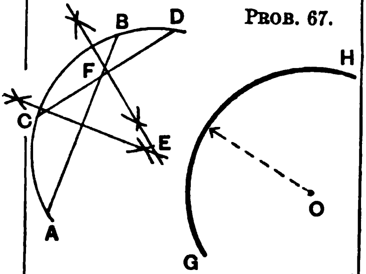

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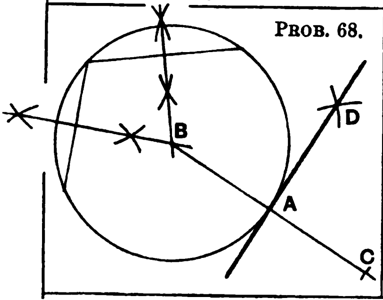

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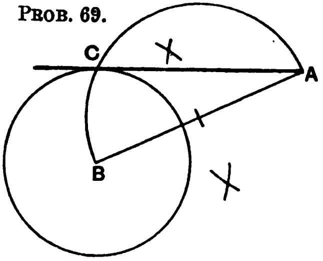
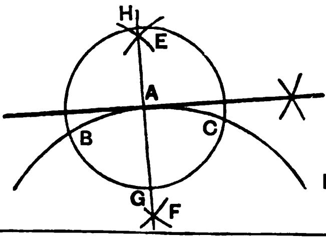
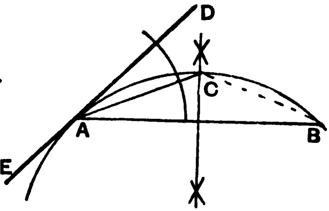


PROB. 70.



PROB. 70*a*.



PROBLEM 71.—To draw a tangent to a circle which shall be parallel to a given straight line.

Let A B be the given straight line. Find the centre, C, of the circle. From C draw a perpendicular to A B. Through D, the point where the perpendicular cuts the circumference, draw E F at right angles to C D.

Note.—In all problems where the centre of the circle is not given, it must be found as shown in Problem 64.

PROBLEM 72.—To draw two tangents to a circle to meet at a given angle.

Let the given angle in this case equal 60°.

From the centre, B, draw any line, B C. Take any point, C, and on each side of the line B C construct an angle equal to *half* the given angle—in this case 30°. Now apply Problem 71, and draw perpendiculars B E, B D. Through F and G draw parallels to E C and D C.

PROBLEM 73.—To draw a common tangent to two equal circles which shall not cross the line joining their centres. (This is commonly called an **exterior tangent.**)

Join the centres A and B. At A and B draw perpendiculars A C, B D. Through the points C and D draw the exterior tangent.

PROBLEM 74.—To draw a common tangent to two equal circles which shall cross the line joining their centres. (Interior tangent.)

Join the centres A and B. Bisect A B in C, and A C in D. With centre D and radius D A describe a semicircle. Join A E, and from B draw B F parallel to A E. (Use set square.) Through E and F draw the interior tangent.

PROBLEM 75.—To draw an exterior tangent to two unequal circles.

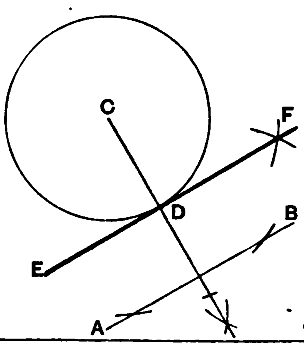
Join the centres A and B. From C set off C D equal to B E, the radius of the smaller circle. With centre A and radius A D (the *difference* of the radii of the given circles) describe a circle. Bisect A B, and describe a semicircle. From B draw B F, a tangent to the small, described circle. Through F draw A G, and from B draw B H parallel to A G. Through the points G and H draw the exterior tangent.

PROBLEM 76.—To draw an interior tangent to two unequal circles.

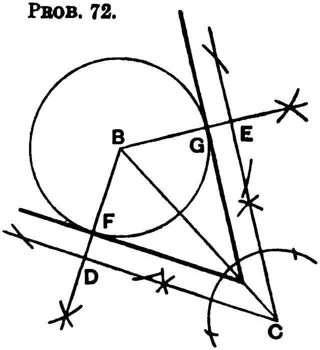
Join the centres A and B. From C set off C D equal to B E. With centre A and radius A D (the *sum* of the radii of the given circles) describe a circle. Bisect A B, and describe a semicircle. From B draw B F, a tangent to the large, described circle. Join A F, and from B draw B G parallel to A F. Through the points G and H draw the interior tangent.

Note.—In Problems 73, 74, 75, 76, two tangents might be drawn, if required, as shown in dotted line on Problem 74.

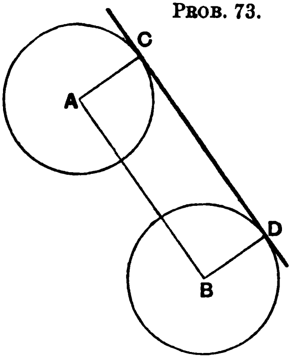
PROB. 71.



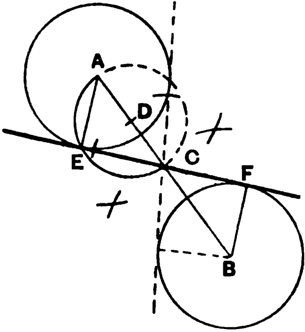
PROB. 72.



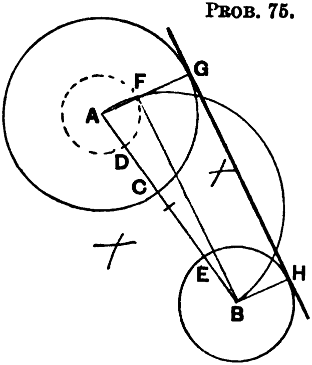
PROB. 73.



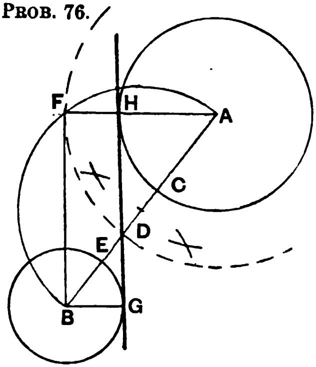
PROB. 74.



PROB. 75.



PROB. 76.



CHAPTER VIII

REGULAR POLYGONS

PROBLEM 77.—To construct ANY regular polygon on a given line, A B (in this case a pentagon).

Produce A B, and with centre A and radius A B describe a semicircle. Divide the semicircle into as many equal parts as the required figure has sides (in this case five) with the dividers. Join A with 2, giving another side of the polygon. (Always join A with the second division for any polygon.) Bisect the two sides, A B and A 2, by lines meeting at O, giving the centre of the polygon. With centre O and radius O B describe a circle. Mark off B E and E D equal to A B. Join B E, E D, D 2, forming the required pentagon.

Notes.—1. The accuracy of this construction depends upon the correct division of the semicircle and the care with which the centre is obtained.

2. A semicircle may be divided into three equal parts by marking off its radius three times. To get 6 equal parts, bisect each third part; to get 9, trisect each third part. To get 4 equal parts, bisect each half; by bisecting again, 8 will be obtained. If the set squares are perfectly accurate, the semicircle may be divided into 4 by using the 45°, and into 8 by using the 60°, set square.

PROBLEM 78.—To construct a regular hexagon on a given line, A B. (Special method.)

With centres A and B and radius A B describe arcs intersecting in O. With centre O and the same radius describe a circle, and set off A B round it. Join the points, thus forming the required hexagon.

Note.—The side B C may be obtained by using the 60° set square, and the hexagon quickly set up by this means.

PROBLEM 79.—To construct a regular octagon on a given line, A B. (Special method.)

Erect perpendiculars at A and B. Produce A B both ways. With centres A and B and radius A B describe quadrants. Bisect each quadrant, obtaining B E and A F, two more sides of the octagon. From E and F draw parallels to A C and B D, and make them equal to A B. Join E F and G H. Make K D and L C equal to J B. Join the points H, D, C, G.

Notes.—1. The lines from A and B through M and N will also give the points H and G.

2. B E, A F, H D, G C may be readily obtained by using the 45° set square.

PROBLEM 80.—To construct a regular polygon by using the protractor.

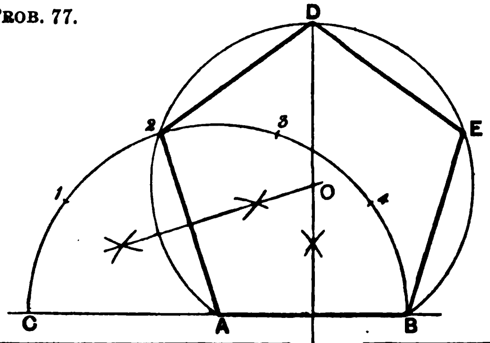
The number of degrees in each angle of a regular polygon may be readily found as follows:—*From twice as many right angles as the figure has sides subtract four right angles, and divide this result by the number of angles the figure contains.* (Euc. I. 32, Cor.)

Suppose the regular polygon to be a pentagon.

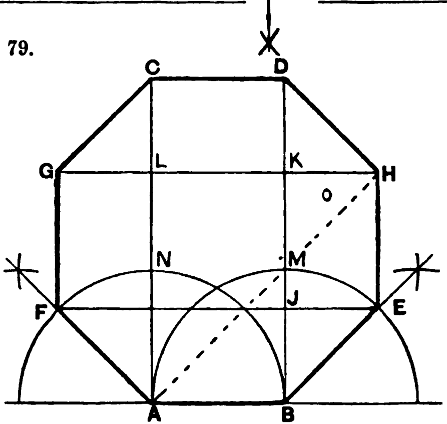
The pentagon has *five* sides; therefore take *ten* right angles. Deduct *four*, leaving *six* right angles. The solution will then be $\frac{6 \times 90^\circ}{5} = 108^\circ$. The angles for other polygons may be found in the same manner. For an octagon the solution would be $\frac{12 \times 90^\circ}{8} = 135^\circ$.

Place the protractor on A B, with its centre on B (see page 2). Mark 108°, reading from the left. Place the centre on A, and mark 108°. Make A D and B C equal to A B. With centres D and C and radius A B describe arcs intersecting in E. Join D E, E C

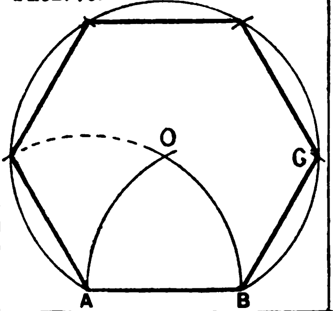
Prob. 77.



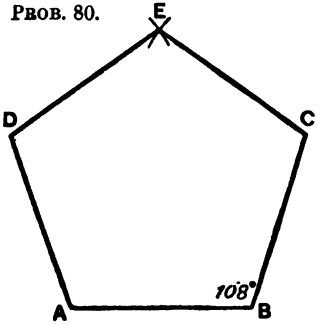
Prob. 79.



Prob. 78.



Prob. 80.



PROBLEM 81.—To inscribe ANY regular polygon in a circle.

Draw the diameter A B. (If the centre of the circle be not given, draw a chord, and bisect it.) Divide A B into 7 equal parts. With centres A and B describe arcs intersecting at C. From C, through the second part, rule C D, cutting off A D, one of the sides of the polygon. Set off A D round the circle, and join the points.

Notes.—1. The greatest care must be exercised in dividing the line and in drawing the line from C exactly through the point 2.

2. The circumference of the circle may be divided into the number of parts required with the dividers, and the points joined to form the figure.

3. The hexagon, octagon, and duodecagon are more easily inscribed in a circle by special methods.

PROBLEM 82.—To inscribe a regular hexagon in a circle. (Special method.)

Draw the diameter A B. With centres A and B set off the radius on each side. Join the points.

PROBLEM 83.—To inscribe a regular octagon in a circle. (Special method.)

Draw the diameter A B. Bisect it at right angles by the diameter C D. Bisect each quadrant thus formed, cutting the circumference into 8 equal portions. Join the points thus obtained.

PROBLEM 84.—To inscribe a regular duodecagon in a circle. (Special method.)

Draw two diameters, at right angles to each other, as before.

With centres A, B, C, D, describe arcs passing through the centre of the circle and cutting the circumference. Join the points thus obtained, forming the required polygon.

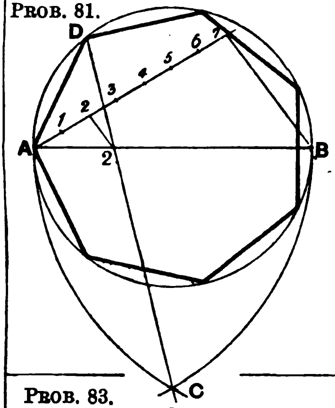
PROBLEM 85.—To complete a regular polygon, two adjacent sides and the included angle being given.

Let A B, B C, be the two sides, and A B C the included angle (in this case 120°). Bisect the two sides. Describe a circle passing through the points A, B, C. Set off the length of one side round the circumference, and join the points thus obtained.

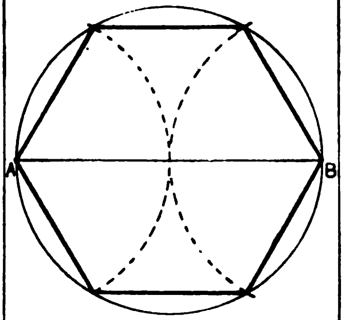
PROBLEM 86.—To inscribe ANY regular polygon in a circle by using the protractor.

If all the angles of a polygon be joined with the centre, there will be formed as many equal isosceles triangles as the polygon has sides, having their vertical angles at the centre equal. All the angles at the centre equal four right angles. (Euc. I. 15, Cor. 2.) Hence, if 360° be divided by the number of sides the regular polygon has, the magnitudes of the central angles will be obtained. For example, it is required to inscribe a regular pentagon in a circle by this method. The central angle in this case will be $\frac{360^\circ}{5} = 72^\circ$. Draw any radius, O A. Make the angle A O B equal to 72°, with the protractor. Set off the distance A B round the circumference, and join the points. A B C D E is the required pentagon.

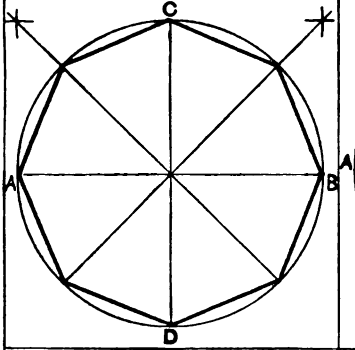
PROB. 81.



PROB. 82.



PROB. 83.



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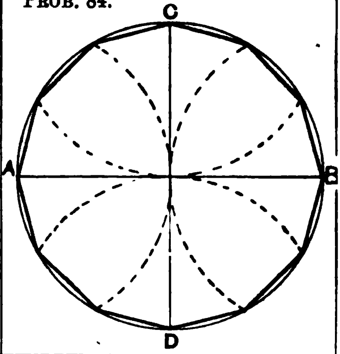
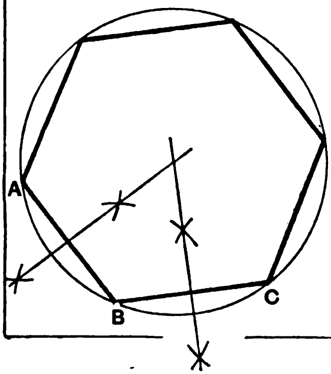


PROB. 85.



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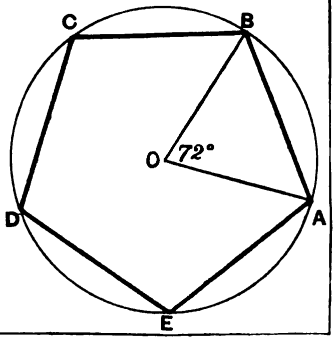


PROBLEM 87.—To describe ANY regular polygon about a circle (say a pentagon).

Divide the circumference of the given circle into as many equal parts as the polygon has sides, in the same manner as for the inscribed polygon. (*Prob.* 81.) From the centre, O, draw lines through each point. Draw A B, one of the sides of the inscribed pentagon. Draw the tangent C D parallel to A B. Make O E, O F, O G, equal to O D. Draw D E, E F, F G, G C, tangential to the circle. Then C D E F G is the required pentagon.

PROBLEM 88.—To construct ANY regular polygon, the length of any diagonal being given.

Let A B be the length of one of the *longer* diagonals of a regular heptagon.

On any base, C *d*, construct a regular heptagon. (*Prob.* 77.) From C draw the diagonals, and produce them, if necessary. Mark off C F equal to A B on one of the longer diagonals. Draw F E, E D, F G, G H, H J, parallel to the corresponding sides of the heptagon.

Notes.—1. If the length of the shorter diagonals be given, proceed in an exactly similar manner.

2. If one of the longer diagonals of a hexagon, octagon, or duodecagon be given, bisect it, describe the circumscribing circle, and proceed as in Problems 82, 83, and 84.

PROBLEM 89.—To construct ANY regular polygon, having the diameter given (say a pentagon).

Note.—The **diameter** divides the polygon into two equal parts. It is the line passing through the centre of the polygon, and terminated at the middle points of two opposite sides in a polygon having an *even* number of sides. When the sides are *uneven*, the diameter is drawn through the centre, from one angle to the middle of the opposite side.

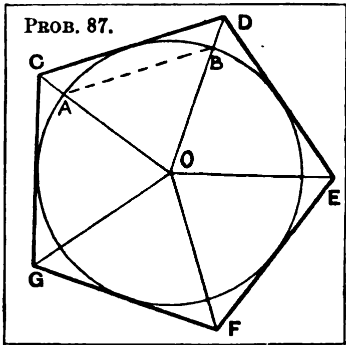
Let A B be the given diameter. Through B draw a line at right angles to A B. Set off B *c*, B *d*, any equal distances on each side of B. Describe a pentagon on *c d*. Through *e* and *f* draw lines from B. From A draw A E and A F parallel to *a e* and *a f*, and from E and F draw E D and F C parallel to *e d* and *f c*. Then C D E A F is the required pentagon.

PROBLEM 90.—To construct a regular pentagon, the diagonal being given. (Special method.)

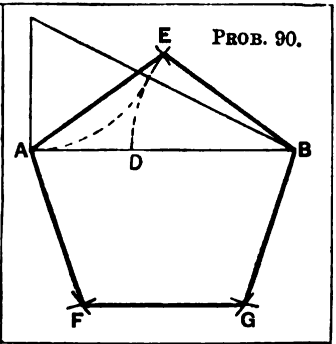
Let A B be the diagonal. An important property of the pentagon is, that if the diagonal be divided into an extreme and mean proportion (*Prob.* 80), *the greater segment* of the diagonal so divided will equal the *side* of the pentagon.

Divide A B into an extreme and mean ratio at D. Then B D, the greater segment, will be the side of the pentagon. With centres A and B and radius B D describe arcs intersecting at E. Draw B E, E A, two sides of the pentagon. With centre B and radius B A describe an arc. With centre A and radius A E intersect the arc in F. With centres F and B and radius F A describe arcs intersecting in G. Draw A F, F G, G B.

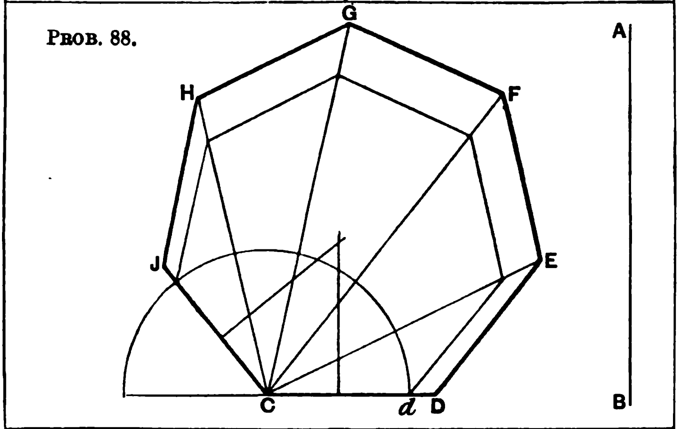
Prob. 87.



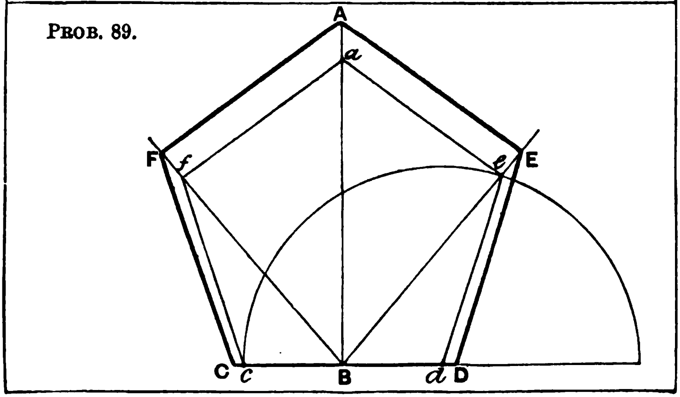
Prob. 90.



Prob. 88.



Prob. 89.



CHAPTER IX

SCALES

Drawings are not usually made of the same size as the objects which they represent. It would be manifestly inconvenient to draw the plan of a building its proper size, as the drawing would be too large for use; but if a drawing were made on which every yard of the object was represented by an inch, then we should have a diagram showing the relative proportion of the parts, but drawn to a different *scale*. As every inch on the drawing represents a true length of one yard, the drawing would be on a *scale of 1 inch to 1 yard, or* $\frac{1}{36}$, because each line on the drawing is $\frac{1}{36}$ part of its true length. This fraction is called the **representative fraction**, and shows the ratio each line on the drawing bears to the object delineated.

The scale of a drawing may be stated in *words*, as 1 *in. to* 1 *yd.*; by its *representative fraction*, as $\frac{1}{36}$; or by *drawing a line divided into equal parts*, each representing the unit used.

When the scale shows equal divisions only, it is called a **plain scale**.

Scales, to be of use, should fulfil the following conditions:—1. Divided with great *accuracy*, and carefully numbered. 2. Long enough to measure the principal lines of the drawing. 3. The zero (o) must always be between the unit and its subdivisions. 4. The name of the scale should be written on it, and the representative fraction shown. 5. Numbered decimally in the primary divisions.

PLAIN SCALES

PROBLEM 91.—To construct a scale of $\frac{1}{2}$ in. to 1 ft., or $\frac{1}{24}$, to measure 6 ft.

Rule two parallel lines about $\frac{1}{12}$th of an inch apart. Set off 6 half inches. Divide the first part into 4, showing spaces of 3 in. Figure the scale as shown, placing the *zero* between the feet and the subdivision into inches.

PROBLEM 92.—To construct a scale of $\frac{3}{4}$ in. to 1 mile, showing miles and furlongs, and measuring 4 miles.

Draw two lines as before. Set off 4 spaces of $\frac{3}{4}$ in. each. Divide the first part into 8, showing the furlongs, and number as shown.

Note.—The representative fraction in this scale is $\frac{\frac{3}{4}\text{ in.}}{1\text{ m.}}=\frac{\frac{3}{4}\text{ in.}}{63360\text{ in.}}=\frac{1}{84480}$. Every line of the object drawn to this scale would be 84480 times as long as that of the drawing.

PROBLEM 93.—Draw a scale of $\frac{1}{60}$, to show yards and feet, and measuring 5 yds.

The scale has to be long enough to measure lines 5 yds. long; therefore, its total length will be $\frac{5}{60}$ or $\frac{1}{12}$ of 1 yard, that is, 3 in. Draw two lines, 3 in. long. Divide, and number as shown.

PROBLEM 94.—Construct a scale of ·875 in. to 10 ft., to measure 40 ft.

If 10 ft. are represented by ·875 in., then 40 ft. would be represented by ·875 × 4 = 3·5 in. Take a line 3½ in. long, divide it into 4 equal parts, giving distances of 10 ft. Divide the first part into 10 equal parts, showing feet.

Representative fraction $=\frac{\cdot875\text{ in.}}{10\text{ ft.}}=\frac{\cdot875\text{ in.}}{120\text{ in.}}=\frac{\frac{7}{8}\text{ in.}}{120\text{ in.}}=\frac{7}{960}$.

PROBLEM 95.—Draw a scale showing $5\frac{1}{2}$ yds. to 1 in., to measure 20 yds.

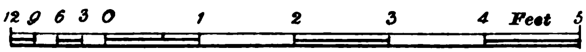
If 5½ yds. be represented by 1 in., then 2 in. will represent 11 yds. Draw a line, and set off A B, 2 in.

Divide this distance into 11 equal parts. Each of these parts will represent 1 yard. Add 9 more parts, to make up the 20, and complete the scale as shown.

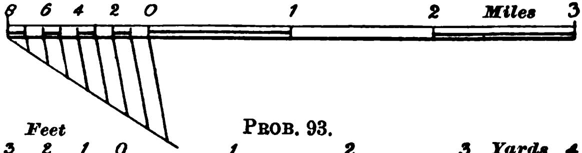
PROBLEM 96.—One inch represents 13 ft. 4 in. Draw the corresponding scale, divided generally into 10-ft. lengths, with one such length subdivided into single feet. (*Sc.*, 1885.)

The representative fraction $=\frac{1\text{ in.}}{13\text{ ft. }4\text{ in.}}=\frac{1}{160}$. The *unit* is a distance of 10 ft. If 160 in. be represented by 1 in., then 10 ft., or 120 in., will be represented by $\frac{120}{160}=\frac{3}{4}$in. Set off distances of $\frac{3}{4}$ in., representing 10 ft. Subdivide the left-hand division into 10 equal parts, showing single feet.

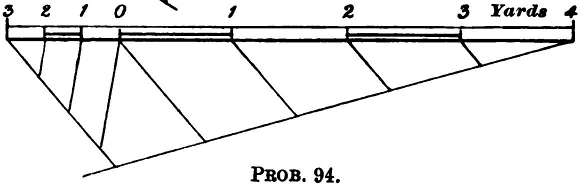
PROB. 91.



PROB. 92.



PROB. 93.



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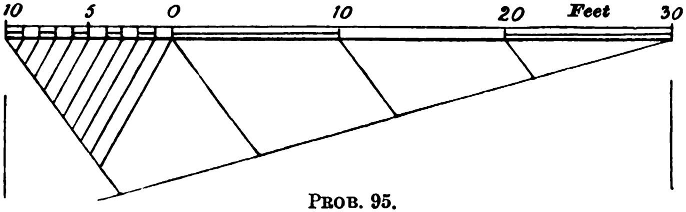
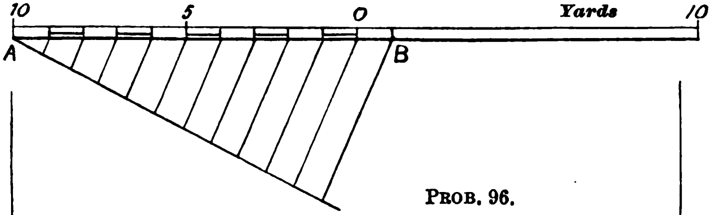
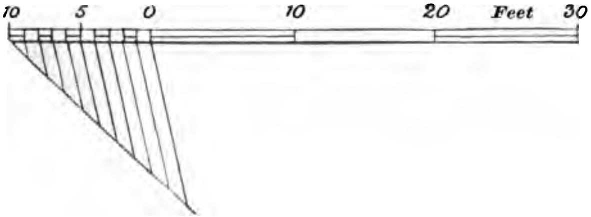


PROB. 95.



PROB. 96.



PROBLEM 97.—The line A B represents $3\frac{1}{2}$ yds. Construct a scale, showing yards and feet, to measure 10 yards.

Divide A B into 7 equal parts, showing $\frac{1}{2}$-yards. Produce the line to the required length. Subdivide the left-hand division into 3 equal parts, showing feet.

PROBLEM 98.—The given line, A B, is 2 ft. 5 in. long by scale. Produce it so as to make it 6 ft.

Draw A C at any angle, and set off 2 ft. 5 in. to any convenient scale; $2\frac{5}{12}$ in. will do. Join B C, and rule parallels cutting A B into 2 ft. 5 in. Add on the required number of feet.

DIAGONAL SCALES

By means of the diagonal scale very minute distances may be measured with great accuracy. The principle of its construction is as follows:—If the rectangle A B C D, on the opposite page, be divided into 8 equal parts by parallels to A B, and the diagonal D B be drawn, then a number of similar triangles will be formed. (Euc. vi. 2.) If D 4 is half D A, then 4 a will be half A B. In the same manner, D 1 is $\frac{1}{8}$ of D A; therefore, 1 b will be $\frac{1}{8}$ of A B.

From a *plain scale* we get two dimensions, such as yards and feet; from a *diagonal scale* we may obtain three dimensions, as yards, feet, and inches.

PROBLEM 99.—Draw a diagonal scale showing inches and tenths.

Draw a line A B, and set off inches. At A erect a perpendicular, and set off 10 equal parts to any convenient unit, and from each part draw parallel lines to A B. Erect verticals at each primary division. Draw the diagonal from 0 to 10.

Note.—We may obtain from this scale inches and tenths, thus:—Suppose $1\frac{7}{10}$, or 1·7 in., be required. Place the dividers where *vertical* 1 meets *horizontal* 7 (point e), and open until point f is reached. $ab = 1\frac{1}{10}$, or 1·1 in.; $cd = 1\frac{2}{10}$, or 1·2 in.; $gh = 2\frac{8}{10}$, or 2·8 in.

PROBLEM 100.—Draw a diagonal scale showing inches and hundredths of an inch.

Draw the 11 parallels, and set up the verticals as before. Now divide A 0 into 10 equal parts, and join the first part on the left with 10 at the end of the top line. Rule parallels from each of the parts as shown.

Note.—Each part on the line A 0 shows tenths of an inch, and the distance 9 a on the second line from the top will be $\frac{1}{10}$ of a tenth—that is, $\frac{1}{100}$ of an inch. The distance 1 $b = 1\frac{3}{10}$, or 1·3; $cd = 1\frac{31}{100}$, or 1·31; $ef = 1\frac{86}{100}$, or 1·86.

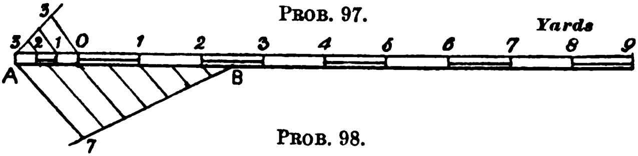
If 1·59 inches were required from the scale, place one point of the dividers on the point g, where the vertical 1 meets the horizontal 9; open the dividers to the point h, where the diagonal 5 meets the horizontal 9.

PROBLEM 101.—Draw a scale of $\frac{1}{48}$th to show yards, feet, and inches, and show a distance of 2 yds. 2 ft. 7 in. on the scale.

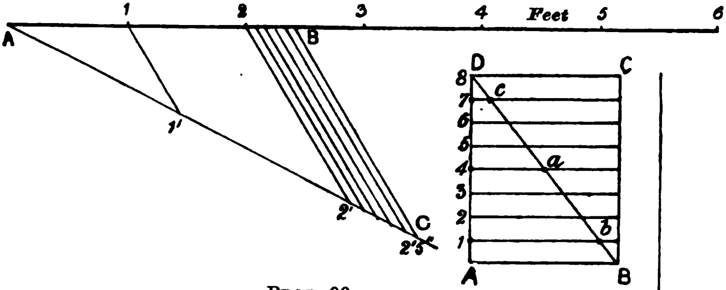
A scale of $\frac{1}{48}$ would be $\frac{3}{4}$ in. to the yard. Draw a line, A B, and set off distances of $\frac{3}{4}$ in. Divide the firstpart into 3, showing feet. Draw the vertical A 3, and set off 12 equal parts. Rule parallels, and complete the scale.

To obtain the required distance, place one point of the dividers on a, where vertical 2 meets the horizontal 7, and extend the other point to b, where the diagonal 2 intersects the horizontal 7.

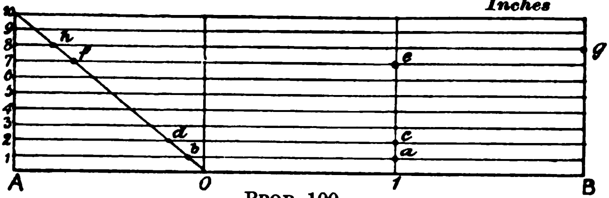
PROB. 97.



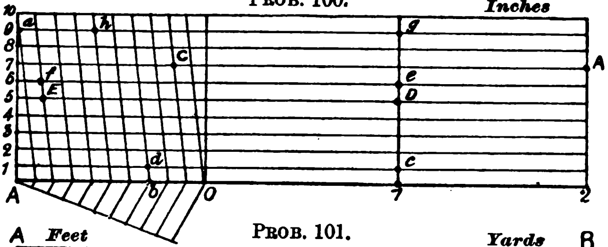
PROB. 98.



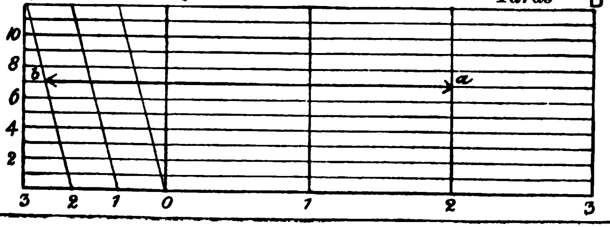
PROB. 99.



PROB. 100.



PROB. 101.



THE SCALE OF CHORDS

This scale is used to measure angles, and is usually marked on a scale by the letters 'C' or 'CHO.'

PROBLEM 102.—To construct a scale of chords.

Describe the quadrant A B C. Trisect the arc A C, and subdivide each part again into three, thus dividing the arc into 9 equal parts of 10° each. Make A D equal to A C. From centre A describe arcs from each of the divisions on to the line A D. Complete the scale as shown.

Notes.—1. The distance from A to each division on the scale is the chord of the angle containing that number of degrees. 2. The divisions get smaller as the angle increases towards 90°. 3. The chord of the arc of 60° is always equal to the radius.

PROBLEM 103.—To construct angles of 50° and 100° by means of the scale of chords.

Draw any line, A B. With centre A and *radius* o *to* 60° from the scale, describe an arc. Now take the distance from o to 50°, and intersect the arc in C. Then B A C is the angle of 50°. **For 100°**, take the distance from o to 50°, and set off from C, giving B A D, the angle of 100° (50° + 50° = 100°) Another way would be to obtain E for 90°, and cut off E D equal to the distance from o to 10° (90° + 10° = 100°).

PROBLEM 104.—To measure the size of an angle by means of the scale of chords.

Let B A C (*Prob.* 103) be the given angle. With radius o to 60° from the scale, describe the arc B C. Take the distance B C, and apply to the scale, giving 50°.

THE SECTOR

This instrument is shown on the opposite page. The most important of the scales marked on it are the **line of lines**, marked **L**, and the **line of polygons**, marked **POL**. The following problems show some of its uses.

PROBLEM 105.—To bisect a line.

Open the sector until the transverse distance from 10 to 10 on L equals the given line. Then the distance from 5 to 5 will be half the line.

Notes.—1. Measure with the dividers from the inside line, where the dots are marked. 2. The transverse distances between 8 and 4, or 6 and 3, &c., will also give the bisection.

PROBLEM 106.—To divide a straight line into 7 equal parts.

Open the sector until the transverse distance from 7 to 7 on L equals the given line. Then the transverse distance from 1 to 1 will be $\frac{1}{7}$th of the given line.

PROBLEM 107.—To find a fourth proportional to three given lines, A, B, C.

From the centre O, on L, set off O D equal to A. Open the sector until the transverse distance at D equals B. Then, if O E be set off equal to C, the transverse distance at E will be the required fourth proportional.

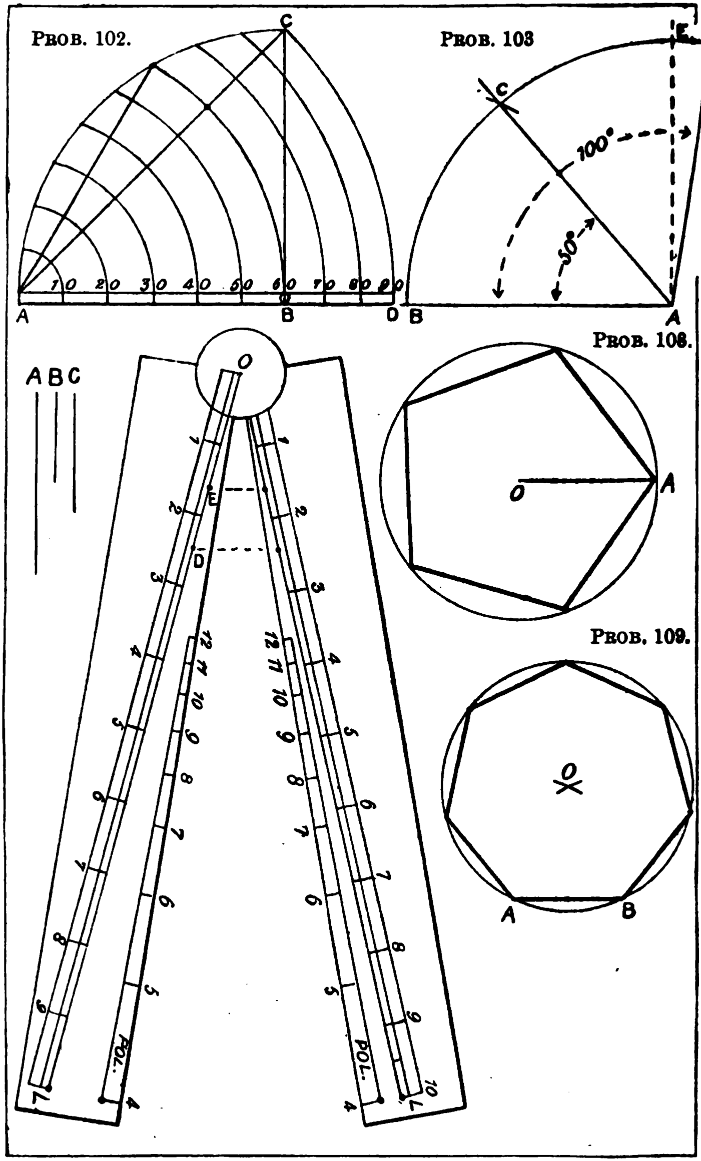
PROBLEM 108.—To inscribe a regular pentagon in a circle.

Let O A be the radius of the circle. Open the sector until the transverse distance from 6 to 6 on **POL** equals O A. Then the transverse distance at 5 will equal the side of the pentagon.

Notes.—1. Always make the radius of the circle equal the distance from 6 to 6. 2. If a heptagon were required, then the distance from 7 to 7 would equal the side.

PROBLEM 109.—To construct a regular heptagon on a given line.

Let A B be the given line. Open the sector until the transverse distance from 7 to 7 equals A B. With A and B as centres, and the distance from 6 to 6 as radius, describe arcs intersecting at O. With centre O and the same radius describe a circle. Set off the distance A B round it.


PROB. 102.
C
A
B
D
10 20 30 40 50 60 70 80 90
PROB. 103
E
D
C
100°
50°
B
A
A B C
O
E
D
1 2 3 4 5 6 7 8 9 10
12 11 10 9 8 7 6 5 4
POL.
L
PROB. 108.
O
A
PROB. 109.
O
A
B

CHAPTER X

THE USE OF SCALES IN THE CONSTRUCTION OF IRREGULAR POLYGONS, AND IN THE REDUCING, ENLARGING, AND COPYING OF PLANE FIGURES.

PROBLEM 110.—To construct an irregular polygon having given the lengths of the sides and the magnitude of the angles.

Sides, A B = 1·3″, B C 1·2″, C D $1\frac{1}{4}''$, D E $1\frac{5}{8}''$, E A 1·4″. Angles, A B C 140°, B A E 100°. First make a rough freehand sketch, lettered and figured, as a guide. Draw A B and make it 1·3″ ($1\frac{3}{10}$) long from the scale. Make the angles A B C and B A E equal to 140° and 100° respectively, either with the protractor or scale of chords. Make B C 1·2″ and A E 1·4″.

With centre C and radius $1\frac{1}{4}''$ describe an arc. With centre E and radius $1\frac{5}{8}''$ intersect the arc in D. Draw E D, C D.

Note.—The angles of figures are usually lettered in order, starting from the first letter.

PROBLEM 111.—To construct an irregular polygon, the lengths of the sides and diagonals being given.

Sides, A B = 1·9″, B C = 1″, C D = 1·85″, D E = 1·75″, E A = $1\frac{1}{8}''$. Diagonals, A C = 2·17″, B E = 2·4″. Make a rough sketch of the figure as before. Draw A B 1·9″ long. With centre B and radius 1″ describe an arc, and with centre A and radius 2·17″ intersect the arc in C. (This distance may be taken from the diagonal scale showing hundredths of an inch; it is marked by the letters A C on the figure for Problem 100.) Draw the triangle A B E in a similar manner. With centres C and E describe arcs of 1·85″ and 1·75″ radii intersecting in D. Draw B C, C D, D E, E A. (The distance C D is shown on the same scale by the letters D E.)

Note.—Problems 111 and 112 are drawn half size.

PROBLEM 112.—To construct an irregular polygon, having given two sides, the lengths of the diagonals drawn from one angle, and the angles between them.

Sides, A B = 1·15″, B C = 1·3″. Diagonals, B D = 2·4″, B E = 2·3″, B F = 2″. Angles, A B F = 30°, F B E = 45°, E B D = 35°, D B C = 25°.

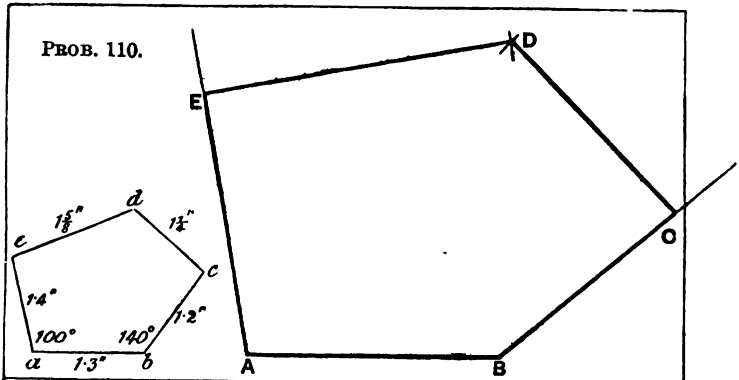
After making a sketch, draw A B = 1·15″. With the protractor set off angles A B F, F B E, E B D, D B C. Make B C, B D, B E, B F equal to the lengths given. Draw C D, D E, E F, F A.

PROBLEM 113.—To construct an irregular polygon having given a point O within the polygon, the distance from the point to each angle, and the angles round the point.

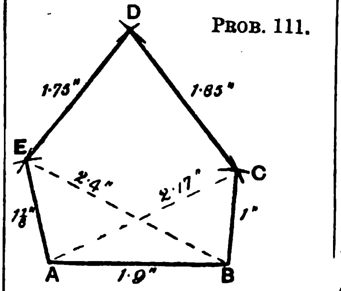
O A = 50′, O B = 55′, O C = 60′, O D = 25′, O E = 75′, O F = 40′. Angles.—A O B = 65°, A O F = 50°, F O E = 75°, E O D = 30°, D O C = 55°. Scale, 45 ft. = 1 inch.

First construct the scale, showing distances of 5 ft. After making a sketch, fix a point O, draw O B, and set out the angles B O A, A O F, F O E, E O D, D O C with the protractor. Take the lengths of the lines given from the scale and join the points, A, B, C, D, E, F, with each other.

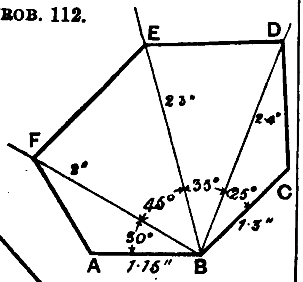
PROB. 110.



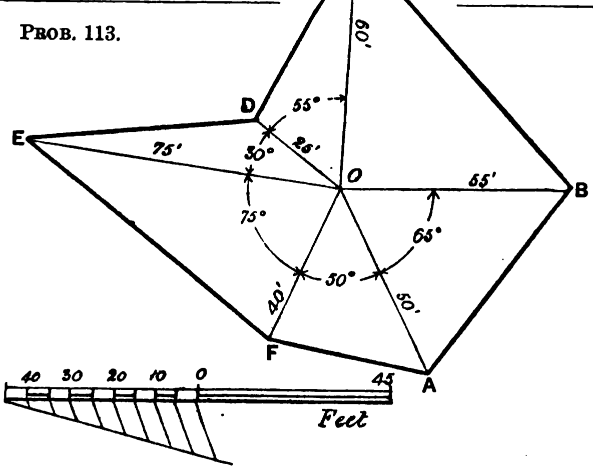
PROB. 111.



PROB. 112.



PROB. 113.



PROBLEM 114.—To construct an irregular polygon from a rough diagram, the dimensions on the diagonal and the offsets or ordinates being given.

A E = 3 ch. 50 l., A b = 75 l., A g = 90 l., A c = 2 ch. 20 l., A f = 2 ch. 90 l., A d = 3 ch. Offsets, b B = 1 ch. 10 l., g G = 80 l., c C = 25 l., f F = 1 ch. 30 l., d D = 90 l.

Scale, ¾ in. to represent 1 chain.

$$\textit{Representative Fraction} = \frac{\frac{3}{4}\text{ in.}}{66\text{ feet}} = \frac{\frac{3}{4}\text{ in.}}{792\text{ in.}} = \frac{1}{1{,}056}.$$

First construct the scale. As no distances in the figure are required smaller than 5 links it will only be necessary to divide the unit into 20 parts. This will be more accurately performed by means of a diagonal scale. Draw 5 parallel lines. Set off 4 spaces of ¾ in. each and draw verticals. Divide the first space into 5 equal parts giving distances of 20 links. Draw the diagonals, thus obtaining distances of 15, 10, and 5 links.

Next draw A E, 3 ch. 50 l. from the scale ($a\,b$ on the scale), and set off A b ($c\,d$ on scale), A g, A c, A f, A d. At the points b, g, c, f, d draw the offsets, taking the distances from the scale. Join the points A, G, F, E, D, C, B.

PROBLEM 115.—To construct an irregular figure from a given figured rough sketch. Scale, 20 ft. = 1 in.

$$\textit{Representative Fraction} = \frac{1\text{ in.}}{20\text{ ft.}} = \frac{1}{240}.$$

First draw the scale. As the longest distance to be measured is under 40 ft. the scale need be only 2 in. long. Divide the first inch into 20 parts.

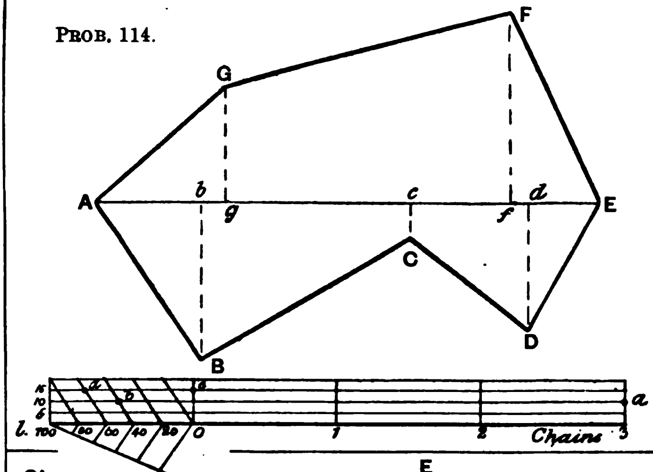
Draw any line A B. At A draw A C at an angle of 90° and make it 27 ft. long by scale. Make an angle of 30° at C and draw C D 15 ft. by scale. At D make an angle of 100° and draw D E 35 ft. long. Proceed in a similar manner for the other lines and angles. From G draw G B at right angles to A B.

PROBLEM 116.—Construct a six-sided polygon $a\,b\,c\,d\,e\,f$, such that $e\,f = \frac{a\,e}{2} = \frac{2}{3}\,a\,f$; $d\,e = a\,d$; and $a\,b = \frac{2}{3}\,b\,c$. The rest of the data are given on the figure. Scale ½″ = 10′ 0″.

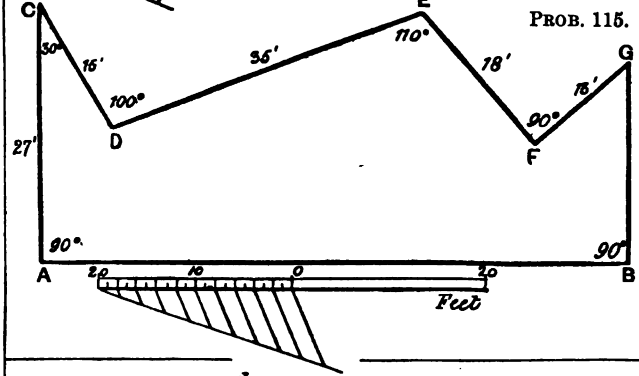
First construct a plain scale showing feet, and long enough to measure $a\,e$. Then draw $a\,e$ 56′ by scale. On $a\,e$ construct the isosceles triangle $a\,d\,e$, making the sides $d\,a$ and $d\,e$ 49′ long. To obtain point c, draw a line parallel to $a\,e$ and 22′ from it, and with centre e and radius 42′ from the scale intersect the parallel. Draw $d\,c$. ($a\,b = \frac{2}{3}\,b\,c = 18'$.) With centre a and radius 18′ describe an arc, and with centre c and radius 27′ intersect the arc in b. Draw $c\,b, a\,b$. ($e\,f = \frac{a\,e}{2} = 28'$, and $\frac{2}{3}\,a\,f = e\,f$, therefore $a\,f = 42'$.) With centre e and radius 28′ describe an arc, and with centre a and radius 42′ intersect the arc in f. Draw $a\,f, f\,e$.

Notes.—1. A figure similar to this was set in the science examination 1889.
2. The figure is drawn half its proper size.

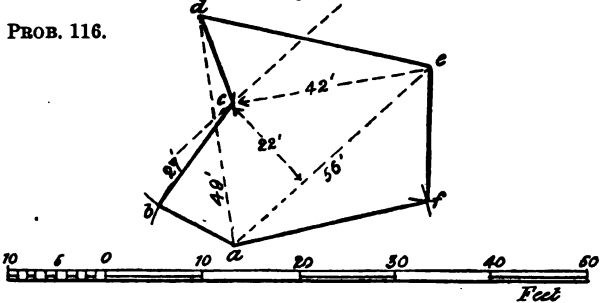
Prob. 114.



Prob. 115.



Prob. 116.



PROBLEM 117.—To make a reduced drawing of a given figure.

Let A B C D E F be the given figure. It is required to reduce it so that each side shall be ⅔ of its length on the given figure.

Method 1.—Draw B F, B E, B D. Divide A B into 3 equal parts. B *a* will be one of the required sides. Draw *a f*, *f e*, *e d*, *d c* parallel to the corresponding sides of the figure.

Note.—The principle is the same as in Prob. 88.

Method 2.—Divide A B into 3 equal parts as before. Take B *a* and place it in the same straight line or parallel to A B. Draw *a f*, *b f* parallel to A F, B F, thus obtaining point *f*. From *f* draw a parallel to F E and from *b* draw a parallel to B E giving point *e*. Proceed in a similar manner for the other sides.

Method 3.—Trisect each side and construct the figure by applying the principle of Prob. 63.

Notes.—1. Two other figures are given showing the application. The sides of both rectangle and triangle are reduced two-thirds.

2. A figure may be reduced to any given size by this method.

PROBLEM 118.—To make an enlarged drawing of a given figure.

Let A B C D E F be the given figure. It is required to enlarge it so that A B may be equal to the given line *a b*.

Draw A C, A E, A D and produce them. Set off *a b* from A. From *b* draw *b c* parallel to B C, and proceed as in the previous problem.

PROBLEM 119.—To enlarge or reduce a drawing by means of a PROPORTIONAL SCALE.

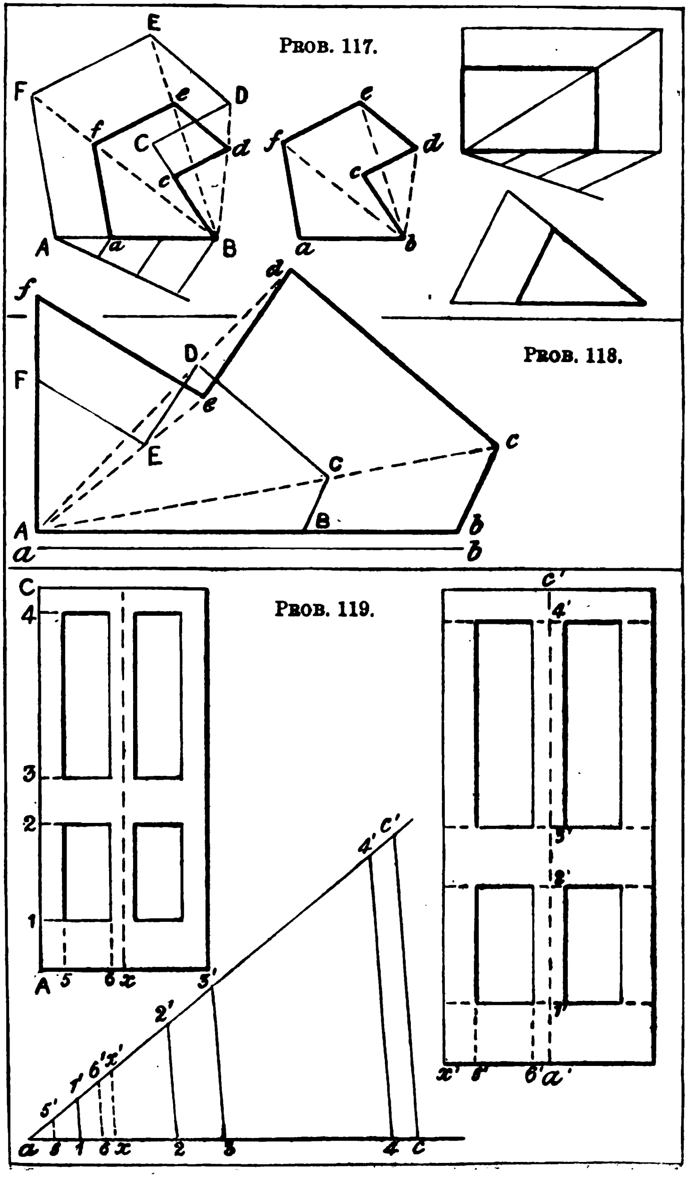
It is required to enlarge the given drawing so that A C shall equal 2½ ins.

Construct a proportional scale by drawing two lines at any angle to each other, set off *a c* equal to A C and *a c′* equal to the required distance, 2½ ins. Join *c c′*. Set off the distances A 1, A 2, A 3, A 4 on *a c*. From each point rule parallels to *c c′* giving the proportionate distances for the new drawing.

Draw the centre line 2½ ins. long and on it set off the distances *a* 1′, *a* 2′, &c. Through *a′* and *c′* draw perpendiculars. Set off A *x* on *a c* and obtain *a x′* the width of half the door. Set this distance off on each side of *a′* and erect perpendiculars. Through 1′, 2′, 3′, 4′ draw parallels. For the width of the panels set off A 5, A 6 on *a c* and proceed as before.

To reduce a drawing proceed in a similar manner. The only difference will be that the *given* dimensions will be set off on the *longer* line of the scale and the *required* dimensions will be obtained from the *shorter* line.

PROBLEM 119a.—To reduce or enlarge a drawing by squaring. See Prob. 31.



Prob. 117.

Prob. 118.

Prob. 119.

PROBLEM 120.—To copy figures.

The copying of given figures is an exercise of considerable importance in helping to form habits of accuracy and quickness in a draughtsman. The proper construction lines must be first obtained, and always in drawing symmetrical figures remember to set out the centre lines first. Several examples are here given, with the proper method of proceeding.

Figure 1 represents a Greek fret. It will be found that the pattern is formed upon ten equi-distant horizontal lines intersected by vertical lines forming squares. Draw the pattern in firmer line, keeping the *shade* line darker.

Note.—The figure is given half size. The same method of construction must be employed when copying any fret.

Figure 2.—Rule the parallels first. Draw the verticals, and with the 60° set square obtain the equilateral triangles as shown. Keep the proper relation between thick and thin lines. Make your copy double the size of the given figure.

Figure 3.—Draw the figure to the dimensions given.

It will be seen that a square of 3″ sides divided into 9 equal squares will contain the inner lines of the four squares of which the figure is composed. Draw the diagonals, and about each square describe another square at the required distance. In thickening, or inking in the pattern, be careful to notice the portions that overlap.

Figure 4.—Draw the pattern to the dimensions given. As the sides of the small squares and the distances between them are equal, draw a line and set off 5 equal spaces of ¾″ each. On this distance construct a square and divide it into 25 equal squares.

To obtain the other lines divide the alternate squares as shown.

Figure 5.—Draw the figure, making the centres of the circles 1¼ inch apart. Radius of the large circle ⅜″, of the smaller one ⅛″. Draw the centre line, set off the centres and describe the circles. For the centres of the connecting arcs describe equilateral triangles as shown.

PROB. 120.

FIG. 1.

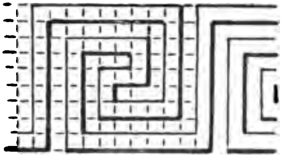


FIG. 2.

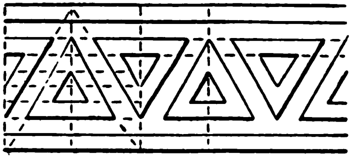


FIG. 3.

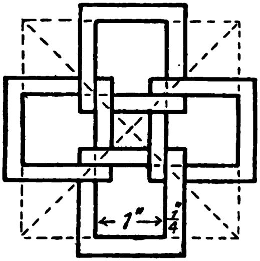


FIG. 4.

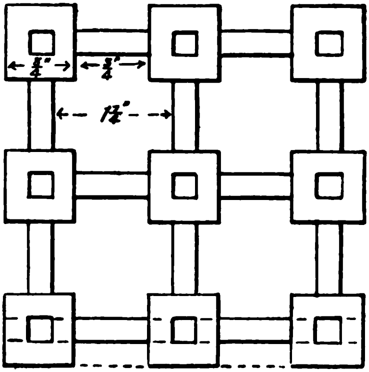


FIG. 5.

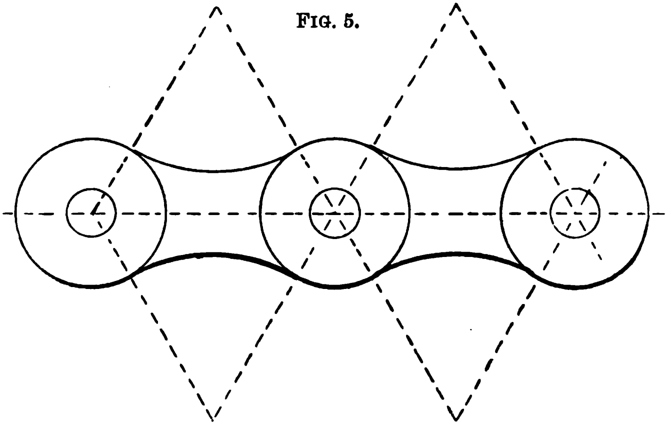


Figure 6.—Copy the double spiral, which is composed of semicircles ¼″ apart; the diameter of the smallest semicircle is ½ in.

Draw a line A B and set off 12 equal spaces of ¼″, and with centres 3 and 9 describe three concentric semicircles above the line from 3, and below the line from 9. The centres for the connecting semicircles will be midway between 2, 3, and 9, 10. Describe the arcs which have equal radii before commencing the next.

Figure 7.—Rule the horizontal and vertical lines as shown, and from each centre describe two concentric circles. The finished pattern should be thickened or inked in. Let the centres for each circle be ⅝″ apart.

Figure 8.—Draw the figure from the given dimensions.

Draw a line A B and set off distances of ⅜″, 1¼″, ⅜″, 1¼″, ⅜″, 1¼″, ⅜″. Mark the centre for each of the circles and describe them, omitting the portions which are omitted in the copy.

Note.—The given figure is drawn to a smaller scale.

Figure 9.—Draw the figure from the given dimensions.

Commence with the centre lines A B and C D.

Set off on C D distances of ⅛″, ¼″, ⅛″, ½″, ⅛″, ¼″, ⅛″. Describe the circles. Set off 1⅛″ on each side of the centre of A B and draw the vertical lines E and F. Describe the semicircles and complete the figure.

FIG. 6.

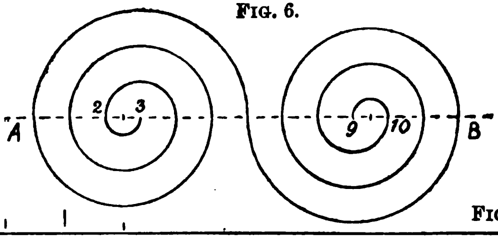


FIG. 7.

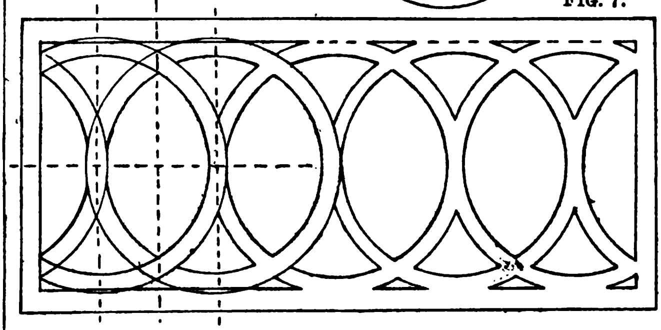


FIG. 8.

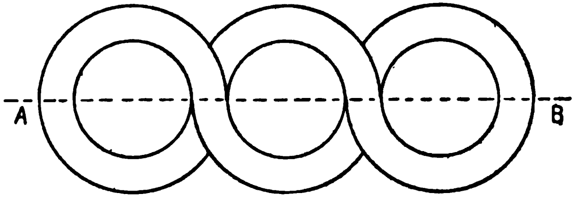
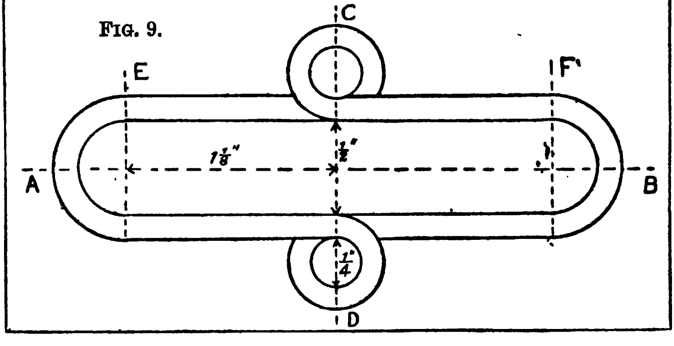


FIG. 9.



EXERCISES

Note.—Any figures relating to the following exercises will be found on page 62.

CHAPTER VII

1. Describe a circle 1¼" radius, and from a point A, 3" from its centre, draw a line touching the circle.

2. Describe two circles with radii ¾" and 1", having their centres 2½" apart. Draw a right line to touch both these circles.

3. To a circle of 1·25" radius, draw two tangents which shall contain an angle of 60°. (*Sc.*, 1871.) (*Prob.* 72.)

4. Describe a circle of 1⅛" radius, and draw tangents intersecting each other from two points in the circumference 100° apart.

5. Describe a circle 1·5" radius. Draw a straight line 1·75" long within the circle. On this line construct an isosceles triangle having its vertical angle in the circumference of the circle.

6. Draw a circular arc of large radius, and draw a tangent to it without finding the centre of the circle.

CHAPTER VIII

1. Construct a regular hexagon having its diagonal 3 inches.

2. On a base of 1¼" construct a regular heptagon.

3. Describe a circle with a radius of 2·9". About it describe a regular nonagon.

4. On a side of ¾" set up a regular octagon by means of the set square.

5. In a circle of 3·5" diameter inscribe a regular pentagon.

6. Construct a regular pentagon whose diagonal shall be 3". (*Sc.*, 1870.) (*Prob.* 90.)

7. Construct a regular hexagon on a base of ·75", and on the same base construct a similar hexagon having its side 1·25" long. (*Prob.* 117.)

8. Copy the hexagons (Fig. 1) which are drawn to a scale of ¾" to 1′ 0", and make them ½" to 1′ 0". (*Art.*)

Note.—The figure is given half its proper size and the construction lines are indicated.

CHAPTER IX

1. Construct a scale 5 ft. long, 1⅛ in. to 1 ft. to show inches. What is its representative fraction?

2. Construct a scale 2 inches to 1 yard to show 5½ feet. What is its representative fraction?

3. The given line (Fig. 2) is 2 ft. 6 in. long. Produce it to measure 3 ft. 6 in (*Art.*)

4. Convert A B 2 ins. long into a diagonal scale showing $\frac{1}{100}$ of the line. (*Art.*)

5. Construct a plain scale ¾ in. to 1 ft., and mark upon it a distance of 5 ft. 7 in. by the scale. (*Art.*)

6. Convert the given scale (Fig. 3) into a diagonal scale reading inches. State the representative fraction of this scale. (*Sc.*, 1889.)

Note.—The first division will require dividing into 10 equal parts, and twelve parallels must be ruled to obtain the inches.

7. Construct a scale of chords on a radius of 4·25″ to read to 5°. By means of this scale plot an angle of 75°. (*Sc.*, 1889.)

8. Given a scale of yards (Fig. 4). Deduce from it a scale of feet to read to 1′ 0″ and show 70′ 0″.

Note.—The given scale is ¾ its proper size and shows 20 yds. or 60 feet. Divide it into 6 equal parts, showing distances of 10 feet, and add one space on to obtain 70 feet. Subdivide the first part into 10 to obtain one foot.

9. The line *a b* (Fig. 5) represents 8′ 9″. Construct a scale reading inches and showing 10′ 0″. The scale to be correctly figured. (*Sc.*, 1886.)

Note.—Draw a line at any angle from *a*, and set off 8¾ to any unit. Divide *a b* and add the distance required as in Probs. 97 and 98.

10. Construct a triangle with sides 10′ 6″, 14′ 0″, and 16′ 8″. Scale ⅛″ to 1′ 0″. (*Sc.*, 1884.)

Note.—First construct the scale. As divisions equal to ½ of ⅛″ will be needed, a diagonal scale should be used.

11. Transfer the given scale (Fig. 6) to your paper and complete it neatly, with figuring, &c. Write down the representative fraction (*Sc.*, 1888.)

Note.—The scale measures 3·75″, therefore its representative fraction will be $\frac{3{\cdot}75 \text{ in.}}{50 \text{ yds.} \times 3 \times 12}$. Reduce to lowest terms.

12. Make a scale 6″ long to read feet and inches. Fraction $\frac{1}{16}$. (*Sc.*, 1882.)

13. A length of 100 yds. is found to measure 3·6″ on a drawing. What is the fraction of the scale? Construct a scale to read yards, making it not less than 4 ins. long. (*Sc.*, 1881.)

14. Give the representative fraction of a scale on which 8½″ represents 2247′ 0″. Construct a scale of $\frac{1}{20}$, reading inches. (*Sc.*, 1879.)

15. Construct a scale of $\frac{1}{468}$ showing yards. Scale to be properly figured and not less than 7″ long. (*Sc.*, 1878.)

Note.—1 in. stands for 468 ins., that is 13 yards.

16. Construct a scale of $\frac{1}{19}$ to read inches and show 10′ 0″. (*Sc.*, 1877.)

Note.—To find the length of the scale. If 19 feet be represented by 1 foot, what distance will be represented by 10 feet? 19 ft. : 10 ft. as 12 ins. : required length, whence required length $= \frac{12 \times 10}{19} = 6\frac{6}{19}$ ins. Draw a line $6\frac{6}{19}$ ins. long and divide it into 10 equal parts showing feet. Subdivide the first unit into 12 equal parts showing inches.

17. Draw a scale of 12·5 feet to 1 inch and give the fraction. (*Sc.*, 1876.)

18. A line 100 ft. long is represented on a drawing by a line 4″ long. Make a scale of feet for the drawing and give the representative fraction.

19. A line 50 yds. long is measured from **A** to **B** at right angles to a line joining **A** with another point **C**. The angle subtended by the line **A C** is found to be 47°. Find by construction the distance from **A** to **C**. Scale 20 yds. to an inch.

20. Describe a circle of 1½″ radius. Within it inscribe a regular nonagon by means of the sector. (*Prob.* 108.)

CHAPTER X

1. On a base 2″ long construct a figure similar to the given figure (Fig. 7).

2. Construct an irregular pentagon, sides 2½″, 2″, 1½″, 1″, and ½″. The angle between the longest and shortest sides to be 100° and the diagonal joining the extremities of the two longest sides 2½″. (First draw a rough sketch.)

3. Draw any irregular four-sided figure, no side less than 1¼″. Construct a similar figure whose sides are 1½ times those of the first figure. (*Sc.*, 1879.)

4. The tops of two vertical poles are 85′ 0″ apart. The poles are 40′ 0″ apart. The height of one pole is 12′ 0″. Determine the height of the other pole. Scale 1″ to 10′ 0″. (*Sc.*, 1879.)

5. Draw a circle of 1·25″ radius with centre **O**. The corners of the polygon inscribed in this circle are so that the angles at the centre are as follows: **A O B** = 60°, **B O C** = 70°, **C O D** = 50°, **D O E** = 80°, **E O F** = 50°. Write down the lengths of **A B**, **B C**, **C D**. (*Sc.*, 1871.)

6. Construct an irregular polygon from the following dimensions. Sides **A B** = 2″, **A F** = 1·8″. Diagonals, **A D** = 3·5″, **A E** = 3″. Angles, **A B C** = 33°, **B A C** = 40°, **B A D** = 59°, **B A E** = 118°, **B A F** = 130°.

7. Construct an irregular pentagon having its sides 2″, 2¼″, 2⅜″, 2½″, 2¾″ respectively and with two of its angles right angles.

8. Draw to a scale of 40 yds. to an inch an irregular polygon **A B C D E F**. **A B** = 140 yds., **B C** = 113 yds., angle **A B C** = 130°, **C D** = 82 yds., **A D** = 200 yds., **D E** = 133 yds., **A E** = 140 yds., **E F** = 67 yds., **F A** = 90 yds.

Note.—A diagonal scale will be required, showing the inch divided into 40 parts.

9. The sides of a quadrilateral figure **A B C D** are as follows:—**A B** = 1″, **B C** = 1″, **C D** = 1¼″, **D** = 1½″, and the diagonal **B D** = 1½″. Construct the figure and obtain a similar figure whose perimeter is 4″.

Note.—Find a fourth proportional to the perimeter of the constructed figure (4¾″), the perimeter of the required figure (4″), and the side **A B** of the constructed figure (1″). This will give the side of the required figure. Set off this distance on **B A** and proceed as in Prob. 117.

10. The given figure (Fig. 8) represents a Maltese cross. Two dimensions and an angle are given. Draw the cross from the figured dimensions to a scale of ⅛″ to 1′ 0″. (*Sc.*, 1888.)

Note.—The chief construction lines are indicated on the figure, which is drawn to a smaller scale. Commence with the inner square 5′ 6″ side. Draw the diagonals and diameters of the square, and produce them. From the centre set off 7′ 3″ on each side and draw parallels. On each side of the diagonal produced set off 20°. From *a* draw a line parallel to the vertical diameter to meet the diagonals. Complete the figure as shown.

11. Copy the given figure (Fig. 9), making the diameter of the circle 4″ and the width between the parallel lines ¼″.

Note.—Obtain the points as for the inscribed pentagon by using the protractor.

12. Copy the figure (Fig. 10) to the dimensions given.

Note.—Commence with the dotted lines.

FIG. 1.

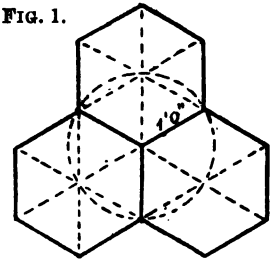


FIG. 7.

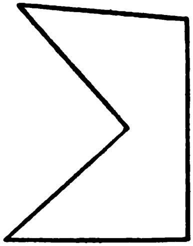


FIG. 2.

FIG. 3.

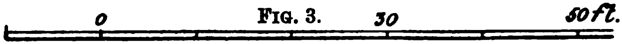


FIG. 4.

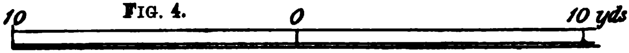


FIG. 5.

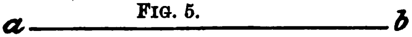


FIG. 6.

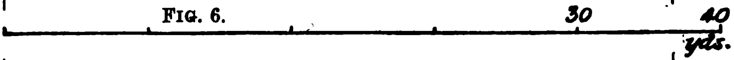


FIG. 8.

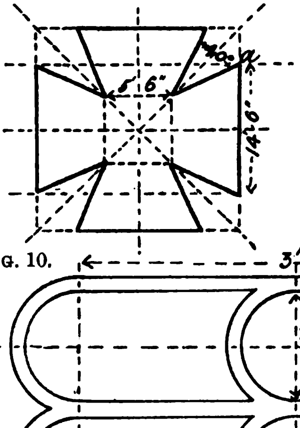


FIG. 9.

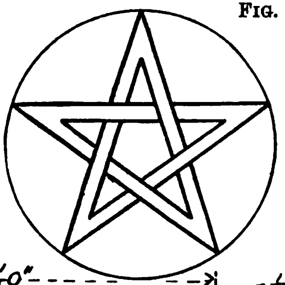
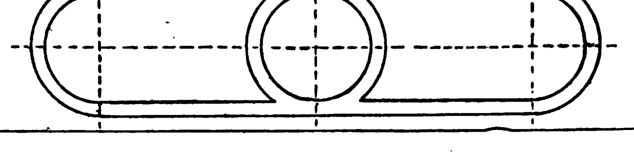


FIG. 10.



CHAPTER XI

CIRCLES TOUCHING LINES AND CIRCLES

PROBLEM 121.—To describe a circle passing through a given point and touching a given straight line in a given point.

Let A be the given point, and B the point in the given straight line.

At B draw B O perpendicular to the given line. Join A B.

At A construct the angle O A B equal to the angle O B A. With centre O and radius O B describe the circle.

Note.—The centre must lie in the line B O, because the given line will be a tangent to the circle.

PROBLEM 122.—To describe a circle passing through a given point, touching a given straight line, and having a given radius.

Let A be the given point, B the given line, and C the radius.

Draw a line parallel to B and at a distance equal to C.

With centre A and radius equal to C intersect this line in O.

With centre O and radius equal to C describe the circle.

PROBLEM 123.—To describe a circle of a given radius to touch two converging lines.

Let A B and C D be the two lines and E the radius.

At a distance equal to E draw lines parallel to A B and C D.

From O where the two parallels intersect, with radius equal to E, describe the circle.

PROBLEM 124.—To describe a circle touching three given straight lines which make angles with each other.

Let A B, A C, C D be the given lines.

Bisect the angles at A and C by lines meeting at O. Draw O E perpendicular to C D. With centre O and radius O E describe the circle.

PROBLEM 125.—To describe a circle touching two converging lines and passing through a given point between them.

Let A B, A C be the two converging lines and D the given point.

Bisect the angle B A C by the line A X; the centre of the circle must lie in this line. From any point E draw E F perpendicular to A B and describe a circle touching A B and A C. Join A with the given point D. Draw E G, and from D draw D H parallel to G E. With centre H and radius H D describe the circle.

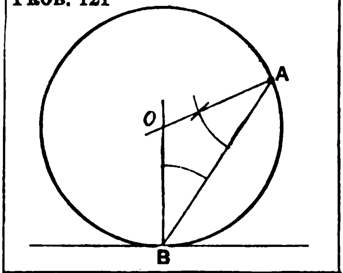
PROBLEM 126.—To describe a succession of circles touching each other and two converging lines.

Let A B, A C be the two lines. Draw A D bisecting the angle B A C.

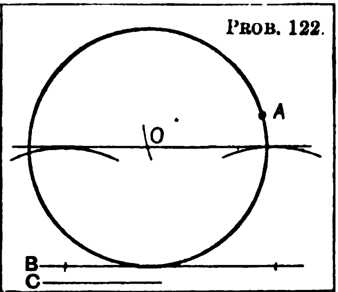
Take any point E, draw E F perpendicular to A C and describe a circle touching the two lines. Draw G H tangential to the circle at J.

Make G K equal to G F. From K draw K L parallel to F E for the centre of the next circle. Proceed in a similar manner for the other circles.

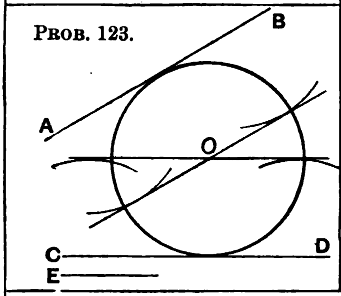
Prob. 121



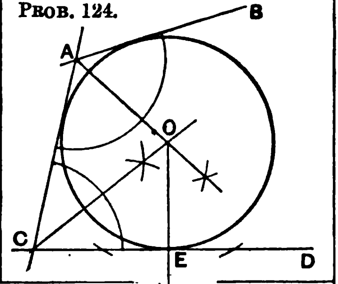
Prob. 122.



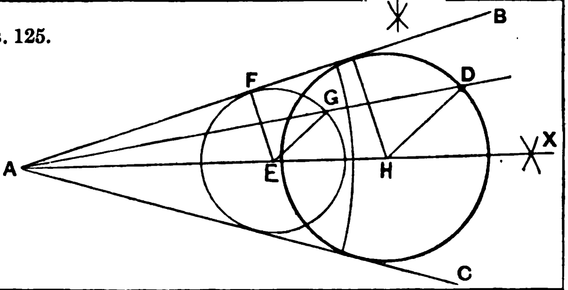
Prob. 123.



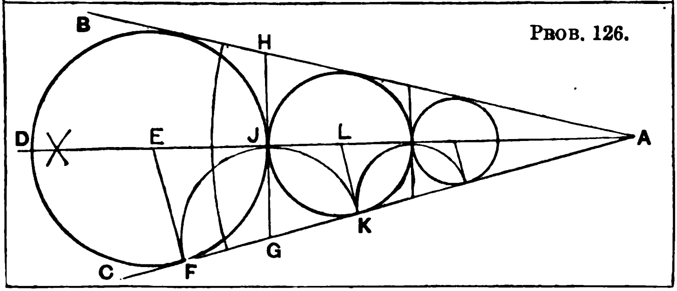
Prob. 124.



Prob. 125.



Prob. 126.



PROBLEM 127.—To inscribe a circle in a triangle.

Let A B C be the triangle.

Bisect any two angles, and proceed as in *Prob*. 124.

PROBLEM 128.—To inscribe two equal circles in an isosceles triangle, touching each other and two sides of the triangle.

Let A B C be the triangle. Draw the perpendicular A D dividing the triangle into two equal triangles. Find the centre O of the triangle A B D. For the centre of the other triangle join C E, and from O draw a line parallel to B C and meeting C E in P. From centres O and P describe the circles.

PROBLEM 129.—To inscribe three equal circles in an equilateral triangle, each touching one side and two circles.

Let A B C be the triangle. Divide it into three equal triangles by bisecting the sides. Find the centre of the triangle D B C and inscribe a circle. Set off D 2, D 3 each equal to D 1. With centres 2 and 3 describe the circles.

Note.—If parallels to the sides of the triangle be drawn through 1, 2, and 3, the angles 4, 5, and 6 of the triangle thus formed will be the centres for three more equal circles which may be inscribed in the triangle.

PROBLEM 130.—To inscribe four equal circles in a square, each touching one side and two circles.

Let A B C D be the given square.

Draw the diagonals and the diameters of the square.

Find the centre of the triangle A O B as in the preceding problem, and inscribe a circle. Set off the centres as shown, and describe the circles.

PROBLEM 131.—To inscribe in any regular polygon as many equal circles as the figure has sides, each touching one side and two circles.

Let the polygon in this case be a pentagon.

Divide the figure into as many equal triangles as it has sides, and inscribe a circle in each triangle as in the preceding problems.

PROBLEM 132.—To inscribe three circles in ANY triangle, each touching one side and two circles.

. Let A B C be the triangle. Bisect the angles at A and B by lines meeting at O, and join O C. In each of the triangles thus formed inscribe a circle by *Prob*. 127.

PROBLEM 133.—To inscribe three equal circles in a hexagon, each touching one side and two circles.

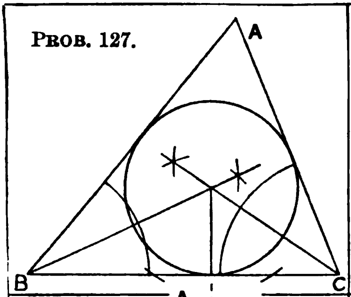
Draw the diameters of the figure. Produce two of these, and form a triangle as shown. Inscribe the circles.

Note.—Problems 128 to 134 all depend on Problem 127.

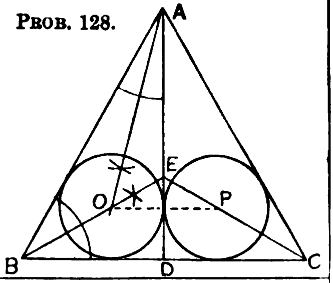
PROBLEM 134.—To inscribe a circle in a square.

Draw the diagonals to find the centre O. For the radius, draw O D perpendicular to the side. With centre O and radius O D describe the circle.

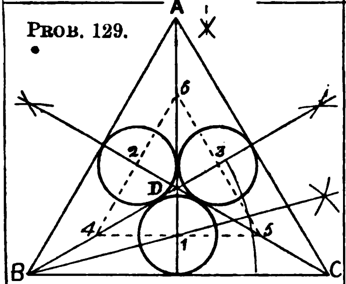
Prob. 127.



Prob. 128.



Prob. 129.



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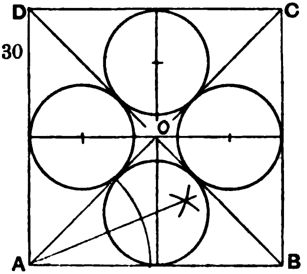
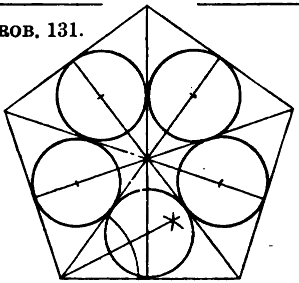
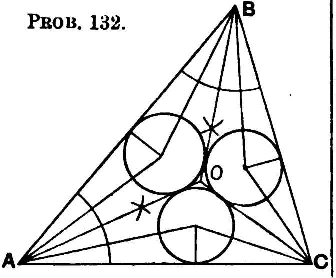


Prob. 131.



Prob. 132.



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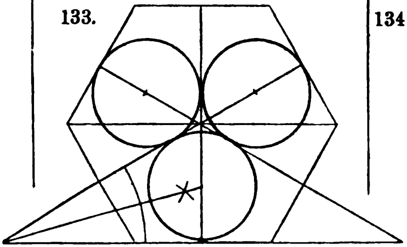

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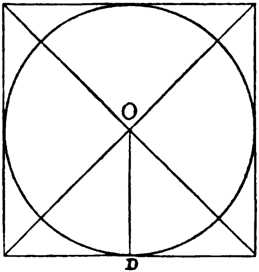


PROBLEM 135.—To inscribe four equal circles in a square, each touching two sides and two circles.

Draw the diagonals and the diameters of the given square. Draw the diagonals A B, B C, C D, D A, giving the centres of each of the four squares. For the radius of the circles join E and F.

PROBLEM 136.—To inscribe a circle in any regular polygon.

Bisect two of its angles as in the given pentagon. O will be the centre, and O A the radius.

Notes.—1. Bisecting two of the sides by perpendiculars will also give the centre.

2. If the polygon has an even number of sides, the radius must be found by drawing a line from the centre perpendicular to one of the sides.

3. To describe a circle about the figure, O will be the centre and O B the radius.

PROBLEM 137.—To inscribe a circle in a rhombus.

Draw the diagonals to obtain the centre. From O draw O A perpendicular to one of the sides for the radius.

PROBLEM 138.—To inscribe a circle in a trapezion.

Draw the diagonal A B. Bisect one of the other angles. The point O is the centre. For the radius draw a perpendicular to one of the sides.

PROBLEM 139.—To inscribe three equal circles in an equilateral triangle, each touching two sides and two circles.

Bisect each of the angles by the lines A E, B D, and C F, thus dividing the triangle into three equal trapezions. Bisect the angle C F B, obtaining centre 1. Set off centres 2 and 3. To obtain the the radius join 1, 2. With centres 1, 2, 3 inscribe a circle in each trapezion.

Note.—To inscribe in **any regular polygon** as many circles as the figure has sides, each touching two sides and two circles; divide the polygon into as many equal trapezions as the figure has sides, and inscribe a circle in each as above.

PROBLEM 140.—In ANY regular polygon having an even number of sides, to inscribe half as many equal circles as the figure has sides, each touching two sides and two circles.

Let the given polygon be an octagon. Draw the diagonals. Inscribe a circle in the trapezion A B C O. Set off the other centres, and proceed as shown.

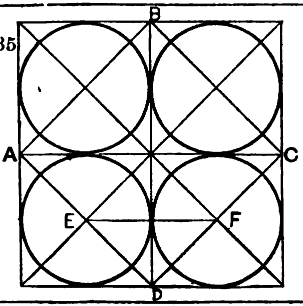
PROBLEM 141.—To inscribe a circle in a sector.

Let A B C be the sector. Bisect the angle B A C by the line A D. Through D draw a tangent to meet A B, A C produced. Bisect one of the angles of the triangle thus formed. With centre O and radius O D inscribe the circle.

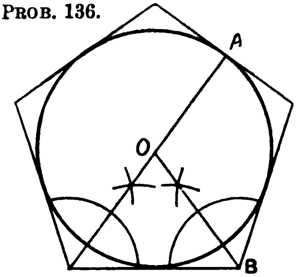
PROBLEM 142.—To describe a circle to touch the arc of a sector externally and the two radii produced.

Bisect the angle as before, and produce A D. Draw a tangent at D. Bisect the exterior angle at E by the line E O. With centre O and radius O D describe the circle.

135



PROB. 136.



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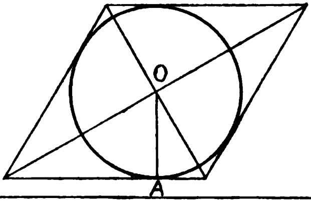

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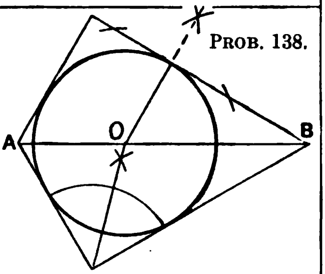

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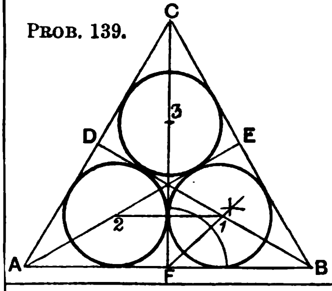

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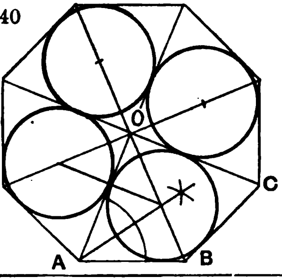

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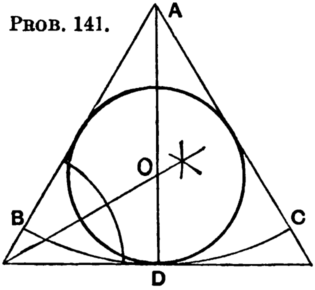

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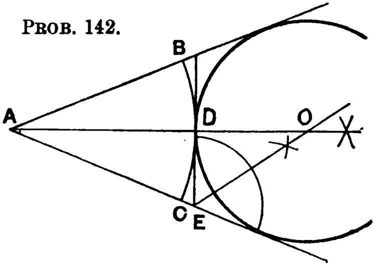


PROBLEM 143.—To inscribe ANY number of equal circles in a circle.

Let it be required to inscribe four equal circles in the given circle.

Divide the circle into twice as many sectors as circles required. In this case divide the circle into eight equal parts. (Use the 45° set square.) In the sector O A B inscribe a circle (*Prob.* 141). With centre O and radius O C mark off the centres D, E, F, for the other circles.

PROBLEM 144.—To describe ANY number of equal circles about a given circle. Say six.

Divide the given circle into twice as many sectors as there are circles required, and produce the diameters. At A draw a tangent B C.

Bisect the exterior angle at C. D is the centre for one of the circles. Set off the other centres and describe the circles.

PROBLEM 145.—To describe about ANY regular polygon as many circles as the polygon has sides, each touching one side and two circles.

Let the given polygon be a hexagon. From each angle of the hexagon draw lines through the centre, and produce them both ways. Bisect the exterior angle at A by the line A C. C will be the centre for one of the circles. Set off the centres, and proceed as in the preceding problem.

Note.—Circles may be described about a triangle or a quadrilateral by applying the same principles.

PROBLEM 146.—To describe a circle having a given radius, and touching another circle either externally or internally in a given point.

Let A be the given radius, and B the point in the given circle. Find the centre O. Draw O B and produce it. Set off B 1 and B 2 equal to the given radius A. The circle described from 1 will touch externally, that from 2 internally.

PROBLEM 147.—To describe a circle touching a given circle at a given point and passing through another given point either within or without the circle.

1st. Let A be the given point in the circle, and B the given point *within* the circle. Find the centre O of the given circle and join O A. Draw A B and bisect.

The point C, where the bisecting line intersects the radius O A, is the centre for the required circle.

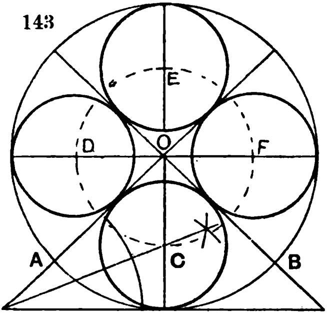
2nd. Let A be the given point in the circle, and B the given point *without.*

Proceed as in the first case, the bisecting line meeting O A produced.

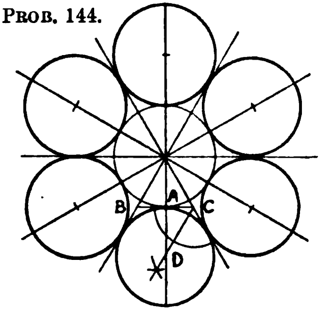
PROBLEM 148.—To describe a circle touching a given circle and a given straight line, and having a given radius.

Let A be the given radius and B C the given line. Draw a line parallel to B C at a distance equal to A. Find the centre O of the given circle. Draw any radius O 1 and produce it. Set off 1 2 equal to A. With centre O and radius O 2 intersect the parallel line in 3. With centre 3 and radius A describe the circle.

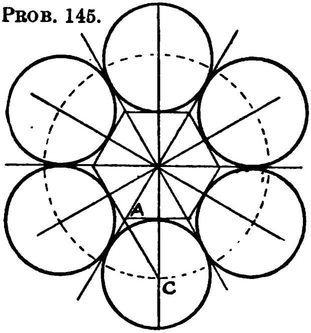
143



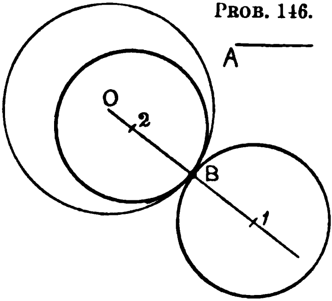
Prob. 144.



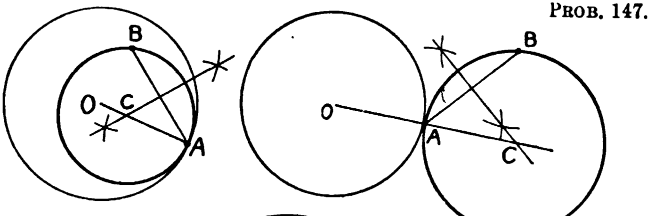
Prob. 145.



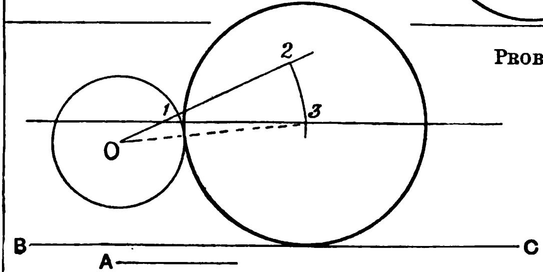
Prob. 146.



Prob. 147.



Prob. 148.



PROBLEM 149.—To describe a circle touching a given circle in a given point, and also touching a given straight line.

1st. Externally. Let A be the given point, and B C the given straight line. Find the centre D of the given circle. Join D A and produce it. At A draw a tangent A E to the circle. Set off E F equal to E A. At F draw F O perpendicular to B C. O is the centre for the required circle.

2nd. To include the given circle. Draw the radius D A, and produce it in the opposite direction. Draw the tangent A E. Set off E F equal to E A. Draw F O perpendicular to F E, and from centre O with radius O F describe the circle.

PROBLEM 150.—To describe a circle which shall touch a given circle and a straight line at a given point.

1st. Externally. Let A be the point in the given line.

Find B the centre of the given circle, draw B C perpendicular to the given line, and produce it to D. At A draw a perpendicular. Draw D A cutting the circumference of the circle in E. From B through E draw a line meeting the perpendicular in F. With centre F and radius F A describe the circle.

2nd. To include the given circle. Find the centre B, and draw the perpendiculars from B and A as before. Join A D, and produce to cut the circumference in E. Draw E B, and produce to meet the perpendicular in F. With centre F and radius F A describe the circle.

PROBLEM 151.—To describe a circle of a given radius, touching two given circles externally.

Let A be the given radius. Find the centres B and C of the given circles, draw radii B D, C E and produce them. Set off D F, E G equal to A. With centre B and radius B F describe an arc. With centre C and radius C G intersect in O. With centre O and radius A describe the circle.

Note.—If the arcs were continued, the centre for another circle would be obtained.

PROBLEM 152.—To describe a circle of a given radius, touching two given circles and including them.

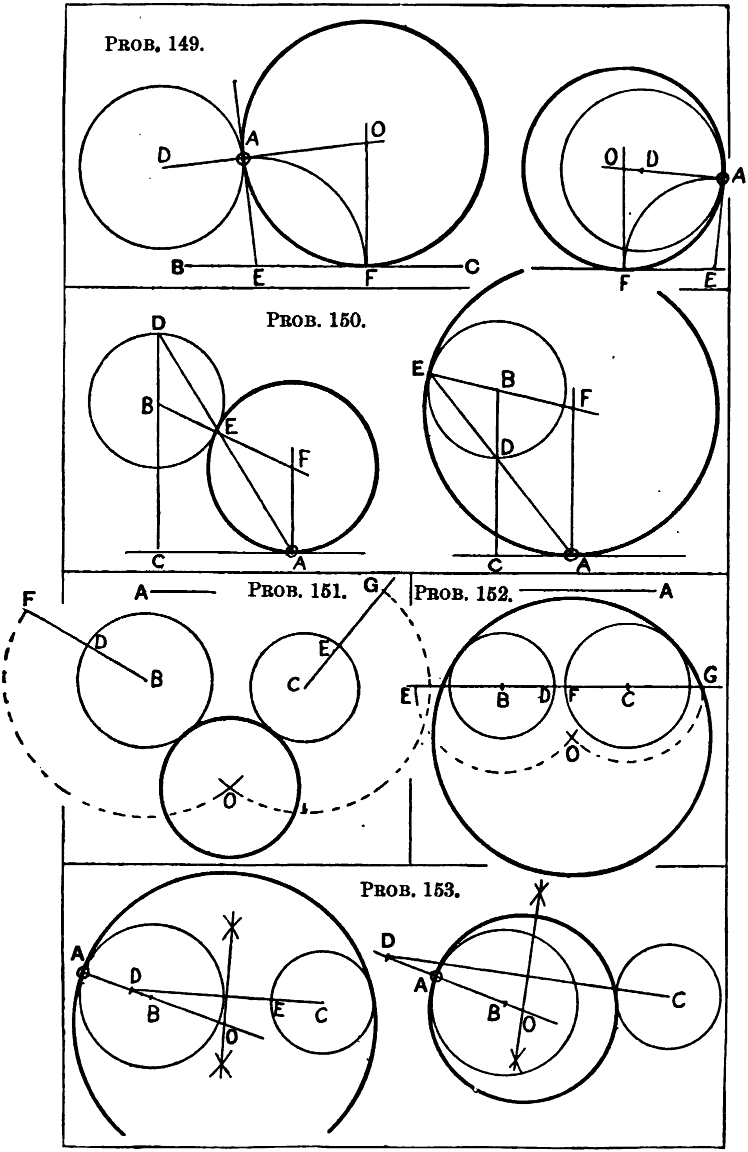
Join the centres of the given circles and produce each way. Set off D E and F G equal to the given radius A. With centres B and C, and radii B E and C G describe arcs intersecting in O. With centre O and radius A describe the circle.

Note.—As in the preceding problem, two circles may be described.

PROBLEM 153.—To describe a circle to touch two given circles, one of them in a given point.

1st. To include both circles. Find the centres B and C. Join A B and produce it. Make A D equal to C E, the radius of the other circle. Draw C D and bisect. O will be the required centre, and O A the radius.

2nd. To include one circle. Find the centres. Draw A B, and produce it both ways. Set off A D equal to the radius of the other circle. Join D C, and bisect it. With centre O and radius O A describe the circle.



PROB. 149.

PROB. 150.

PROB. 151.

PROB. 152.

PROB. 153.

PROBLEM 154.—To describe three circles touching each other, their radii being given.

Let A, B, C be the radii. Draw any line and from the extremity A set off A C, C B equal to A and B, and describe two of the circles. Set off C E, C D equal to C. From centres A and B with radii A D and B E describe arcs intersecting in O. With centre O and radius C describe the third circle.

PROBLEM 155.—To describe three circles touching each other, the position of their centres being given.

Let A, B, C be the centres of the circles. Join the points, and find the centre of the triangle thus formed. From this centre draw a perpendicular D E to one of the sides.

With centre B and radius B E describe a circle. With centre C and radius C E describe the second circle. With centre A and radius A F describe the third circle.

Note.—The centres must not be in the same straight line.

PROBLEM 156.—To inscribe within a given circle two other circles of given radii, touching each other and the given circle.

Let A and B be the given radii. Draw the diameter, and find the centre F of the given circle. Set off C D equal to A, and describe one of the circles. From points E and J set off E G and J H equal to B. With centre D and radius D G describe an arc. With centre F and radius F H intersect the arc in O. With centre O and radius B describe the other circle. If the arcs be continued on the other side of C J, the centre for another circle will be obtained.

PROBLEM 157.—To describe a circle which shall pass through two given points and touch a given circle.

Let A and B be the two given points. Describe any circle passing through A and B, and cutting the given circle in points C and D. Draw C D to meet A B produced in E. From E draw a tangent E F to the given circle (*Prob.* 69). F will be the point where the described circle will touch the given circle. Describe a circle passing through the points A, B, and F.

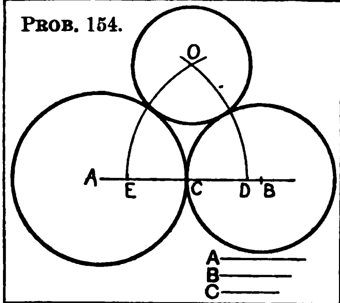
PROBLEM 158.—To describe a circle which shall pass through two given points and touch a given straight line.

1st. When the line joining the two points is parallel to the given line. Join the two given points A and B. Bisect A B by a perpendicular line meeting the given straight line in C. Draw A C and bisect it. With centre O and radius O C describe the circle.

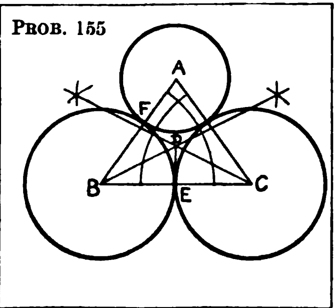
2nd. When the line joining the two points is at an angle to the given line.

Join the points A and B, and produce the line to meet the given line in the point C. Find a mean proportional between A C and C B by bisecting A B, describing a semicircle with D C as radius, and drawing the perpendicular A E. Make C F equal to A E. At F draw the perpendicular F O to meet the perpendicular from D. With centre O and radius O F describe the circle.

Prob. 154.



Prob. 155



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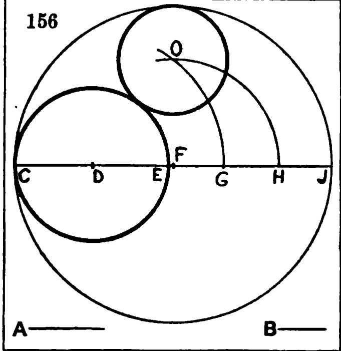

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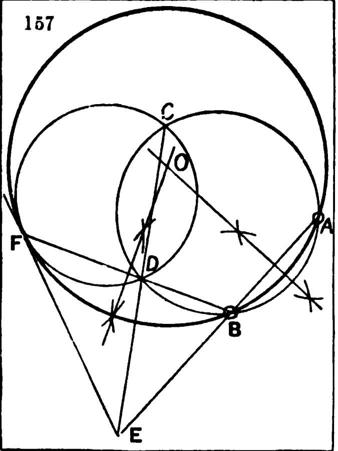
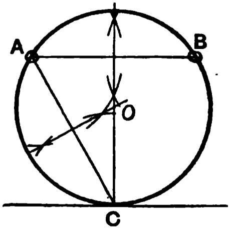
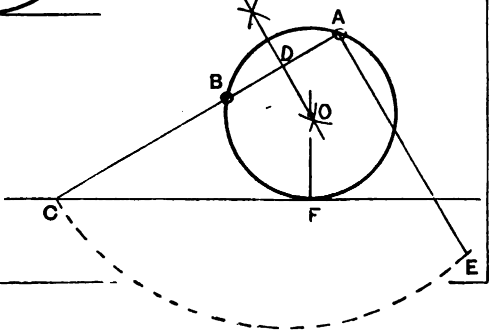




Prob. 158.



PROBLEM 159.—From a given point without a circle to draw a straight line cutting the circumference in two points, so that the segment intercepted between the two points shall equal a given distance.

Let A be the given point and B the given distance.

From a point C in the circumference set off C D equal to B, and draw the chord C D. Find the centre O of the given circle, and from it draw O E perpendicular to C D. With centre O and radius O E describe a circle. From A draw A G touching the described circle. Then F G will equal the given distance B, because all chords of a circle equidistant from the centre are equal. (Euc. iii. 14.)

PROBLEM 160.—Describe a circle of 1·25″ diameter touching each pair of adjacent lines *oa*, *ob*, *oc*, *od*, produced if necessary. Describe two circles touching the three circles.

Describe a circle touching each pair of lines, having a radius of ⅝″ by Problem 123. The only two circles that can touch the three are the circles touching them externally and internally.

From *o* draw a line passing through the centre of one of the circles. With centre *o* and radii *o e*, *o f* describe the circles.

Notes—1. If the angles between each pair of adjacent lines are unequal, then find the centre for the touching circles by bisecting the lines joining the centres of the circles. The point where the two bisecting lines meet will be the centre.

2. In working problems of a similar character to Problems 146–160, the following facts should be constantly borne in mind:

a. 'If two circles touch one another either internally or externally, the straight line, or the straight line produced, which joins their centres passes through the point of contact.' (Euc. iii. 11 and 12.)

b. 'Equal straight lines in a circle are equally distant from the centre; and those which are equally distant from the centre are equal.' (Euc. iii. 14.)

c. 'If a straight line touch a circle, the straight line drawn from the centre to the point of contact is perpendicular to the line touching the circle.' Also the converse of this. 'If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line.' (Euc. iii. 18 and 19.)

3. There is a large variety of problems of this kind, and in cases of difficulty a sketch of the required figure should be made, assuming it to be completed. Then by working backwards step by step, endeavour to find out what principles must be used to secure this result.

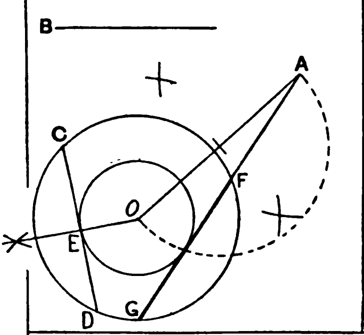
PROBLEM 161.—The given figure is made up of circular arcs, all of ¾″ radius. Draw it full size. (*Sc.* 1889.)

First draw the equilateral triangle A B C of 1⅜″ sides. With centres A, B, C and radius ¾″ describe arcs intersecting at D, E, F, giving three of the centres. The points *a*, *b*, *c* are obtained in each case by describing two circles tangential to those already obtained. Join the points D, E, F, and produce the lines each way. Set off on each line ¾″ as shown, and from the centres thus obtained describe arcs completing the figure.

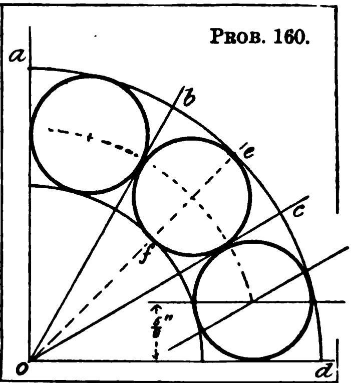
PROBLEM 162.—Draw the given geometrical pattern.

Describe a circle of 3½″ diameter. From the extremity of the diameter set off ⅛″ and describe the inner circle. Divide the circumference into six equal parts, and draw the diameters. From *a* set off 1¾″ and describe one of the smaller circles. Set off the centres for the other two inner circles and describe them. From the same centres describe the remaining circles. Omit the parts where the lines are not continuous.

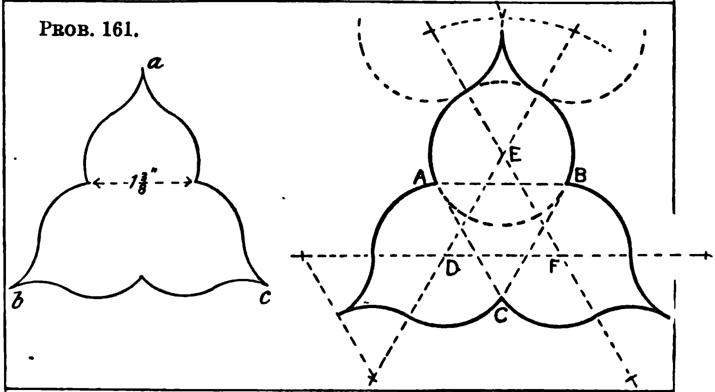
Prob. 159.



Prob. 160.



Prob. 161.



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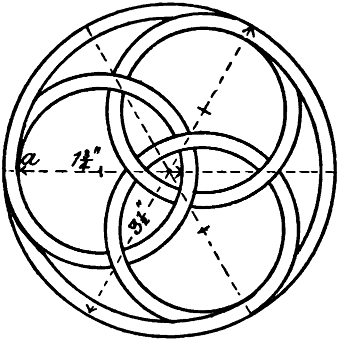


PROBLEM 163.—Draw the geometrical pattern shown, adhering strictly to the figured dimensions, and showing the construction lines.

It will be seen that the figure is formed by six equal circles, each touching two other circles. But when six equal circles are inscribed in a circle, a seventh equal circle may always be inscribed touching the six circles internally. The problem then resolves itself into describing a circle of ¾″ radius, placing six equal circles about it (*Prob.* 144), and omitting those portions of the circles which are not needed. Describe a circle of ¾″ radius, divide its circumference into 12 equal parts. Find the centres for the circles, and complete as shown.

Note.—Problems 161–168 indicate a few of the numerous ways in which the preceding problems on circles may be applied.

PROBLEM 164.—To inscribe a semicircle in an isosceles triangle.

Bisect the angle A C B by the line C D. Bisect the angle C D B by the line D E.

Draw E F parallel to A B, and on it describe a semicircle.

PROBLEM 165.—To inscribe three equal semicircles in an equilateral triangle having their diameters adjacent, and each touching one side of the triangle.

Bisect each angle of the triangle by the lines A D, B E, and C F.

Bisect the angle C F B by the line F G. From G draw G J, G H parallel to A B, B C. Join J and H. On the lines H G, G J, J H describe semicircles forming a *trefoil* of semicircular arcs.

PROBLEM 166.—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching one side of the square.

Draw the diagonals and diameters of the given square. Bisect one of the angles at A, and obtain the inner square. Describe a semicircle on each side of this square, forming a *quatrefoil* of semicircular arcs.

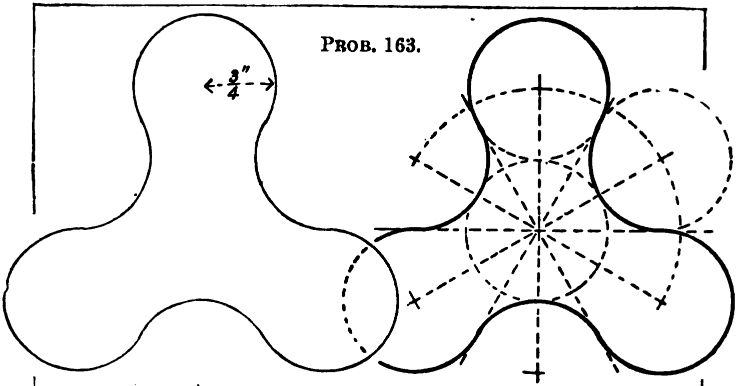
PROBLEM 167.—To inscribe within any regular polygon as many semicircles as the figure has sides, each touching one side and having their diameters adjacent.

Let A B C D E be the given polygon, in this case a pentagon. Divide the polygon into equal isosceles triangles. Inscribe a semicircle in each as before, forming a *cinquefoil* of semicircular arcs.

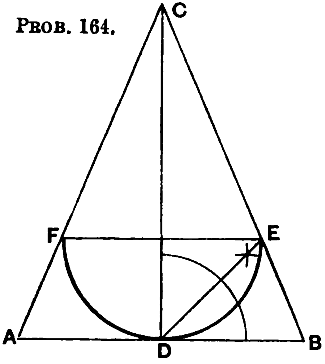
Notes.—1. In Problems 165–173 notice that the foiled figures are made by *semicircular* arcs. In Problems 174–177 they are formed by *tangential* arcs. The problems relating to the inscription and circumscription of circles and foiled figures are exceedingly useful in geometrical design, as they furnish the leading lines for window tracery and ornamental forms of various kinds.

2. Problems 165–168 depend upon the same principles of construction as those used in Problem 164.

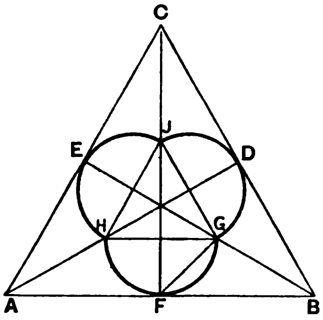
Prob. 163.



Prob. 164.



Prob. 165.



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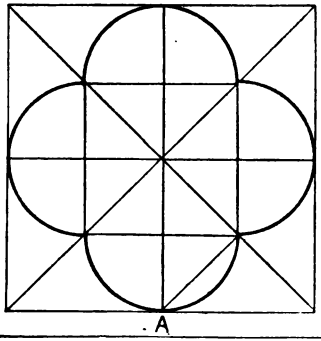

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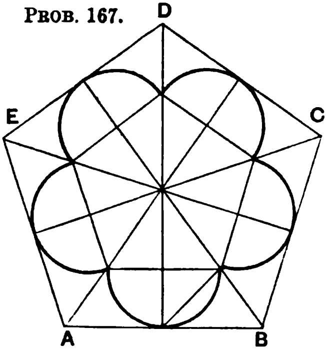


PROBLEM 168.—To inscribe any number of equal semicircles in a circle, having their diameters adjacent, and each touching the circumference.

Divide the circumference of the given circle into twice as many parts as there are semicircles required. (In this case four.) Draw the diameters. Draw a tangent at A and bisect the angle. Set off the distance O B on the alternate diameters, and draw the square. On each side of the square describe a semicircle.

PROBLEM 169.—To inscribe a semicircle in a square.

Draw the diagonals of the square, and on one of them A B describe a semicircle. From O the centre of the square draw O C at right angles to the side of the square. Draw C F, and from D where the line C F cuts the side of the square draw D E parallel to C O. Through E draw a line parallel to A B, and on it describe the required semicircle.

PROBLEM 170.—To inscribe four equal semicircles in a square, having their diameters adjacent, and each touching two sides of the square.

Draw the diagonals and diameters of the given square, and inscribe a semicircle in one of the four squares thus obtained by the preceding problem. Complete the inner square, and describe a semicircle on the other three sides.

PROBLEM 171.—To inscribe a semicircle in a trapezion, or kite.

Draw the diagonals of the given figure. On A B, the shorter diagonal, describe a semicircle. From O draw O C perpendicular to the side of the trapezion. Draw C F, and from D draw D E parallel to C O. Through E draw a line parallel to A B, and upon it describe the required semicircle.

PROBLEM 172.—To inscribe three equal semicircles in an equilateral triangle, having their diameters adjacent, and each touching two sides of the triangle.

Bisect each angle of the triangle by lines dividing the triangle into three equal trapezions. Inscribe a semicircle in the trapezion A F B G by the preceding problem. Complete the inner equilateral triangle, and upon the other two sides describe semicircles.

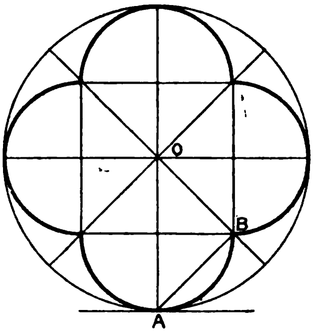
PROBLEM 173.—To inscribe in any regular polygon a number of equal semicircles, having their diameters adjacent, and each touching two sides of the polygon.

Let the given polygon be a regular hexagon. Draw the diameters and diagonals of the figure, cutting it into six equal trapezions, and inscribe a semicircle in each, as shown.

Notes.—1. Problems 170–173 depend upon the same principles of construction as those used in Problem 169.

2. Notice that in all cases the line joining the points C and F is drawn from C to the angle of the quadrilateral which is opposite to the semicircle described upon A B.

PROB. 168.



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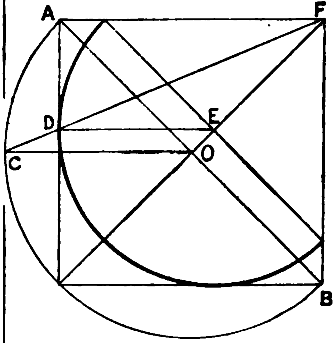

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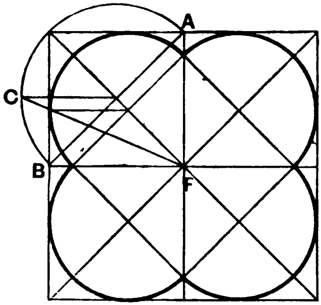

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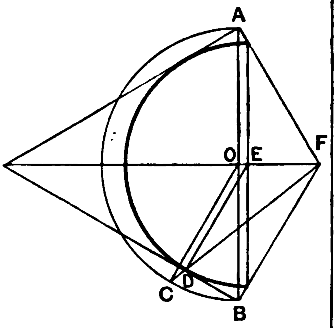

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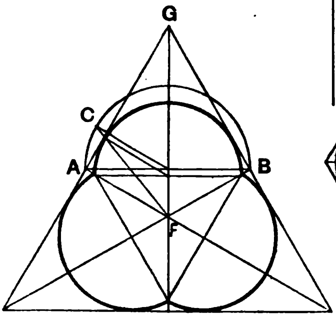

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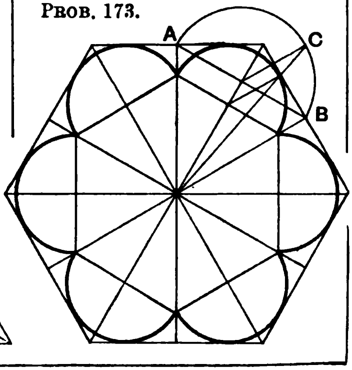


PROBLEM 174.—To describe a trefoil of tangential arcs, the radius of the arc being given.

Construct an equilateral triangle having each of its sides double the given radius. From each angle with the given radius describe the arcs.

PROBLEM 175.—To describe a quatrefoil of tangential arcs, the radius being given.

Construct a square having each of its sides double the given radius and describe the arcs as shown.

Note.—The same principle may be employed in the construction of all foiled figures formed by tangential arcs.

PROBLEM 176.—About any regular polygon, to construct a foiled figure of tangential arcs.

Let the given polygon be a regular hexagon. Bisect one side to obtain the radius, and proceed as in the preceding problems.

Note.—These problems are identical with the problems connected with the describing of a number of equal circles each touching two others. For example, in Problem 140, if the centres of the circles be joined, and the portions of the circles enclosed by the square thus formed be removed, we have the figure of Problem 175 inscribed in an octagon.

PROBLEM 177.—To draw the given geometrical pattern to the figured dimensions.

The inner arcs are identical with *Prob.* 176. Construct a hexagon having each side double the given radius (⅝″), and from each angle describe the tangential arcs. From the same centres with a radius of ¾″ describe the outer arcs.

CHAPTER XII

THE INSCRIPTION AND CIRCUMSCRIPTION OF RECTILINEAL FIGURES

PROBLEM 178.—To inscribe or describe an equilateral triangle in or about a circle.

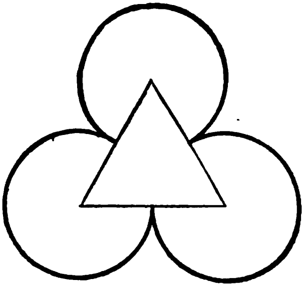
Draw a diameter A B, and find its centre. Set off A C, A D equal to the radius. Draw B C, B D, C D.

For the triangle about the circle find the points of the inscribed triangle, and through each draw a parallel to the opposite side.

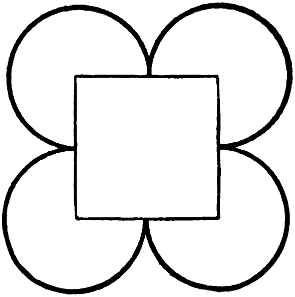
PROBLEM 179.—To inscribe or describe a square in or about a circle.

Draw two diameters at right angles to each other, and join the ends. For the square about the circle, with centres A, B, C, D, and radius A O, describe arcs intersecting without the circle and join the points; or through the points A, B, C, D draw parallels to the diagonals.

Prob. 174.



Prob. 175.



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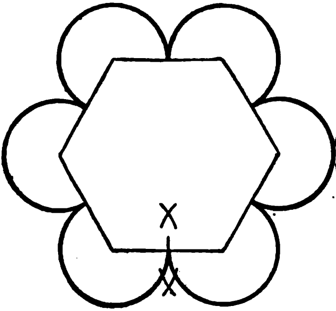

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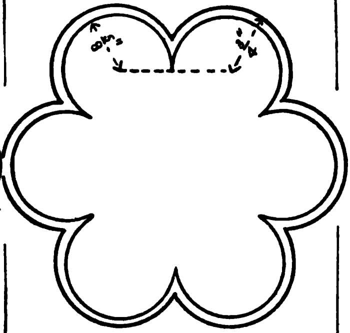

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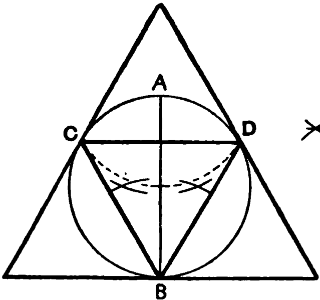

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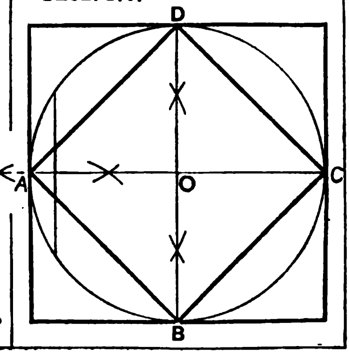


PROBLEM 180.—To inscribe an equilateral triangle in a square.

Draw the diagonal A B. On A B construct an equilateral triangle. From C draw C E, C D, parallel to the sides of the triangle. Join D and E.

PROBLEM 181.—To inscribe an equilateral triangle in a pentagon.

Find B, the middle point of one side, and join it with A, the opposite angle. On each side of A B construct an angle of 30° (*Prob.* 33), and join C D.

Notes.—1. This method may be applied to the square by making an angle of 30° on each side of the diagonal.

2. The triangle may also be obtained, as in Problem 180, by describing an equilateral triangle on E F, and drawing parallels from A.

PROBLEM 182.—To describe an equilateral triangle about a square.

On one side A B of the given square describe an equilateral triangle A B E. Produce E A, E B to meet the side C D produced.

PROBLEM 183.—About a given square to describe a triangle similar to a given triangle.

Let A B C D be the given square and E F G the given triangle. This problem depends upon exactly the same principle as the preceding. On A B construct a triangle similar to the triangle E F G. Produce the sides to meet C D produced.

PROBLEM 184.—About a given triangle to describe another triangle similar to a given triangle.

On A B construct a triangle similar to the triangle D E F.

Through C draw a line parallel to A B, and produce G A, G B to meet it.

PROBLEM 185.—Within a given triangle to inscribe another triangle similar to a given triangle.

On A C, a side of the given triangle, construct a triangle similar to the triangle D E F. Draw G B. From H draw H J and H K parallel to G C and G A. Join J K. Then H J K is the required triangle.

PROBLEM 186.—Within a given circle to inscribe a triangle similar to another triangle.

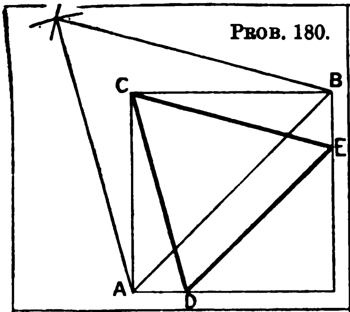
At any point D in the circumference of the given circle draw a tangent. Make the angle E D F equal to the angle A B C, and the angle G D H equal to the angle B C A. Join F H. Then D F H is the required triangle, the angle at F being equal to the angle at C, and the angle at H to the angle at B. (Euc. iv. 2.)

PROBLEM 187.—Within a given circle to inscribe a quadrilateral figure similar to a given one.

Note.—A quadrilateral figure can only be inscribed in a circle when the sum of the opposite angles equals two right angles.

Let the given quadrilateral have angles of 100°, 70°, 80°, and 110°, as shown.

Through any point E draw a tangent F G. Make the angle H E G equal to the angle A D C, G E J equal to C A B, and F E K equal to C A D. Draw J H, H K.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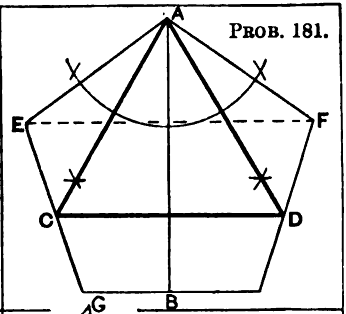
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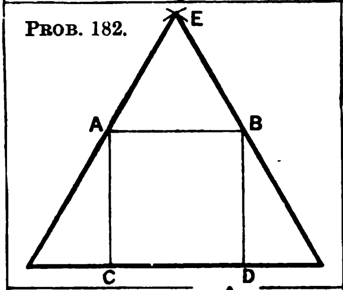
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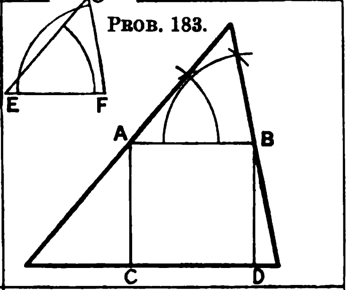
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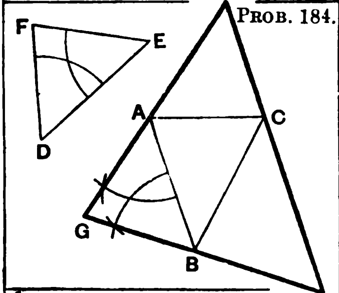
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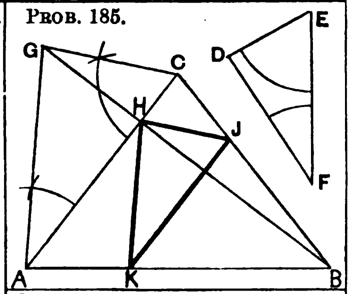
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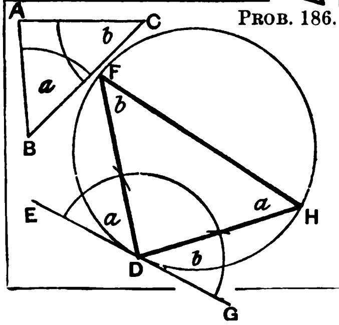
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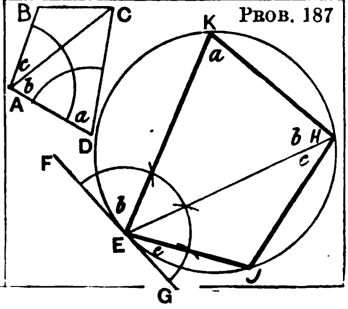

PROB. 187

PROBLEM 188.—About a given circle to describe a triangle similar to a given triangle.

Produce the base A B of the given triangle. Find the centre O of the given circle, draw any radius O G, and produce it. Construct the angle F O G equal to the exterior angle C B D, and the angle H O G equal to the exterior angle C A E. Produce O H, O F, and draw tangents as shown. (Euc. IV. 3.)

Note.—Angle F O G + angle F J G = angle C B D + angle C B A = two right angles, because the angles at F and G are right angles.

PROBLEM 189.—In a given square to inscribe an isosceles triangle, the base being given.

Draw the diagonals A C and B D of the given square, and on A C set off A F equal to the given base E. Draw F G parallel to A B and G H parallel to A F. Draw D H and D G.

Note.—G H = A F. (Euc. I. 34.)

PROBLEM 190.—To describe a square about an isosceles triangle,

Bisect the base B C of the given triangle by the line A D. On B C describe a semicircle. Then A D will be a diagonal of the required square. Draw D C and D B of indefinite length, and from A draw parallels to meet them.

PROBLEM 191.—In a given hexagon to inscribe an isosceles triangle, the base being given.

Draw the diagonal A B of the given hexagon. Draw C D, one of the diameters, at right angles to A B. Set off D F equal to E. Draw F G parallel to B D, and G H parallel to C D. Draw A G and A H.

PROBLEM 192.—Within a given circle to inscribe an isosceles triangle, the base being given.

Draw two diameters of the given circle, A B and C D, at right angles to each other. Make O F equal to half the given base E. Draw F G parallel to A B, and G H parallel to C D. Draw A G and A H.

Note.—A similar method may be used for inscribing an isosceles triangle in a square, rhombus, or polygon.

PROBLEM 193.—To inscribe a square in a triangle.

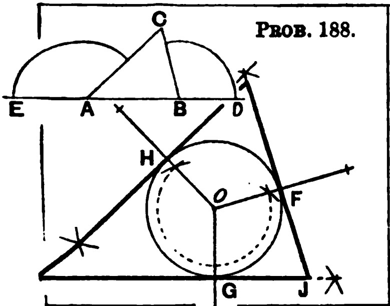
Draw C D perpendicular to A B. From C draw C E parallel to A B and equal to C D. Draw A E. From F draw F G parallel to A B, and F H parallel to C D. From G draw G J parallel to C D. Then G F H J is the required square.

PROBLEM 194.—To inscribe a square in a trapezion.

Draw the diagonals A B, C D. Draw C E parallel to A B, and equal to C D. Draw A E. From F draw F G parallel to A B, and F H parallel to C D. From G draw G J parallel to C D, and join J and H.

PROBLEM 195.—To inscribe a square in a pentagon.

If two sides of the pentagon be produced until they meet, a trapezion will be formed. The same construction as in *Prob.* 194 will then apply.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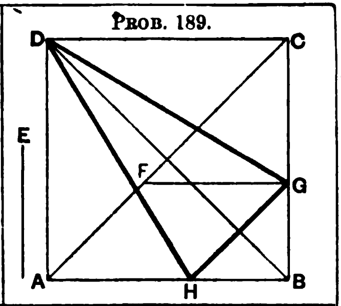
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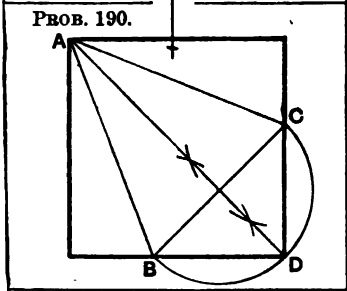
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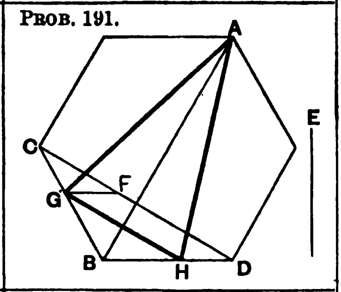
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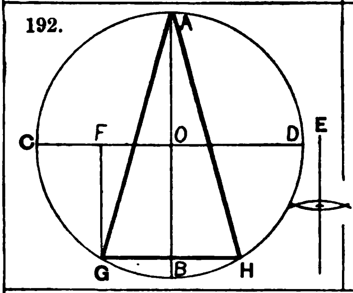
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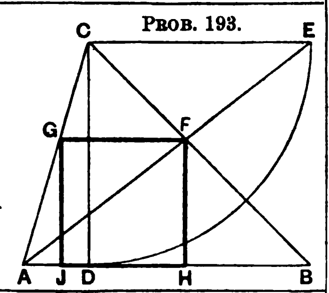
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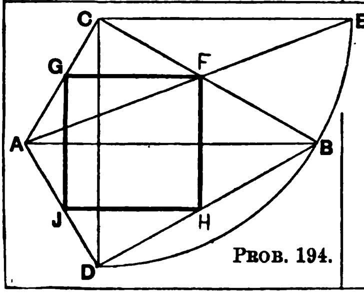
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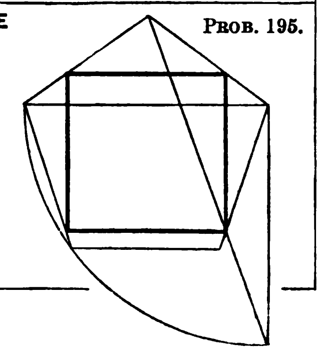

Prob. 195.

PROBLEM 196.—To inscribe a square in a sector.

Join B and C. Draw C D perpendicular to C B and equal to it.

Draw A D, and from E draw E G parallel to B C, and E F parallel to C D.

Draw G H and F H parallel to E F and E G.

PROBLEM 197.—To inscribe a square in a segment.

Bisect the chord A B of the segment. Draw B D equal to A B and perpendicular to it. Draw D C. From E draw E F parallel to B D and E G parallel to A B. Draw G H parallel to E F.

PROBLEM 198.—Within a given square to inscribe another square, one angle to touch a side at a given point.

Let A be the position of one angle. Draw the diagonals of the given square.

With centre O and radius O A describe a circle. Join the points A, B, C, and D.

Note.—If the length of the diagonal be given, proceed in a similar manner, taking half the given diagonal as radius.

PROBLEM 199.—To inscribe a square in a rhombus.

Draw the diagonals, and bisect the angles thus formed.

Join A, B, C, and D.

PROBLEM 200.—To inscribe a square in a hexagon.

Draw the diagonal A B, and bisect it at right angles by the diameter C D.

Complete as in the preceding problem.

Note.—To inscribe a square in an octagon, join the alternate corners.

PROBLEM 201.—To inscribe a rhombus in a parallelogram, having one of its angles at a given point.

Let A be the given point. Draw the diagonals of the parallelogram.

From A draw A B passing through the centre O of the parallelogram.

Bisect A B by a line C D at right angles to it. Draw A C, C B, B D, and D A.

PROBLEM 202.—To inscribe a regular hexagon in an equilateral triangle.

Bisect each of the angles of the given triangle by the lines A E, B F, and C D. With centre O and radius O A describe a circle. Draw D E, E F, and F D.

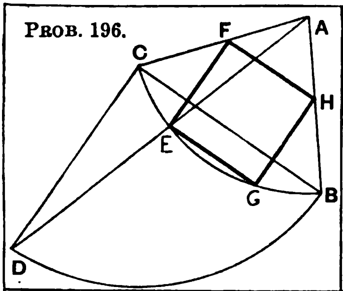
PROBLEM 203.—Within a given triangle to inscribe a rectangle, the length of one side being given.

Let A B C be the given triangle and D the given side of the rectangle.

Set off A E equal to D. Draw E F parallel to A C.

From F draw F G parallel to A B. Draw F H and G J perpendicular to A B.

Prob. 196.



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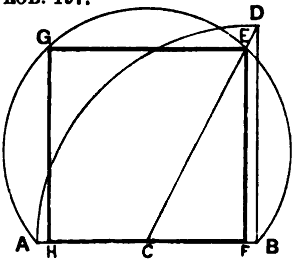

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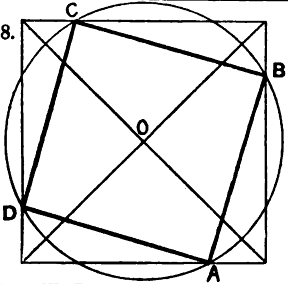

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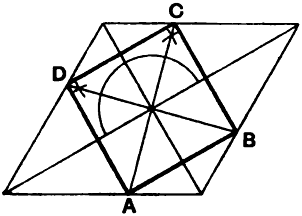

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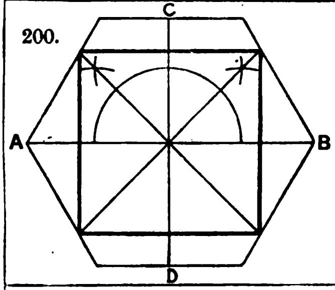

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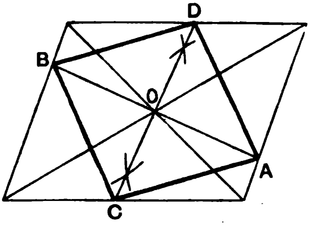

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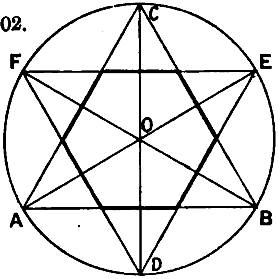

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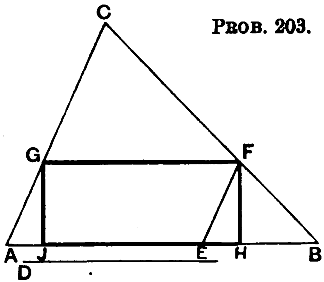


PROBLEM 204.—Within any given quadrilateral to inscribe a parallelogram, having given the position of one angle.

Let E be the position of one angle. Draw the diagonals A C, B D. Draw E F parallel to A C, E H and F G parallel to B D. Join G and H. E F G H will be the required parallelogram.

PROBLEM 205.—Within any given quadrilateral to inscribe a parallelogram, having given the length of one side.

Let E be the length of one side. Draw the diagonals A C, B D. On one of them set off A F equal to E.

From F draw F H parallel to A B. Draw H G parallel to A C, H K and G J parallel to B D. Join J K.

Notes.—1. G H = A F. (Euc. I. 34.)

2. The same construction will apply for inscribing a rectangle in a square, rhombus, or trapezoid.

PROBLEM 206.—Within a given triangle or any regular polygon to inscribe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

Let A B C be the given triangle, and D the length of one of the sides of the required triangle. Bisect the angles and obtain the centre E. Set off A F equal to D. Draw F G parallel to A E, G H parallel to A B, H J parallel to A C, and G J parallel to B C.

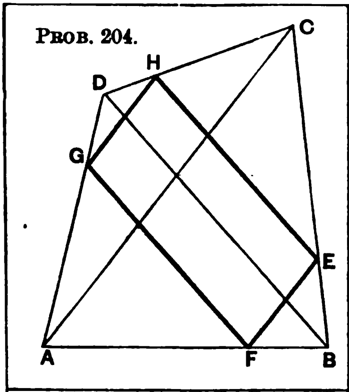
Note.—The construction for the inscription of a hexagon within a hexagon is also shown, A being the length of one side of the inscribed figure.

PROBLEM 207.—About a given triangle or any regular polygon to describe another similar figure, having its sides parallel to and equidistant from those of the given figure, the length of one side being given.

Let A B C be the given triangle, and D the length of one of the sides of the required triangle.

Find the centre as before, and produce the lines bisecting the angles. Produce A B, and from A set off A F equal to D. Draw F G parallel to A E, G H parallel to A B, H J parallel to A C, and G J parallel to B C.

Note.—The construction for the description of a square about a square is also shown.



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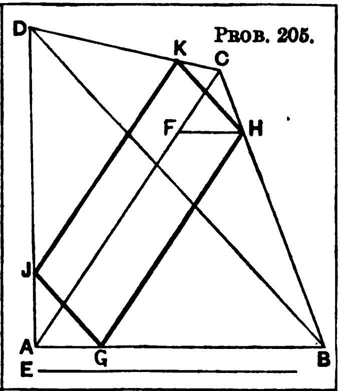

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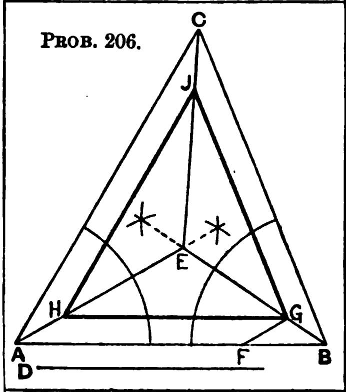

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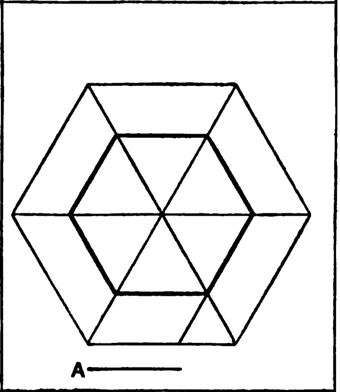

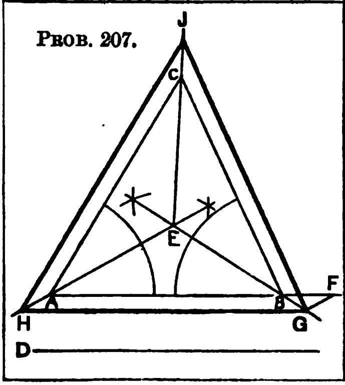

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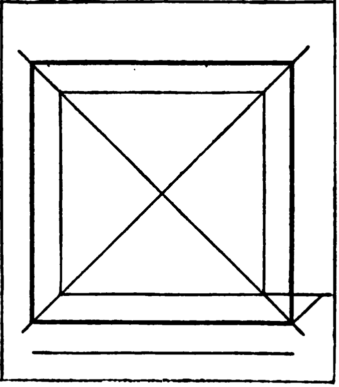


EXERCISES

CHAPTER XI

1. To describe a circle touching two lines and passing through a point between them. (*Art.*) (*Prob.* 125.)

2. Describe a circle of 3″ diameter, and within it inscribe six equal semicircles. (*Art.*)

3. Describe two circles touching each other and having radii of 1″ and ¾″ respectively. Draw a third circle having a radius of ½″ to touch the other two circles. (*Art.*) (*Prob.* 152.)

4. Within a given equilateral triangle of 3″ sides inscribe three equal semicircles, having their diameters adjacent and each touching one side of the triangle. (*Art.*)

5. Within a given circle of 1½″ radius inscribe four equal circles. (*Art.*)

6. Within a square of 2⅞″ sides inscribe four equal semicircles each touching two sides of the square. (*Art.*)

7. Within a square of 2½″ sides inscribe four equal circles, each touching two sides and two circles. (*Art.*)

8. Describe a circle of ¾″ radius which shall touch a given circle and a given straight line. (*Art.*) (*Prob.* 148.)

9. Construct a quatrefoil of tangential arcs of ½″ radius. (*Art.*)

10. Describe a circle to enclose two other given circles. (*Art.*)

11. Describe a triangle with sides of 3″, 2″, and 2½″ respectively, and within it inscribe three circles each touching one side and two circles. (*Prob.* 132.)

12. Within a given isosceles triangle inscribe two equal circles touching each other and two sides of the triangle. (*Art.*)

13. Two parallel lines **A B**, **C D** are 2″ apart and 1½″ long. Describe a circle to pass through points **A** and **B** and touch **C D**. (*Art.*)

14. Within a given pentagon inscribe five tangential arcs each touching two sides of the pentagon. (*Art.*) (*Note*, *Prob.* 139.)

15. About a square of 1″ sides describe four equal circles each touching a side of the square and two circles. (*Art.*) (*Prob.* 145.)

16. Describe a circle ¾″ radius to pass through a point **A** and touch a given straight line. (*Art.*)

17. Construct a rhombus of 2¾″ sides and within it inscribe four equal circles each touching one side and two circles.

18. Describe a triangle about a circle of 1″ diameter, having angles of 30° and 105°.

19. About a circle of ½″ radius place five equal circles each one touching two others and the given circle.

20. Draw two right lines meeting at an angle of 38°. Describe a circle of ¾″ radius to touch these lines. (*Sc.*, 1877.)

21. Draw three equal circles of ·75″ radius, each touching the other two. (*Sc.*, 1870.)

22. Two circles of 1″ and 5″ radius respectively have their centres 2·5″ apart. Draw a circle of 1·5″ radius to touch both, but to contain the smaller one (*Sc.*, 1871.) (*Prob.* 158.)

23. Draw three circles, each touching the other two, their radii being ·5″, ·75″, and 1″, respectively. (*Sc.*, 1876.) (*Prob.* 154.)

24. Construct an equilateral triangle of 2¼″ sides. On each side as diameter describe a circle. Circumscribe the three circles by a circle. (*Sc.*, 1877.)

25. Describe a circle of 2½″ radius touching two given circles of 1″ and ¾″ radius, and having their centres 2¼″ apart. (*Sc.*, 1877.)

26. Inscribe a circle in a rhombus of 2″ side and 2¼″ diagonal. (*Sc.*, 1878.)

27. Draw three circles of 1″, 1·25″, and 1·5″ diameter, each touching the other two externally. Draw a circle which shall be touched internally by the largest and smallest of these three circles. (*Sc.*, 1881.) (*Prob.* 152.)

28. Draw two circles touching the same straight line at points 2·5″ apart and touching one another, the radius of the smaller circles to be 1″. (*Sc.*, 1882.) (*Prob.* 150.)

29. Construct a square of 4¼″ side, and place in it four equal circles, each touching one side and two diagonals. (*Sc.*, 1883.)

30. Draw two lines cutting each other at 57°, and describe four circles of 2⅛″ diameter, each touching both lines. (*Sc.*, 1883.)

31. Describe a circle passing through two given points *a* and *b* and touching a given line *c d*. The line joining *a b* is not parallel to *c d*. (*Sc.*, 1884.) (*Prob.* 158.)

32. Describe a circle passing through a point *p* and touching a line *a b* in a given point *c*. (*Sc.*, 1886.) (*Prob.* 121.)

CHAPTER XII

1. Draw a triangle and within it inscribe a square.

2. Within a square of 2″ sides inscribe the largest possible isosceles triangle having its base ½″ long. (*Art.*) (*Prob.* 189.)

3. Draw a trapezion with sides of 1″ and 2″ respectively, and within it inscribe a square. (*Art.*)

4. Within a square of 2″ sides inscribe the largest possible equilateral triangle. (*Art.*)

5. About a regular pentagon of 1″ sides describe a similar figure, having its sides parallel and equidistant to those of the given figure, and 1½″ in length. (*Art.*)

6. Within an equilateral triangle of 3″ sides inscribe a similar figure, base 1¼″. (*Art.*)

7. About a circle of 1½″ diameter describe an equilateral triangle. (*Art.*)

8. Within a circle of 1¼″ radius inscribe a triangle having angles of 30° and 60°. (*Art.*)

9. Construct a parallelogram, sides 2½″ and 1¾″, included angle 50°, and within it inscribe a rhombus having one angle touching one side of the parallelogram at a point ½″ from one of the corners. (*Art.*)

10. About an isosceles triangle describe a square. (*Prob.* 190.)

11. Within a given circle of 3″ diameter inscribe an isosceles triangle having its equal sides 2½″ long. (*Art.*)

12. Construct an isosceles triangle, the two equal sides to touch the circumference of a given circle at two given points **A** and **B**; the angle made by the radii from **A** and **B** to be 100°.

13. Draw a triangle two of whose angles are 50° and 65°, and the radius of the inscribed circle 1″. (*Sc.*, 1870.)

14. Within a square of 3″ sides inscribe an octagon, so that the alternate sides of the octagon shall coincide with the sides of the square. (*Sc.*, 1871.)

15. Construct a quadrilateral base 3″, base angles 90° and 75°, sides 2″ and 2″. Within it inscribe a parallelogram having a side of 2″.

CHAPTER XIII

AREAS

Before commencing the problems on areas, the following principles should be thoroughly understood; and it will be of considerable service to the student to go through Euclid's demonstrations of these principles:—

1. The **area** of a plane figure is the amount of surface enclosed by its boundary, or perimeter. It depends upon both the shape and the perimeter of the figure.

2. **Parallelograms** upon the **same base**, and between the same parallels, are equal. (Euc. I. 35.)

A B C D = A B D E. (Fig. 1.) F G H J = F G K L. (Fig. 2.)

3. **Parallelograms** upon **equal bases**, and between the same parallels, are equal. (Euc. I. 36.)

M N O P = Q R S T, because each of them is equal to M N S T. (Fig. 3.)

4. **Triangles** upon the **same base**, and between the same parallels, are equal. (Euc. I. 37.)

A B C = A B D. (Fig. 4.)

5. **Triangles** upon **equal bases**, and between the same parallels, are equal. (Euc. I. 38.)

A B C = D E F. (Fig. 5.)

6. If a parallelogram and a triangle be upon the same base and between the same parallels, the parallelogram shall be double of the triangle. (Euc. I. 41.)

A B C D = twice A B C. (Fig. 6.) E F G H = twice E F J. (Fig. 7.)

7. The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. (Euc. I. 47.)

The sq. C B D E = the sq. A B F G + the sq. A H J C. (Fig. 8.)

Note.—The same principle applies to other figures constructed upon the sides of a right-angled triangle as long as they are similar. (Euc. VI. 31.)

8. The area of a triangle is equal to the area of a rectangle upon the same base, but having half the altitude.

Triangle A B C = rectangle A B E F. A E = $\frac{1}{2}$ C D. (Fig. 9.)

9. **Parallelograms** and **triangles** upon the **same base** have their areas in the same ratio as their altitudes.

A B E F = twice A B C D, because the altitude B E = twice the altitude B G. (Fig. 10.)

A B D = three times A B C, because the altitude D E = three times the altitude C E. (Fig. 11.)

10. **Parallelograms** and **triangles** of the **same altitude** are to one another as their bases. (Euc. VI. 1.)

E B F G = $\frac{2}{5}$ A B C D, because E B = $\frac{2}{5}$ A B. (Fig. 12.)

D B E = $\frac{1}{4}$ A B C, because D B = $\frac{1}{4}$ A B. (Fig. 13.)

11. The **areas** of **similar** figures are proportional to the squares on their homologous, or corresponding, sides. (Euc. VI. 19, 20.)

A B C D E : F G H J K as $AB^2 : FG^2$. (Fig. 14.)

12. The **areas** of **circles** are proportional to the squares on their diameters.

FIG. 1.

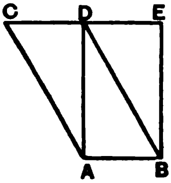


FIG. 2.

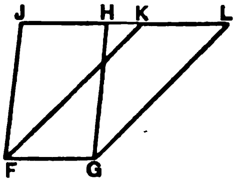
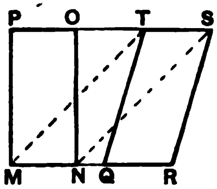


FIG. 3.



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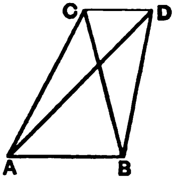

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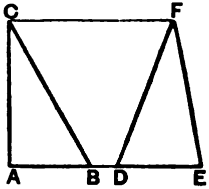

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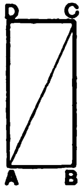
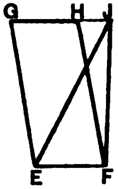


FIG. 7.



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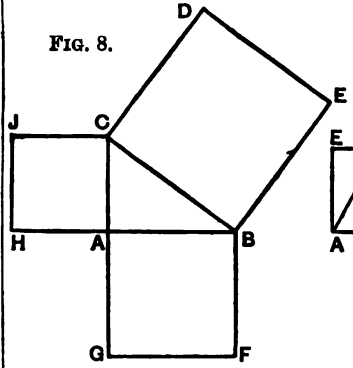

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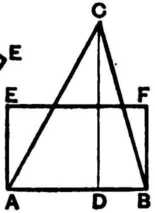


FIG. 10.

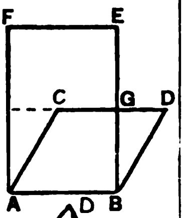


FIG. 11.

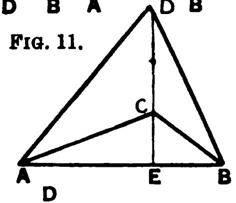
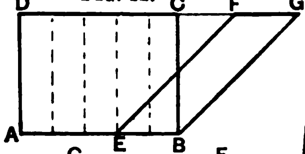


FIG. 12.



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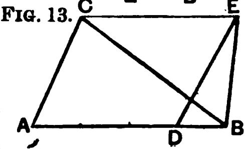

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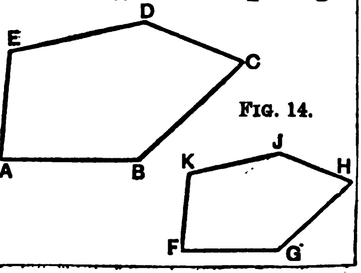


PROBLEM 208.—To construct a triangle equal in area to any parallelogram.

Set up twice the altitude of the given figure and join with the extremities of the base as shown in the first two figures, or double the base and keep the same altitude as in the third figure. The triangle A B C in each case is equal to the given parallelogram. The second figure shows the construction if an isosceles triangle be required.

PROBLEM 209.—To construct a parallelogram equal in area to a triangle.

Draw the altitude of the given triangle, and bisect it by the line D E parallel to A B. At A and B draw perpendiculars. Then A B D E is the rectangle equal to the given triangle. A B F G is a rhombus equal to A B C.

PROBLEM 210.—To construct a triangle equal in area to a given trapezium.

Let A B C D be the given trapezium. Draw the diagonal D B, and from C draw C E parallel to D B to meet A B produced in E. Draw D E. Then A D E is the required triangle.

Note.—D B E = D B C (Euc. I. 37). Add A B D to each. Then A D E = A B C D.

PROBLEM 211.—To construct a triangle equal in area to an irregular pentagon.

Draw D A, D B. From E draw E F parallel to D A and meeting A B produced in F. Draw D F. From C draw C G parallel to D B and meeting A B produced in G. Draw D G. Then F D G is the required triangle.

PROBLEM 212.—To construct a triangle equal to any irregular polygon.

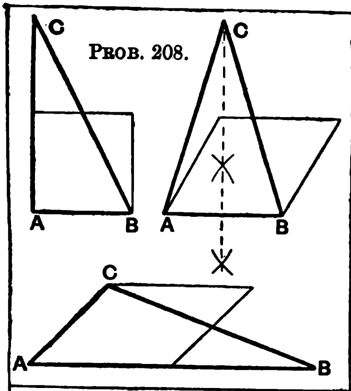
Let A B C D E F be the given polygon. Draw E A. From F draw F G parallel to A E. Draw E G. (The figure G B C D E = A B C D E F.) Draw D B, and from C draw C H parallel to D B. Join D and H. (G H D E = G B C D E.) Draw E H, and from D draw D J parallel to E H. Join E and J. Then G E J is the required triangle.

PROBLEM 213.—To construct an isosceles triangle equal to a trapezium, one side to be common to both figures.

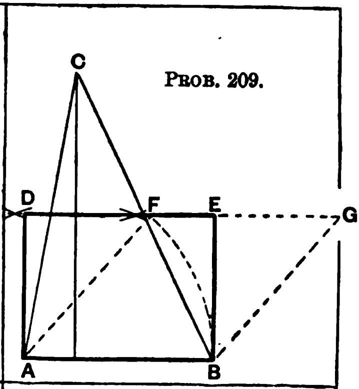
Let A B C D be the given trapezium and A B the side common to both figures. Draw A D, and from C draw C E parallel to A D and meeting B D produced. Join A and E. Then the triangle A B E is equal to the trapezium. To get an isosceles triangle equal to it, bisect the base by the perpendicular F G, and through E draw E G parallel to A B. Draw G A and G B. Then A B G is the required triangle.

Note.—In the case of a pentagon proceed in a similar manner, first converting the pentagon into a trapezium on A B.

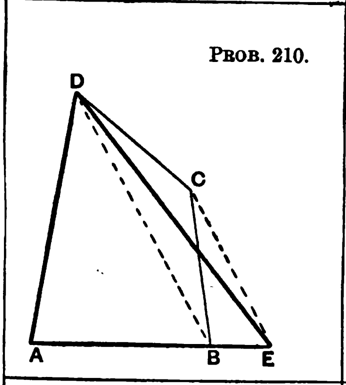
PROB. 208.



PROB. 209.



PROB. 210.



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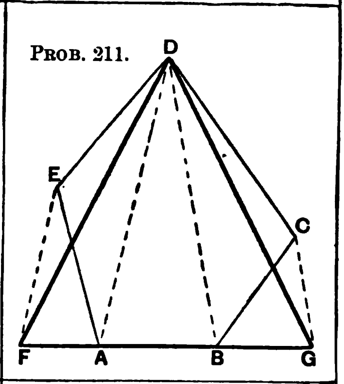

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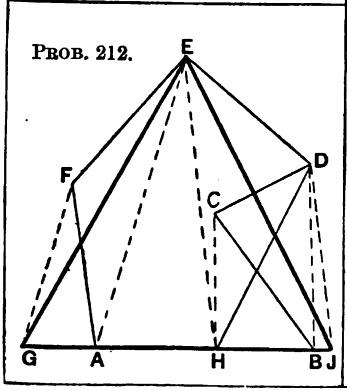
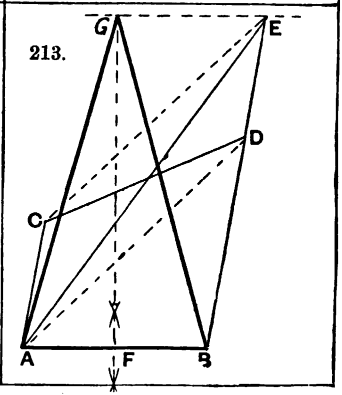


213.



PROBLEM 214.—To construct a triangle equal in area to the sum of two given triangles.

Let A B C and D E F be the given triangles. Make the triangle C E F equal to the given triangle D E F, forming an irregular pentagon, A B E F C. Draw A F, and from C draw C H parallel to A F, and meeting A B produced. Join F and H. Draw F B, and from E draw E G parallel to F B. Join F and G. Then F G H is the required triangle.

PROBLEM 215.—To construct a triangle equal to any regular polygon.

Let A B C D E be a regular pentagon. Find the centre, and divide the polygon into five equal triangles. Make the base G H equal to five times the base A B. Draw F G, F H. Then F G H is the required triangle.

Notes.—1. Another method is to proceed as in Problem 211.

2. Where the polygon has a larger number of sides, the altitude of the triangle may be doubled, and the base made equal to half the perimeter of the given polygon.

PROBLEM 216.—To construct a triangle equal in area to a circle. (Approximate.)

Draw A B, the diameter of the circle. Divide the radius A C into 7 equal parts. Draw A D perpendicular to A B, and make it $3\frac{1}{7}$ times A C. Draw B D. A B D is the required triangle.

PROBLEM 217.—On a given base to draw a triangle equal in area to another given triangle.

Let A B C be the given triangle, and D the given base. On A B or A B produced set off A E equal to D. Draw C E, and from B draw B F parallel to C E. Join F and E. Then A F E is the required triangle. (B F C = F B E.)

PROBLEM 218.—To construct a triangle equal in area to a given triangle, having its vertex in a given point, and its base in the same straight line as that of the given triangle.

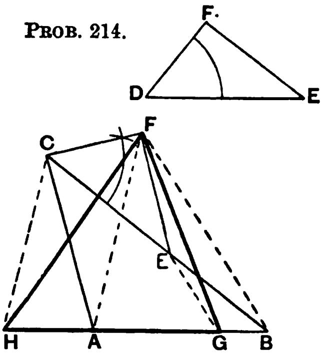
Let A B C be the given triangle, and D the given point. Draw A D. From C draw C E parallel to A B, and join B and E. Then the triangle A B E = the triangle A B C. The Problem now resolves itself into the same as the previous Problem—that is, to construct a triangle on A D equal to the triangle A B E. Draw D B, and from E draw E F parallel to D B. Join D and F. Then A D F is the required triangle.

PROBLEM 219.—To construct a triangle of a given altitude equal in area to another given triangle.

Let A B C be the given triangle, and D the given altitude. Draw C E, the altitude of the given triangle, and on it mark off E F equal to D. Draw F A, and from C draw C G parallel to F A. Join F and G. Draw F B, and from C draw C H parallel to F B. Join F and H. Then G F H is the required triangle.

Note.—A F G = A F C, and B F H = B F C.

PROB. 214.



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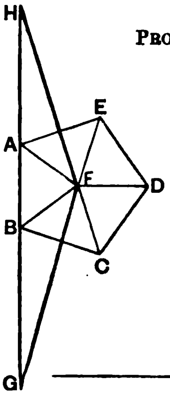
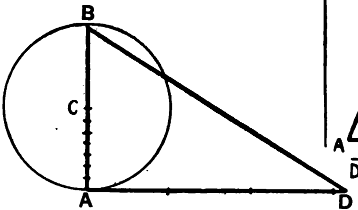


PROB. 216.



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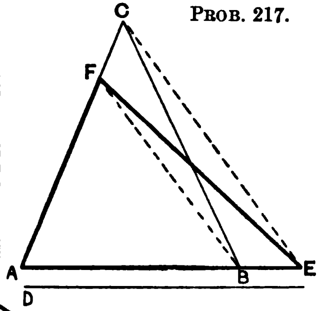
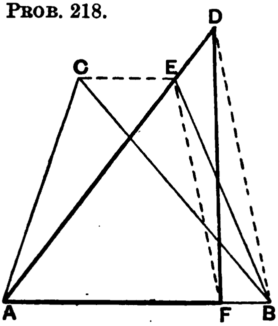
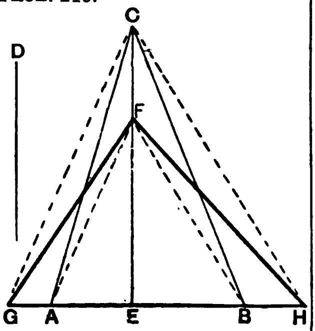


PROB. 218.



PROB. 219.



PROBLEM 220.—To construct a square equal in area to the sum of three squares.

Let A, B, and C be the lengths of the sides of the given squares. Draw E F equal to A, and E D at right angles to E F, and equal to B. Join F and D.

Then the square on D F is equal to the sum of the squares on D E and E F. (Euc. I. 47.) Draw D G at right angles to D F, and equal to C. Join F and G. On G F describe a square.

PROBLEM 221.—To construct a square equal in area to the difference between two given squares.

Let A and B be the sides of the given squares. Draw two lines at right angles to each other. Make C D equal to B. With C as centre and A as radius mark off E. On D E construct the square.

Notes.—1. If the square on C E = the sum of the squares on C D and D E, then the square on D E must be equal to the difference of the squares on C E and C D.

2. The same principles may be applied to the circle, or any rectilineal figure. An equilateral triangle, polygon, &c., may be constructed equal in area to the sum or the difference of two similar figures.

PROBLEM 222.—To describe a circle equal in area to the sum of two given circles.

Draw C D and D E perpendicular to each other, and equal to A and B, the diameters of the given circles. Draw C E, the diameter of the required circle.

Note.—To describe a circle equal in area to the difference between two circles, proceed as in Problem 221. C D will be the smaller diameter, C E the larger, and D E the diameter of the required circle.

PROBLEM 223.—To construct a triangle similar to a given triangle, but having twice its area.

Let A B C be the given triangle. Draw A D perpendicular to A B, and equal to it. Make B E equal to B D. From E draw E F parallel to A C, and meeting B C produced. Then E B F is the required triangle.

Note.—The same principle may be applied to any other rectilineal figure. A trapezium is shown similar to, and double the area of, a given trapezium.

PROBLEM 224.—To construct a trapezium similar to a given trapezium, having half its area.

Let A B C D be the given trapezium. Bisect A B, and describe a semicircle. Draw B H, and make B E equal to it. Draw B D, and from E draw E F parallel to A D, and from F draw F G parallel to D C. Then B E F G is the required figure.

Note.—The same principle applies to other rectilineal figures. The triangle B E F is half the triangle B A D.

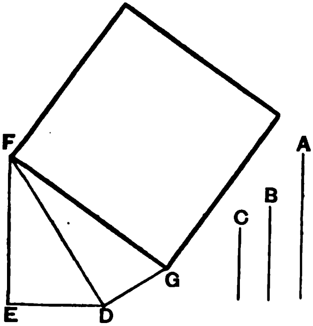
PROBLEM 225.—To construct a square having twice the area of a given square.

Draw the diagonal, and describe a square on it.

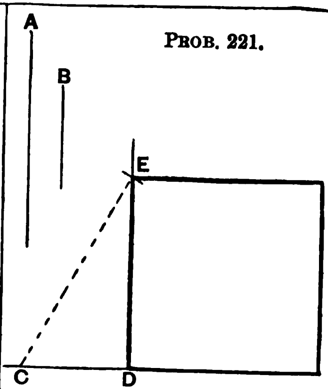
PROBLEM 226.—To construct a square having half the area of a given square.

On half the diagonal describe a square.

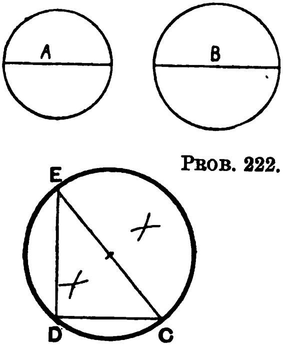
Prob. 220.



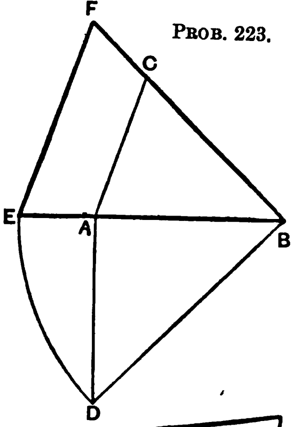
Prob. 221.



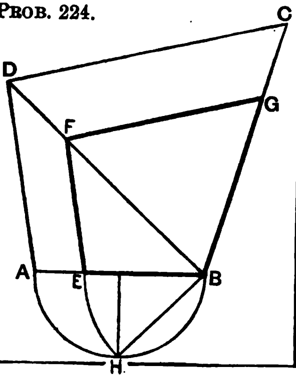
Prob. 222.

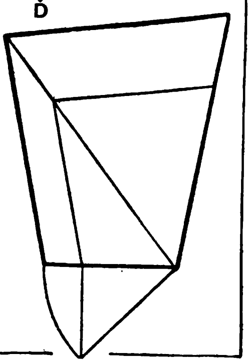


Prob. 223.



Prob. 224.





PROBLEM 227.—To construct a square equal in area to any given parallelogram.

Let A B C D be the given parallelogram. Produce A B, and make B F equal to the altitude of the parallelogram. Find B G, a mean proportional between A B and B F. (*Prob.* 29.) On B G describe the square.

PROBLEM 228.—To construct a square equal in area to a triangle.

Make a rectangle equal to the triangle. (*Prob.* 209.) Find a mean proportional between the base and the height of the rectangle, and on it describe the square.

PROBLEM 229.—To construct a triangle equal in area to a trapezium.

Draw the diagonal. On one side of it make a rectangle equal to the triangle A D C, and on the other side a rectangle equal to the triangle A B C. Proceed as in the preceding problem.

PROBLEM 230.—To construct a square equal in area to any polygon.

First obtain a triangle equal in area to the polygon, as shown in Problems 211 and 212; then construct a rectangle equal in area to the triangle, and proceed as above.

PROBLEM 231.—To construct a rectangle of a given perimeter, and equal in area to a given square.

Let the perimeter equal 4½ inches, and the side of the given square 1 inch. Divide half the perimeter into 2 parts, whose mean proportional shall equal 1 in., as follows:—Draw A B, 2¼ inches long, and at B draw B C perpendicular to A B, and 1 inch long. Describe a semicircle on A B. Through C draw C D parallel to A B, and from D draw D E parallel to C B. Then A E and E B will be two of the sides of the rectangle. Complete the figure as shown.

PROBLEM 232.—To construct a rectangle equal in area to a square, and having its sides in a given ratio.

Let the ratio of the sides be as 2 : 3. Produce the base of the given square. From A set off to any convenient unit A B and A C, equal to 2 and 3 units respectively. Find A D, the side of a square equal to the rectangle contained by A B and A C. From E draw E F and E G parallel to D C and D B. A F and A G will be the sides of the rectangle. Complete the figure as shown.

PROBLEM 233.—To construct a rectangle equal in area to a square, having the difference between two adjacent sides given.

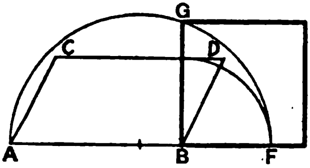
Let A B be the difference between the two adjacent sides. Bisect A B in C. With centre C and radius C D describe a semicircle. Then A F and A E will be the sides of the rectangle.

PROBLEM 234.—On a given base to construct a rectangle equal in area to a given rectangle.

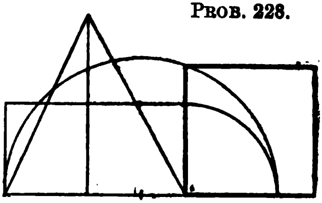
Let A B C D be the given rectangle, and E the given base. Produce A B, and make A F equal to E. Join C and F, and from B draw B G parallel to C F. A G is the other side of the rectangle.

Note.—A C : A G as A F : A B. (Euc. vi. 2.) But when quantities are in proportion the product of the *extremes* equals the product of the *means* (p. 18); therefore A C × A B = A G × A F.

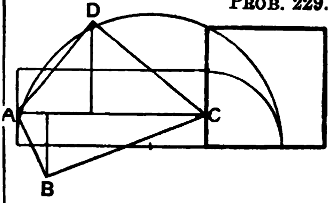
PROB. 227.



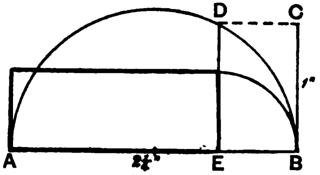
PROB. 228.



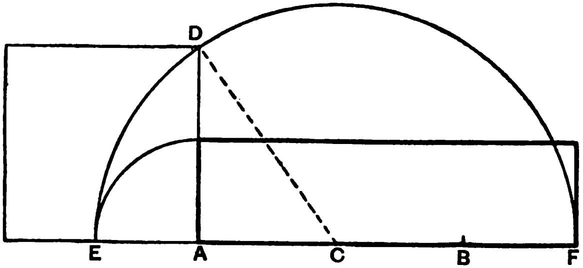
PROB. 229.



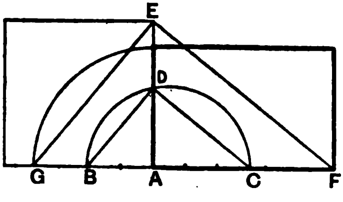
PROB. 231.



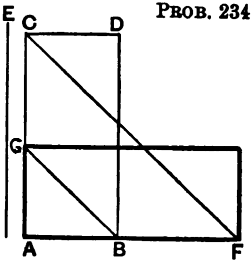
PROB. 233.



PROB. 232.



PROB. 234.



PROBLEM 235.—Another method for Problem 234.

Find a fourth proportional (E *g*) to E F (the given base), A B, and A C. (*Prob.* 27.) Draw E G equal to it, and complete the parallelogram. E F : A B as E *c* : E *g*.

PROBLEM 236.—Another method for Problem 217.

Draw the altitude, C E, of the given triangle. Find a fourth proportional (D *g*) to D F, A B, and C E. D *g* will be the altitude of the required triangle. Set up D G equal to D *g*, and join G and F.

PROBLEM 237.—To construct a parallelogram equal in area to a square, one side and one angle being given.

Let A B C D be the given square, F G the given side, and E the given angle.

At F construct an angle equal to E. Find a third proportional (F *h*) between F G and A B. (*Prob.* 28.) Make F H equal to F *h*, and complete the parallelogram.

PROBLEM 238.—To construct a triangle similar to a given triangle, and equal in area to a given square.

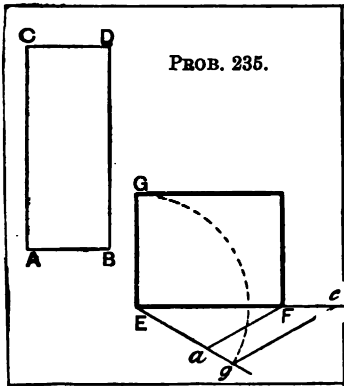
Let A B C be the given triangle, and B H the given square. Place the figures so that their bases are in the same straight line. Find B D, the side of a square equal in area to the given triangle, by reducing the triangle to a rectangle, and finding the mean proportional between the base and the height of the rectangle. (*Prob.* 228.) Draw D C. From E draw E F parallel to D C, to meet B C produced. From F draw F G parallel to A C, and meeting B A produced. Then the triangle G B F is similar to the triangle A B C, and equal to the square B H.

PROBLEM 239.—To construct a triangle similar to one given triangle, and equal in area to another.

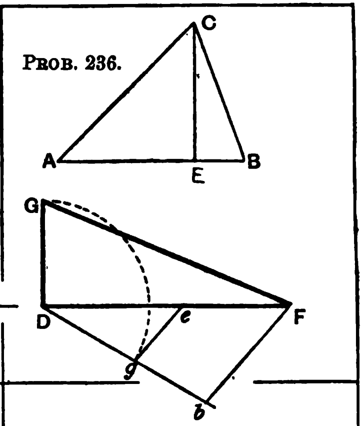
Let A B C be the triangle to which the required triangle is to be similar, and B D E that to which it has to be equal. Place the triangles so that their bases are in the same straight line. Reduce B D E to a triangle having the same area, but with its altitude equal to that of A B C, thus: Draw C F parallel to A B. Join F and D. Draw E G parallel to F D, and meeting A D produced. Join F and G. The triangle B F G equals the triangle B D E. Find B H, a mean proportional between A B and B G. B H is the base of the required triangle. Set off B J equal to B H, and from J draw J K parallel to A C, and meeting B C produced. J B K is the required triangle.

PROBLEM 240.—To construct an equilateral triangle equal in area to a given triangle.

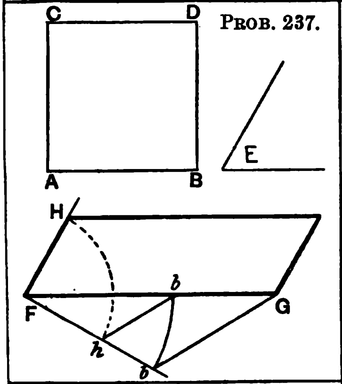
This is an exercise on the previous problem. Let A B C be the given triangle. Construct any equilateral triangle B E D, and place it with its base in the same straight line as A B. Construct the triangle B G F equal to A B C, and having the same altitude as B E D. Find B H, the mean proportional between B G and B D. With B H as the side construct the triangle.



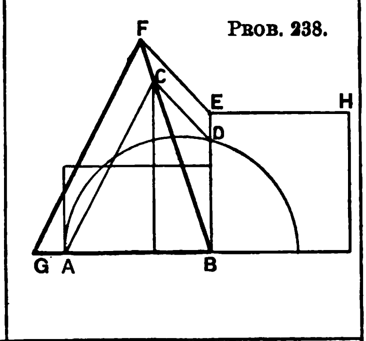
Prob. 235.



Prob. 236.

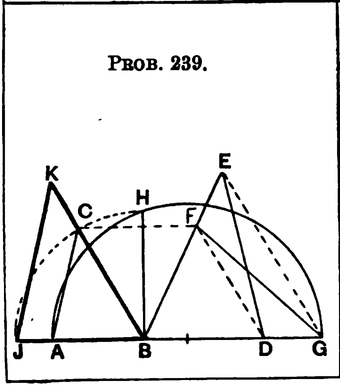


Prob. 237.

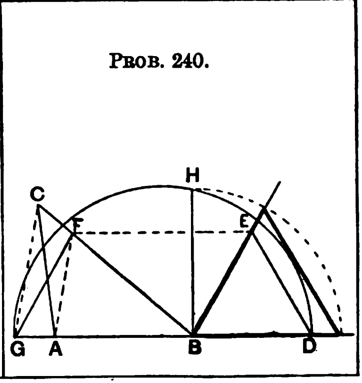


Prob. 238.

Prob. 239.



Prob. 240.



PROBLEM 241.—To construct an isosceles triangle equal in area to a given triangle, and having one of its angles equal to an angle of the given triangle.

Let A B C be the given triangle, and the angle at A the common angle. Find a mean proportional, A E, between A B and A C. Make A G and A F equal to A E. Join F and G. Then A F G is the required triangle.

PROBLEM 242.—To construct a regular polygon equal in area to a triangle.

Let A B C be the given triangle; it is required to construct a regular hexagon equal in area to it. A hexagon may be divided into six equal triangles, having the angles at their apexes 60°. Construct the triangle A B D equal to A B C (by drawing C D parallel to A B), and having the angle at B 60°. Divide A B into 6 equal parts. Then the triangle D E B will equal $\frac{1}{6}$th of the triangle A B D, and also $\frac{1}{6}$th of the required hexagon. If an isosceles triangle be now constructed equal to D E B, and having the angle at B common, it will also equal $\frac{1}{6}$th of the required hexagon. Find a mean proportional, B G, between D B and B E, the sides containing the angle. With centre B and radius B G describe a circle. B J H will be $\frac{1}{6}$th of the hexagon. Mark off J H round the circle, and complete the figure.

PROBLEM 243.—To construct any figure, similar to a given one, and having its area in a given proportion to the area of the given figure.

Let A B C D E be the given figure; it is required to construct a similar figure having $\frac{2}{5}$ths of its area. Divide the base of the given figure into 5 equal parts. Produce A B, and make B F equal to 2 of the parts. Find a mean proportional, B G, between A B and B F—that is, between the numerator and the denominator of the fraction. B G is the base of the figure. Make B H equal to B G, and complete the figure.

Notes.—1. In the case of a circle, divide the **radius** into the required number of parts.

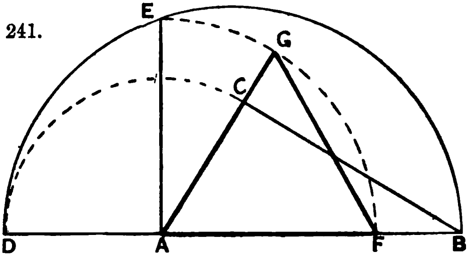
2. The above construction holds good for any proportionate area. Suppose, for instance, a figure is required $\frac{1}{7}$th of a given similar figure; find a mean proportional between the base and $\frac{1}{7}$th of the base.

3. Always divide the line into as many parts as there are units in the denominator of the fraction, or as there are units in the consequent of the ratio. For instance, if the ratio of the area be as 3 : 2, then divide the line into 2 equal parts, and find the mean proportional between the line divided into 2 parts and another line equal to 3 of these parts.

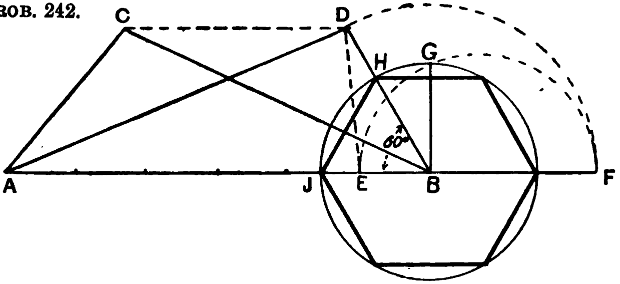
PROBLEM 244.—To describe a parallelogram equal to a given triangle in area and perimeter.

Let A B C be the given triangle. Produce A C, and make C D equal to C B. Through C draw a parallel to A B. Bisect A D in E. With centre A and radius A E obtain A F. Bisect A B in G. Draw G H parallel to A F. Then A G F H is the required parallelogram.

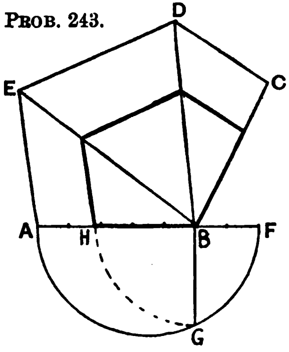
Prob. 241.



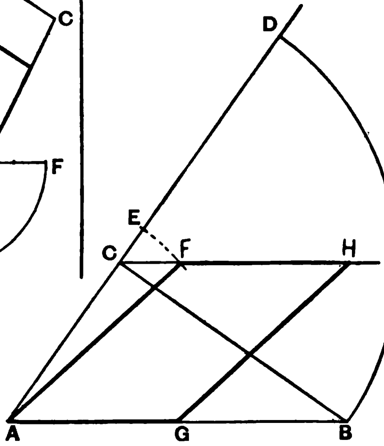
Prob. 242.



Prob. 243.



Prob. 244.



PROBLEM 245.—To divide a triangle into any number of equal parts by lines drawn from one of its angles.

Divide A B into the required number of equal parts (say 3). Draw C 1 and C 2. The three triangles thus formed are equal to each other. (EUC. I. 38.)

PROBLEM 246.—To divide a parallelogram into any number of equal parts by lines drawn from one of its angles.

Divide two adjacent sides into the same number of equal parts as the figure has to be divided into (say 3.) Join D 1, D 1.

PROBLEM 247.—To bisect a triangle by a line drawn from a point in one of the sides.

Let D be the given point. Bisect A B in E, and draw C E. Join D E, and from C draw C F parallel to D E. Join D and F. Then D F bisects the triangle.

PROBLEM 248.—To bisect a parallelogram by a line drawn from a point in one of the sides.

Let E be the given point. Find the centre, and draw E F through it. Then E F bisects the parallelogram.

Note.—This construction will apply for any position of the point.

PROBLEM 249.—To bisect a trapezium by a line drawn from one of its angles.

Let A B C D be the given trapezium. Draw the diagonals. Bisect A C in E. Draw D E, E B, dividing the trapezium into two equal areas. Through E draw F G parallel to B D. Join D and F. Then D F bisects the trapezium.

Note.—The triangle D B F = the triangle D B E.

PROBLEM 250.—To divide a triangle into any number of equal parts by lines drawn from a point in one of the sides.

Let D be the given point. Divide the side C B, in which the given point is situated, into as many equal parts as the triangle has to be divided into (say 3). Draw A D. From 1 and 2 draw 1 E, 2 F, parallel to A D. Draw D E, D F, dividing the triangle as required.

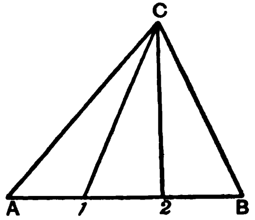
Note.—The triangle D E 1 = the triangle A E 1.

PROBLEM 251.—To divide a parallelogram into any number of equal parts by a line drawn from a point in one of the sides.

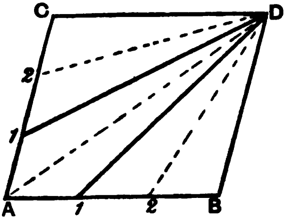
Let E be the given point. Divide A B into the required number of equal parts (say 4) in 1, 2, 3. Draw 1 1, 2 2 parallel to A D and bisect them in F and G. Through F and G draw E H and E J. Bisect the trapezium E J C B by the line E K. (*Prob.* 249.) Then the lines E H, E J, E K divide the parallelogram into 4 equal parts.

Note.—If the point E were placed so that E K would fall upon the side D C, then a line 3 3 might have been bisected, and E K drawn through the points of bisection, as in 1 1 and 2 2.

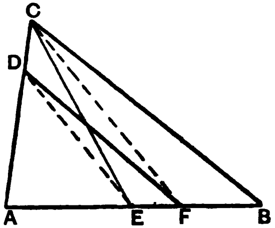
Prob. 245.



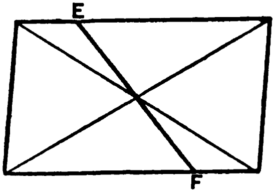
Prob. 246.



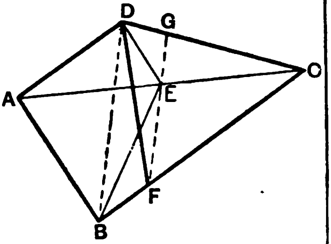
Prob. 247.



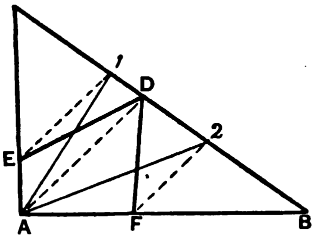
Prob. 248.



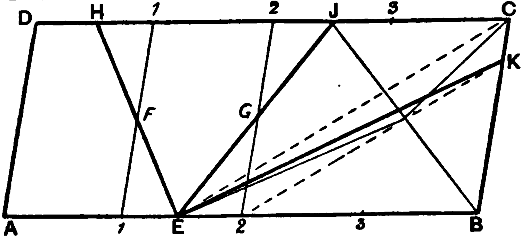
Prob. 249.



Prob. 250.



Prob. 251.



PROBLEM 252.—To divide an irregular polygon into any number of equal parts by lines drawn from one of the angles.

Let A B C D E be an irregular polygon; it is required to divide it into 3 equal parts by lines drawn from the angle at D.

Construct the triangle D F G equal to the polygon. (*Prob.* 211.)

Divide the base, F G, into 3 equal parts. Draw D 1. It is evident that the triangle D 1 F is ⅓rd of the triangle D F G, and consequently ⅓rd of the given polygon. As the point 2 does not fall upon the base A B of the polygon, draw 2 *d* parallel to the diagonal D B. Draw D *d*. The lines D 1 and D *d* divide the polygon as required.

Note.—The triangle D B 2 = the triangle D B *d*. (Euc. I 37.)

PROBLEM 253.—To divide a triangle into any number of equal parts by lines drawn from a point within the triangle.

Let D be the given point. Divide the base A B into as many equal parts as required (say 3). Draw D 1, D 2, and D C. From C draw C E parallel to D 1, and C F parallel to D 2. Draw D E and D F. The lines D C, D E, and D F divide the triangle into 3 equal parts.

Notes.—1. The triangle E 1 C = the triangle E D C. If A E C be added to each, then A 1 C = A E D C. But A 1 C is ⅓rd of the triangle A B C. Therefore A E D C will equal ⅓rd of the triangle A B C.

2. If the line D F does not fall upon A B, then proceed as shown in **Problem 253 a.** Obtain D E as in the preceding problem. Draw 2 F parallel to B C. Join D B, and from F draw F G parallel to D B. Draw D G. Then D C, D E, and D G divide the triangle into 3 equal parts.

Proof.—The triangle B F C = the triangle B 2 C. (Euc. I. 37.) Triangle F G D = triangle F G B. Add to each the triangle C F G. Then C F G + F G D = C F G + F G B—that is, C D G = C F B. But C F B = C 2 B. Therefore, C D G = C 2 B.

PROBLEM 254.—To bisect a triangle by a line drawn parallel to one side.

Bisect C B by the perpendicular E F, and describe a semicircle. With centre C and radius C F describe the arc F G. From G draw the line G H parallel to A B. This line bisects the triangle.

Note.—C F, which equals C G, is a mean proportional between the side C B and its half, C E. (Euc. VI. 8, Cor.) The triangle C G H : triangle C B A as $CG^2 : CB^2$—that is, as C E : C B, or as 1 : 2.

PROBLEM 255.—To bisect a triangle by a line perpendicular to the base.

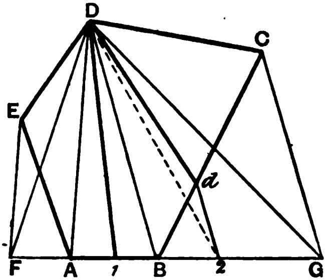
Draw A D perpendicular to B C, and bisect B C in E. Find a mean proportional, C F, between the *larger* segment of the base C D, and the half C E. Make C G equal to C F, and draw G H perpendicular to the base. Then G H bisects the triangle.

PROBLEM 256.—To divide a triangle into any number of equal parts by lines drawn parallel to one of the sides.

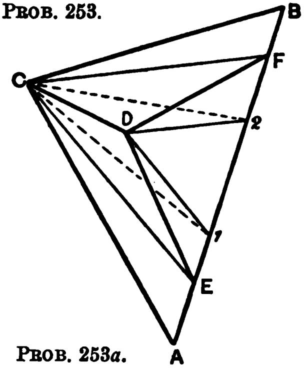
Let the triangle A B C be divided into 3 equal parts. Divide B C or B A into 3 equal parts. Describe a semicircle on B C, and erect perpendiculars at 1 and 2. Make B *d* equal to B D, and B *e* equal to B E. Parallels drawn from *d* and *e* will divide the triangle as required.

Note.—B D is a mean proportional between B C and its third, B 1, and B E is a mean proportional between B C and two-thirds, B 2. B *d* F : B C A as $Bd^2 : BC^2$—that is, as B 1 : B C, or as 1 : 3.

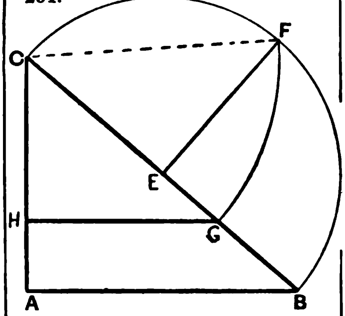
Prob. 252.



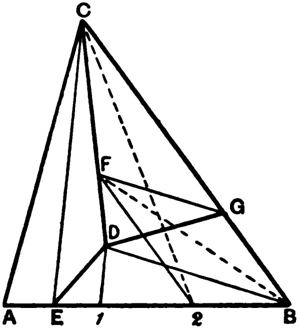
Prob. 253.



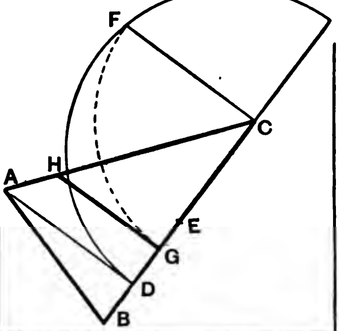
254.



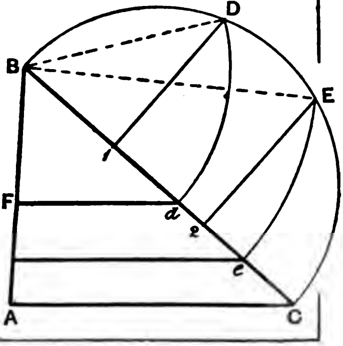
Prob. 253a.



Prob. 255.



Prob. 256.



PROBLEM 257.—To divide a parallelogram into any number of equal parts by lines parallel to the diagonal.

Let it be required to divide the parallelogram A B C D into 5 equal parts by lines drawn parallel to the diagonal D B. Divide D C into 5 equal parts. Describe a semicircle on D C, erect perpendiculars at the alternate parts, 2 and 4, and proceed as in Problem 256.

Note.—Each of the triangles A B D and D B C is divided into 5 equal parts. By drawing the alternate lines only, the whole parallelogram is divided into 5 equal parts.

PROBLEM 258.—To divide a circle into any number of equal parts (say 3) by concentric circles.

Divide the radius A O into 3 equal parts. On it describe a semicircle, and erect perpendiculars at 1 and 2. O B and O C will be the radii of the required circles.

Note.—O B is the mean proportional between the radius O A and its third O 1.

PROBLEM 259.—To divide a circle into any number of parts (say 3) equal in area and perimeter.

Divide the diameter of the circle into twice as many parts as there are equal areas required. With centres 1, 2, 4, and 5 describe semicircles.

PROBLEM 260.—To divide a triangle into 2 parts, having a given ratio to each other, by a straight line drawn through a given point in one of its sides.

Let D be the given point, and the given ratio as 3 : 2.

Divide C B into 3 + 2 equal parts. Join A 2, which divides the triangle into 2 parts in the ratio of 3 to 2. Draw D A, and from 2 draw 2 E parallel to D A. Join D E. Then D E divides the triangle as required.

Note.—The triangle E 2 A = the triangle E 2 D. Therefore, E C D = A C 2.

PROBLEM 261.—To divide a parallelogram into 2 parts, having a given ratio to each other, by a straight line drawn from a given point in one of the sides.

Let E be the given point, and the given ratio as 3 : 1.

Divide A B into 4 equal parts. From 1 draw a line, 1 F, parallel to A D, cutting the parallelogram into 2 parts having the required ratio. Bisect 1 F, and from E draw a line through the point of bisection. This line divides the triangle as required.

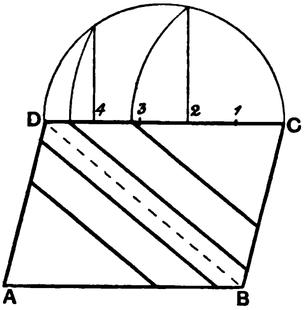
PROBLEM 262.—To construct a square, the area being given.

Let the required area be 2½ square inches. Construct a rectangle, sides 1 inch and 2½ inches. This rectangle will contain 2½ square inches. Find a mean proportional, B E, to the two sides. On B E construct the square.

Notes.—1. It is not necessary to construct the rectangle, but only to find the mean proportional between the dimensions of the two sides which would contain a rectangle of the given area.

2. The figure is drawn to a smaller scale.

PROB. 257.



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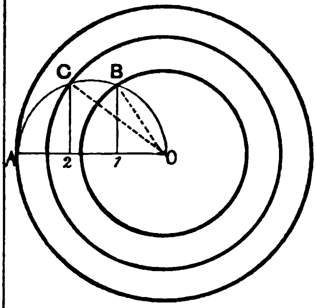

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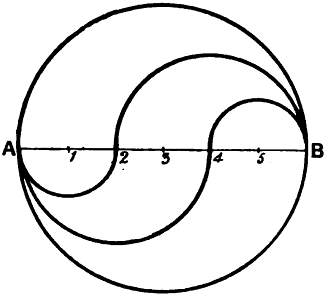

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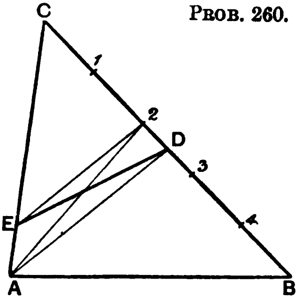

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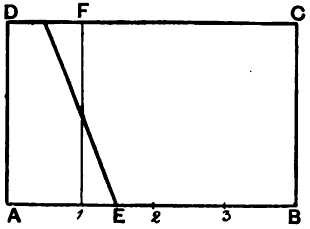

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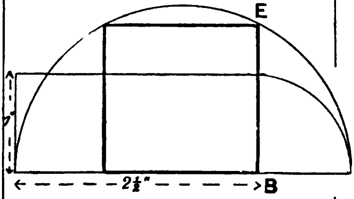


PROBLEM 262 a.—The same as Problem 262. (Another Method.)

Draw A B and B C, 1″ long, perpendicular to each other. Then A C will equal $\sqrt{2}''$. Find C D, $\sqrt{\frac{1}{2}}''$. Draw C E perpendicular to C A, and equal to C D. Then A E will be the side of a square containing 2½ square inches.

PROBLEM 263.—To construct a rectangle having an area of 3 square inches and a height of 2 inches.

Construct a rectangle, A B C D, having the given area. Now obtain another rectangle equal to A B C D, and having the required height of 2 inches, by Problem 234. Make A E equal to 2″. Join E B, and from C draw C F parallel to E B. On A F construct the rectangle.

PROBLEM 264.—To construct an equilateral triangle having an area of 3 square inches.

Construct a rectangle 3″ long and 1″ in altitude. Make the triangle A B C equal to it, and having the angle at B 60°. Find a mean proportional, B E, between the two sides, B C and B A. B E is the length of the side of the required triangle. Make B F and B G equal to B E, and join F and G. Then B F G is the required triangle.

Note.—Any isosceles triangle may be made of a given area on the same principle.

PROBLEM 265.—To construct a rhombus having an angle of 50° and an area of 2½ square inches.

Construct a rhombus, A B C D, having the given angle and an altitude of 1″. Find the side, B E, of a square equal in area to the rhombus, by finding the mean proportional between the base and the height. Make a rectangle of 2½ square inches area. Find B F, the side of a square equal to the rectangle. Join E and C, and from F draw F G, parallel to E C, to meet B C produced. Draw the diagonal B D, and produce it. Draw G H parallel to C D, and H J parallel to D A. Then J B G H is the required rhombus.

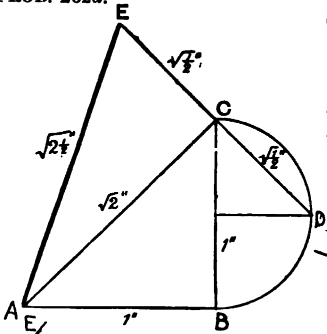
Note.—This important principle may be applied to the solution of other figures under similar conditions.

PROBLEM 266.—To construct a rectangle having an area of 3 square inches, and having its sides in the ratio of 3 to 2.

Construct a rectangle, A B C D, having its sides in the ratio of 3 to 2, and its altitude 1″. Find B E, the side of a square equal to the rectangle. Construct the rectangle B F G C, having an area of 3 square inches, and determine B H, the side of a square equal to it in area. Join E and A. From H draw H J parallel to E A. Then B J is the base of the required rectangle. Draw the diagonal B D, and produce it. Draw J K parallel to A D, and K L parallel to D C.

Note.—Problems 263–266 are drawn to a smaller scale.

PROB. 262a.



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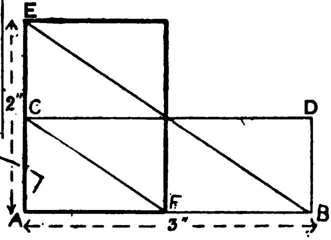

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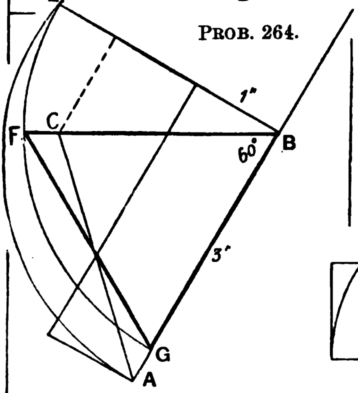

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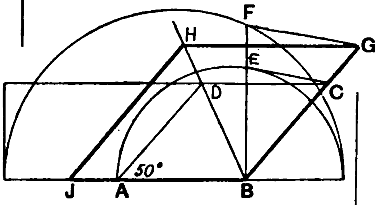

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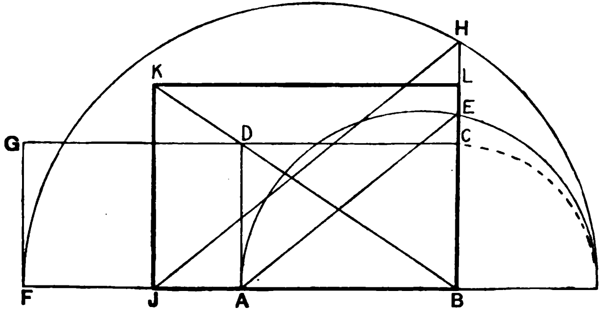


CHAPTER XIV

CURVES

Some of the curves used in geometrical and mechanical drawings, such as the **ellipse**, **parabola**, and **hyperbola**; the **cycloid**, the **involute**, various **spirals**, &c., cannot be described by the ordinary compasses. These curves are obtained by finding a number of points in the required line, and then tracing the curve by hand, or with the help of French curves.

The **ellipse**, **parabola**, and **hyperbola** are termed **conic sections**, because they are formed by plane sections of the cone.

The **ellipse** is the section of a cone formed by an oblique plane passing through both sides of the cone. (Fig. 1.)

The longest line that can be drawn through the centre of the ellipse is called the **transverse**, or **major axis**. (Fig. 1, *a b*.)

The shortest line is called the **conjugate**, or **minor axis**. (Fig. 1, *c d*.) Any line passing through the centre and terminated by the curve is a **diameter**. On the transverse axis, and equidistant from the centre, are situated two points, each of which is a **focus**. If any point in the curve be joined by two lines to the foci, these two lines are, together, equal to the transverse axis.

The **parabola** is the section of a cone formed by a plane cutting the cone parallel to its side. (Fig. 2.)

The **hyperbola** is the section of a cone formed by a plane cutting the cone parallel to its axis. (Fig. 3.)

When the cone is cut by a plane passing through its vertex, the section is a **triangle**; and when the section is parallel to the base, it is a **circle**.

PROBLEM 267.—To describe an ellipse, the transverse and conjugate diameters being given. First Method (by means of a piece of thread).

Let **A B** and **C D** be the diameters. Bisect **A B** and **C D**. Make **O E**, **O F**, each equal to half **C D**. With radius **A O** and centre **E** describe an arc cutting **A B** in **G** and **H**; these points are the **foci**. Take three pins, and stick them in firmly at the points **E**, **G**, and **H**. Tie a piece of thread round these pins, as shown by the lines **E G**, **G H**, **E H**. Remove the pin at **E**, and replace it with a pencil. Move the point of the pencil round, keeping the thread tightly stretched. The curve described by the pencil-point will be an ellipse.

PROBLEM 268.—The same. Second Method (by intersecting lines).

Place the diameters at right angles to each other, as before, and through the extremities draw parallels forming a rectangle. Divide **A E** into any number of equal parts (say 4). Set off these parts on **A F**, **B G**, and **B H**. Join each of these points with **C** and **D**. Divide **O A** and **O B** into the same number (4) of equal parts. Draw lines from **D** through 1, 2, and 3, on each side, to meet **C 1**, **C 2**, **C 3**. In the same manner draw lines from **C**, through the same points, to meet **D 1**, **D 2**, **D 3**. Through the points thus obtained draw the curve.

PROBLEM 269.—The same. Third Method (by intersecting arcs).

Place the axes as before. With radius **A O** and centre **C** obtain the foci. In **E O** take any number of points, as 1, 2, 3. With radius **B 1** and centres **E** and **F** describe four arcs at *a*. With radius **A 1** and the same centres intersect these arcs. With radius **B 2** and centres **E** and **F** describe four arcs at *b*, and with radius **A 2** intersect these arcs. With radius **B 3** and centres **E** and **F** describe arcs at *c*, and intersect them with radius **A 3**. Through the points thus obtained draw the ellipse.

Note.—Describe all the arcs from the foci E and F.

PROBLEM 270.—The same. Fourth Method.

Place the axes as before, and describe a circle on each. Divide one of the quadrants into any number of parts, and obtain corresponding points on the others by producing the radii. From the points 1 and 2 on the smaller circumference draw parallels to **A B**, and from the points 1′ and 2′ on the larger circumference draw parallels to **C D** to meet the lines drawn parallel to **A B**. Through the intersections of these parallels draw the curve.

Note.—All the problems on curves should be drawn to a much larger scale than shown.

FIG. 1.

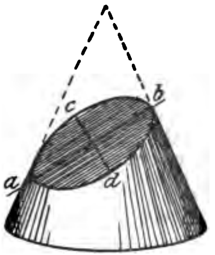


FIG. 2.

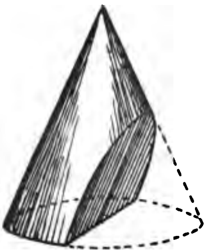
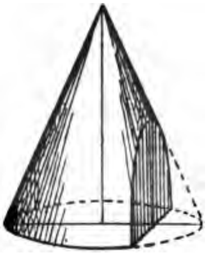
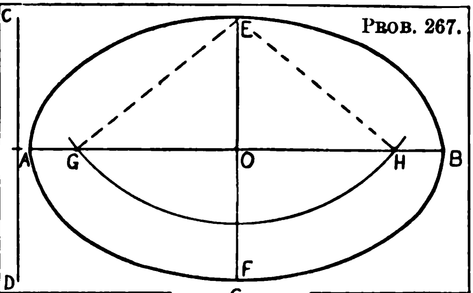


FIG. 3.



PROB. 267.



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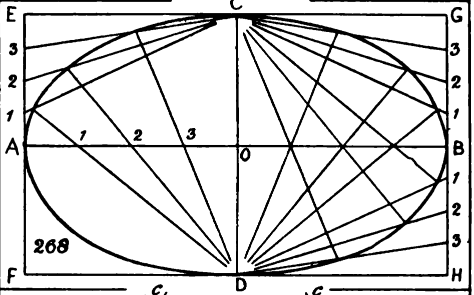
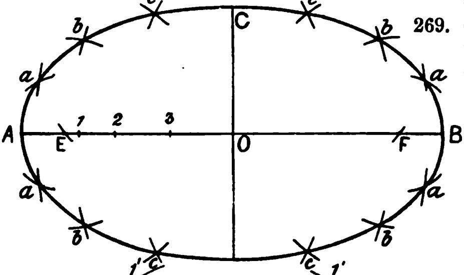


269.



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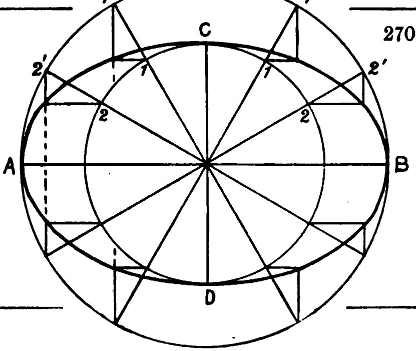


PROBLEM 271.—The same. Fifth Method (by means of a straight-edge or a paper trammel).

This is an exceedingly useful method for practical purposes. Set up the axes A B and C D as before. Take a piece of paper (or a long, flat ruler, if the figure be large), and make E F equal to A O, and E G equal to C O. Place it so that G may be on the transverse, and F on the conjugate diameter. Then E will be a point on the curve. By shifting the paper, and always keeping G on the transverse and F on the conjugate diameter, a number of points in the curve may be obtained. Draw the curve through the points.

Note.—If a point in the curve, as E, and the transverse diameter, were given, the ellipse could be described as follows:—Draw a line, C D, at right angles to A B. With radius A O and centre E cut the line C D in F. Then the distance E G will equal half the conjugate diameter. This being determined, complete the ellipse by any of the methods given.

PROBLEM 272.—To describe an ellipse passing through any three points not in the same straight line.

Let A B C be the given points. Join A B, and bisect it in O.

Join C O, and produce, making O D equal to O C. Through A and B draw parallels to C D, and through C and D draw parallels to A B, forming a parallelogram. Divide A O and A E into the same number of equal parts, and proceed as in Problem 268.

Note.—This problem offers solutions to the following problems:—To describe an ellipse about a triangle (the points A, B, and C, if joined, form a triangle); to inscribe an ellipse in a rhombus or rhomboid.

PROBLEM 273.—To find the centre, axes, and foci of a given ellipse.

Draw any two parallel chords, A B and C D. Bisect them both in E and F. The line drawn through E and F will be a diameter. Find the centre, O. With centre O describe an arc cutting the ellipse in G, H, and J. Draw G H and H J. Parallels to G H and H J, through O, will give the axes.

PROBLEM 274.—A portion of the curve of an ellipse being given, to complete it.

Let C D X be the given portion.

Draw two chords, A B and C D, and bisect them in E and F. Through E and F draw a line. If this line be terminated by the curve of the ellipse at each end, obtain the axes as in Problem 273, find the foci, and complete the ellipse by obtaining points in the curve as already shown. If the line be not terminated, then draw two other parallel chords, and bisect them as in the first pair. The line drawn through the points of bisection will intersect the line drawn through E and F in O, the centre of the ellipse. Make O H equal to O G, and obtain the axes and points in the curve as before.

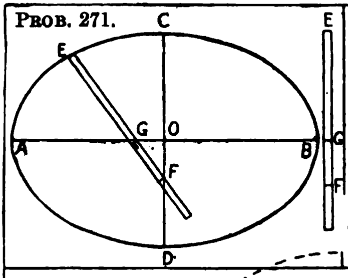
PROBLEM 275.—To draw a tangent to an ellipse at any given point in the curve.

Obtain the transverse diameter and the foci, A and B. (*Prob.* 273.) Join A and B with the given point, C, and produce one of the lines. The line bisecting the angle A C D is the tangent.

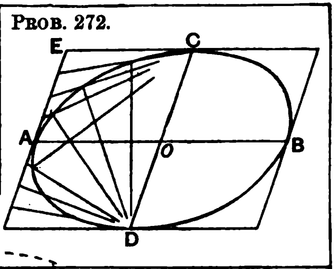
PROBLEM 276.—To draw a normal or perpendicular to the curve of an ellipse.

Join the foci, A and B, with the given point, C, as before, and produce both lines. Bisect the angle thus formed.

PROB. 271.



PROB. 272.



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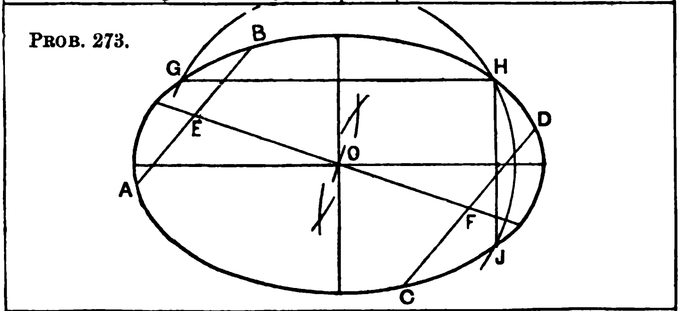

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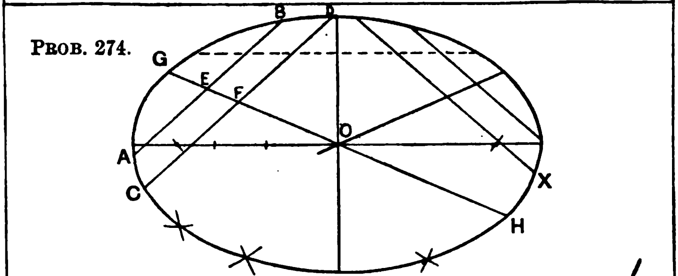

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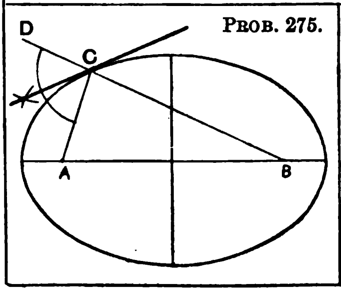
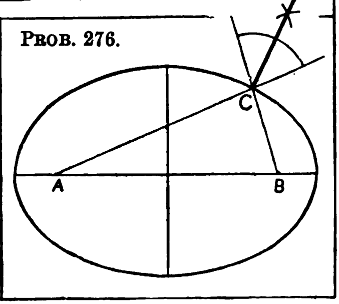


PROB. 276.



PROBLEM 277.—To draw a tangent to an ellipse from a point without the curve.

Find the centre, major and minor axes, and the foci of the ellipse. (*Prob.* 273.) From the given point, P, draw a diameter, A B. Find a third proportional, O C, to O P and O B. Through the extremity, A, of the diameter, draw a tangent. (*Prob.* 275). Through C draw the *double ordinate*, D E, parallel to the tangent, through A. The lines joining P with D and E will be tangents to the ellipse.

Notes.—1. One diameter is *conjugate* to another when it is parallel to the tangents drawn through the extremities of the other. If through the centre O a diameter be drawn parallel to the tangent at A, that diameter will be *conjugate* to A B.

2. An **ordinate** is a line drawn from any point in one diameter parallel to the tangent drawn through its extremity. C D and C E are ordinates; the whole line, D E, is called a double ordinate.

PROBLEM 278.—To describe the curve of a parabola, the axis and a double ordinate being given.

Let A B be the axis, and C D the double ordinate. Through C and D draw parallels to A B, and through A draw a parallel, E F, to C D. Divide E C and F D into any number of equal parts, and join each point with A. Divide B C and B D into the same number of equal parts, and draw parallels to A B. The intersections of the lines drawn from the points 1 1, 2 2, &c., are points on the curve.

PROBLEM 279.—To draw the involute of a given circle.

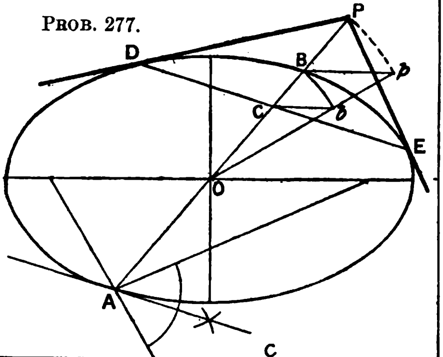
Note.—If a thread be unwound from a circle, and kept constantly stretched, the extremity describes a line which is called the involute.

Let A be the generating point. Draw the diameter A B, and at B the tangent B C. Make B C equal to the semicircle A 3 B by setting off $3\frac{1}{7}$th of the radius B O. As the distance from A to B has been unrolled, C must be a point of the curve. Divide the circle into any number of equal parts (say 12), and B C into as many equal parts as the semicircle contains (6). Draw tangents to the circle at points 1, 2, 3, &c. Make 1 D equal to B 1, 2 E equal to B 2, 3 F equal to B 3, 4 G equal to B 4, &c. To obtain points beyond C, proceed in the same manner; H 8, for example, equals 8 divisions. Through the points D, E, F, &c., draw the curve.

PROBLEM 280.—To describe an Archimedean spiral, the greatest radius and the number of turns or convolutions being given.

Let O A be the greatest radius, and the number of turns be two. Describe a circle with O A as radius. Divide O A into as many equal parts as turns required—that is, 2—in the point *a*. Divide the circle into any number of equal parts (say 8) by radii O B, O C, &c., and divide A *a* into the same number of equal parts. Make O *b* equal to O 7, O *c* equal to O 6, O *d* equal to O 5; proceed in a similar manner for the rest of the points, diminishing each radius by one part. Draw the curve through the points A, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *a*, &c.

PROB. 277.



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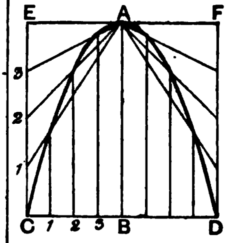

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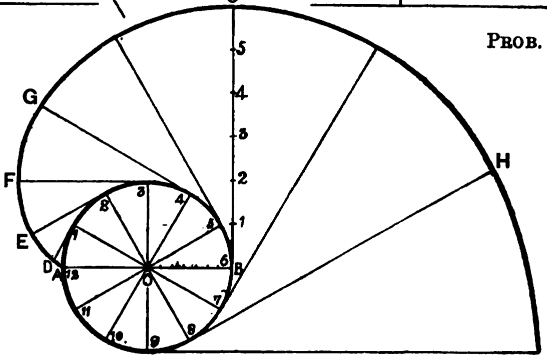

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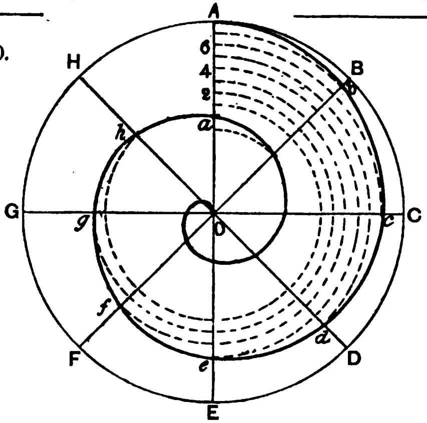


PROBLEM 281.—To draw a common spiral by means of semicircular arcs.

Let the spiral be described upon A B, and consist of three convolutions. Divide A B into 6 equal parts. Bisect 3 4 in O. With centre O and radius O 3 describe a semicircle. With centre 3 and radius 3 4 describe another semicircle. With alternate centres, O and 3, describe the remaining semicircles.

PROBLEM 282.—To draw the spiral known as the Ionic volute by describing arcs of circles.

Divide the diameter, X Y, into 8 equal parts, and upon one part describe a circle forming the *eye* of the volute. Inscribe a square, draw its diameters, and divide each into 6 equal parts as shown in the larger drawing. Number each point consecutively, and with 1 as centre describe an arc to meet 1 2 produced in B. With centre 2 and radius 2 B describe the next arc to meet 2 3 produced. Proceed in a similar manner for the rest of the curve.

Notes.—1. This problem should always be drawn to a large scale.

2. The last arc should finish at the point O, and be described from number 12.

PROBLEM 283.—To describe a continuous curve of tangential arcs passing through a number of given points.

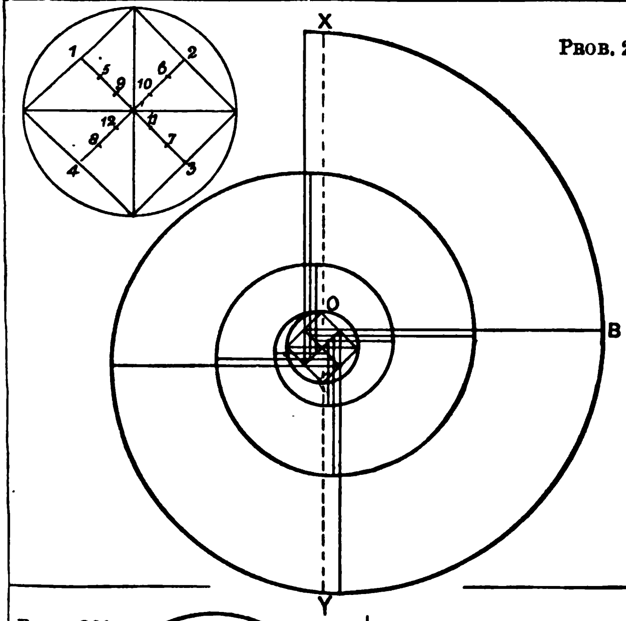
Let A, B, C, D be the given points. Join the points, and bisect the lines by perpendiculars at E, F, and G. Take any convenient point, 1, in E 1, and describe an arc on C D. From 1, through C, draw 1 2 meeting the perpendicular from F. With centre 2 and radius 2 C describe the arc upon C B. Draw 2 B to meet the perpendicular from G. With centre 3 describe the remaining arc.

PROBLEM 284.—To describe an arc of a circle to pass through three points given in position, when the centre is not obtainable.

Let A, B, and C be the three points. Join A B and C B, and produce to E and D. With centres A and C and radius A C describe arcs. On each side of D and E set off equal distances, 1, 2, 3, 4, &c. Join 4′ C and 4 A. The intersection of these lines is a point in the arc. Join 3′ C and 3 A, obtaining another point in the arc. The intersections of corresponding lines on alternate sides of D and E will give other points in the arc.

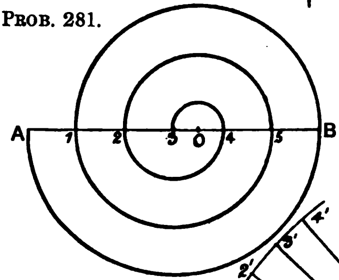
Notes.—1. This construction depends upon the principle that angles in the same segment of a circle are equal. (EUC. III. 21.)

2. Only a few of the problems upon curves are dealt with here. For further information, the student is referred to works on Conic Sections and Mechanical Drawing.

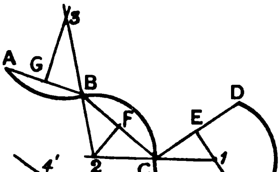


Prob. 282.

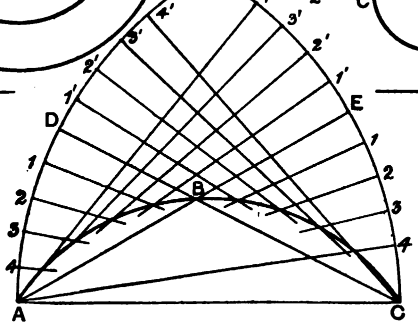
Prob. 281.



Prob. 283.



Prob. 284.



EXERCISES

CHAPTER XIII

1. Construct a square equal in area to a rectangle 2″ by 1½″.

2. Draw a semicircle 3″ in diameter, and divide it into 3 equal parts by means of concentric semicircles.

3. Construct a square equal in area to the sum of three squares of 1″, 1·5″, and 2·25″ respectively. (*Sc.*, 1877.)

4. Construct a square of 2½″ sides, and through one corner draw a line cutting off ⅓ of its area. (*Sc.*, 1886.) (*Prob.* 246.)

5. Draw a triangle, sides 1½″, 2″, and 1½″ respectively. On a base of 1¾″ construct an isosceles triangle of equal area. (*Sc.*, 1889.) (*Prob.* 217 or 236.)

6. Draw any irregular hexagon having one re-entering angle, and construct a similar figure whose sides are to those of the given figure as 7 : 3. (*Sc.*, 1889.)

Note.—Divide one side into 7 equal parts, and on 3 of these parts construct the similar figure.

7. Construct an equilateral triangle having an area of 4 square inches. (*Prob.* 264.)

8. Construct a triangle, sides 2¼″, 2⅓″, and $1\frac{5}{12}$″ respectively, and obtain a rectangle, equal to the triangle in area, upon a base of 1″. (*Sc.*, 1885.) (*Probs.* 209 and 234.)

9. In a square of 3″ sides inscribe another square having ¾ the area of the given square. The corners of the required square must lie in the sides of the given one. (*Sc.*, 1870.)

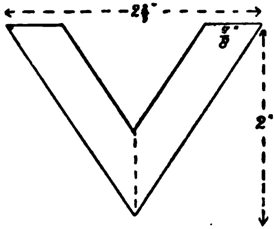
Note.—A mean proportional between the side of the given square and ¾ of the side will be the side of the required square. From this side obtain the diagonal, and from the centre of the given square, with half the diagonal as radius, describe a circle. The points where the circle cuts the sides will be the corners of the required square.

10. Construct a parallelogram having sides 4″ and 1·2″, and the included angle 60°. Determine a rhombus of equal area, and having the same included angle.

Note.—The side of the rhombus will equal the mean proportional between the two sides of the parallelogram.

11. Reduce the given figure (Fig. 1) to a square of equal area. (*Sc.*, 1887.)

FIG 1.



Note.—Divide the figure into 2 equal parts. Obtain a rectangle equal to 1 part, double it, and get a square equal to the rectangle.

12. Divide an equilateral triangle of 2½″ sides into 4 equal parts by perpendiculars to one side.

Note.—Bisect the triangle, and apply Problem 254 to each half.

13. Draw an irregular pentagon, and bisect it by a line drawn from one angle.

Note.—First convert the pentagon into a triangle.

14. The side of a rhombus is 3″ long, and one angle is 75°. Construct the figure, and divide it into 3 equal parts by lines drawn from one angle. (*Prob.* 246.)

15. Construct an equilateral triangle of 1½″ sides, and a rectangle of equal height and area.

16. Construct a square equal in area to an equilateral triangle of 1″ sides.

17. Divide a triangle whose sides are 2½″, 3″, and 4″ respectively, into 3 equal parts, by a line drawn from the middle of the longest side.

18. Construct a regular pentagon of 7 square inches area.

Note.—Obtain a triangle of the required area, and proceed by Problem 242.

19. Construct a rectangle of 6 square inches area, its sides being as 3 : 2. (*Prob.* 266.)

20. Show by construction the $\sqrt{3}$. (*Prob.* 262 *a.*)

21. Taking 1″ as a unit, determine by two different constructions a line equal to $\sqrt{6}$. (*Probs.* 262 and 262*a.*)

22. The sides of a triangle are 3½", 2¾", and 4"; draw the triangle, and obtain an equilateral triangle equal to it. (*Prob.* 240.)

23. Describe a regular hexagon of 1½" sides, and obtain a similar figure having ¾ of its area. (*Prob.* 243.)

24. Draw an equilateral hexagon of 1·2" sides. Reduce it to a triangle, and find its area. (*Page* 94, 8.)

25. Construct a triangle, base 3", and base angles 45° and 75°. On the same base construct an isosceles triangle equal to it in area.

26. Draw a rectangle equal in area to a square of 1·75" side, making the shorter side 1·25" long. (*Sc.*, 1882.)

27. Draw a rhomboid which shall have an area of 4 square inches, two of its sides being 2·5" long, and two of its angles 60°. (*Sc.*, 1881.)

Note.—Construct a square of 2" sides; obtain a rectangle equal to it in area, on a base of 2½", and then a rhomboid equal to the rectangle.

28. Construct a rectangle whose height is 1·4" and area 3 square inches. (*Sc.*, 1878.)

29. Draw a rhombus of 6 square inches area, and having two of its angles 40°. (*Sc.*, 1872.)

30. Having given a circle of 1" radius, draw another ⅓ of its area. (*Sc.*, 1876.)

CHAPTER XIV

1. Construct a semi-ellipse, major axis 3", and half the minor axis 1". (*Art.*)

2. Find the centre, axes, and foci of a given ellipse. (*Art.*) (*Draw the ellipse by means of thread and pins.*)

3. Describe an ellipse by means of intersecting arcs. Axes 3½" and 2". (*Art.*)

4. Construct a rhombus, side 2½", diagonal 4¼". In this rhombus inscribe an ellipse. (*Sc.*, 1889.) (*Prob.* 272.)

5. Describe an ellipse, the longer diameter of which is half as long again as the shorter, and at any point on the curve draw a normal to it.

6. Draw a spiral curve composed of five semicircles, whose diameters are, successively, 1, 1½, 2, 2½, and 3 inches.

7. Draw an undulating, continuous curve of six arcs of circles, each containing 90°, and described with a radius of 1".

8. The major axis of an ellipse is 3" long, and the foci 1" from the centre; describe the curve, and show how to draw a tangent to it.

9. The line *a b* (Fig. 2) represents a piece of thread unwound from the given circle. Draw the curve traced by the extremity *a* when the thread is wound back on to the circle. (*Prob.* 279.)

10. Construct a triangle, sides 3½", 2", and 2¾", and about it describe an ellipse. (*Prob.* 272.)

11. Construct an ellipse having its conjugate diameters each 3½" and intersecting at 70°.

12. Draw a half ellipse with axes 4" and 2·5". Draw a sufficient number of normals to the curve, and produce them externally. Through points on these, ·5" from the curve, draw a second curve parallel to the first.

13. Draw the involute of a circle of 1·75" diameter. The curve to be shown from its starting-point on the circumference of the circle till it cuts the produced diameter at that point. (*Sc.*, 1878, *Ad.*) (*Prob.* 279.)

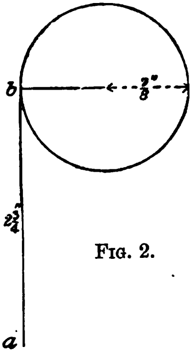


FIG. 2.

CHAPTER XV

THE APPLICATION OF GEOMETRY TO THE CONSTRUCTION OF PATTERNS AND SIMPLE TRACERY

THE following Examples (which should be drawn to a larger scale) show a few of the ways in which Practical Geometry may be applied, Ornamental Design being mostly based on a geometrical foundation.

The Examples on this page are illustrations of straight-lined ornament, and are based on squares. The dotted lines indicate the method of construction. Fig. 10 is termed a plait. Fig. 19 is a Gothic nail-head ornament. The others are examples of various frets. Figs. 9, 17, and 18 are Arabian or Moresque; 16 is Chinese; 5; 12, 13, and 15 are Greek.

FIG. 1.

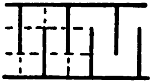


FIG. 2.

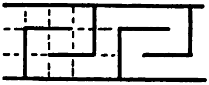


FIG. 3.

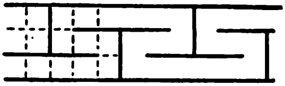


FIG. 4.

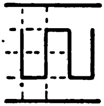


FIG. 5.

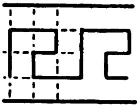


FIG. 6.

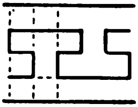


FIG. 7.

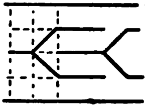


FIG. 8.

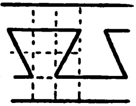


FIG. 9.

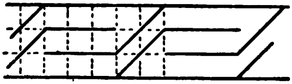


FIG. 10.

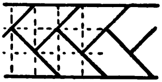


FIG. 11.

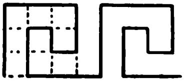


FIG. 12.

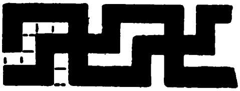


FIG. 13.

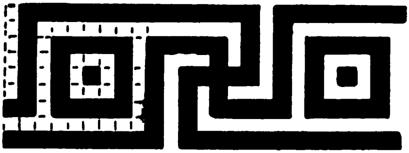


FIG. 14.

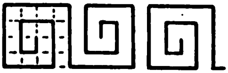


FIG. 15.

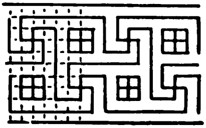


FIG. 16.

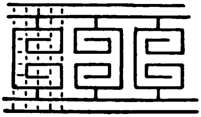


FIG. 17.

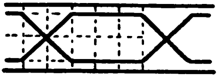


FIG. 18.

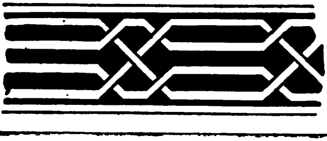


FIG. 19.

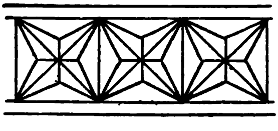


FIG. 20.

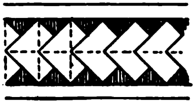


FIG. 21.

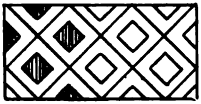


FIG. 22.

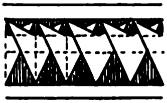


FIG. 23.

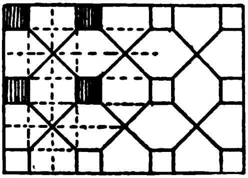


FIG 24.

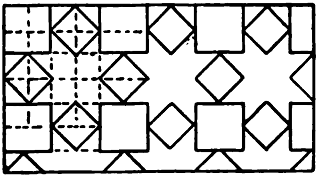


FIG. 25.

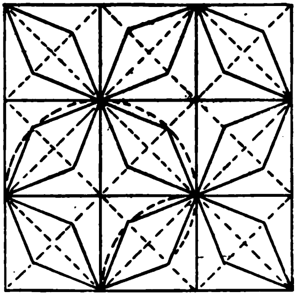


FIG. 26.

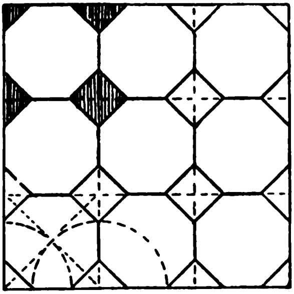


FIG. 27.

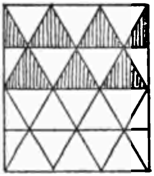


FIG. 28.

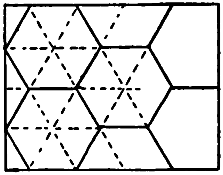
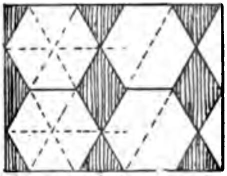


FIG. 29.



Figs. 20–26 are formed on squares. In Fig. 25, circles described from the angles of the square give the required points. In Fig. 26, an octagon is inscribed in each square, as shown. Fig. 27 is formed by equilateral triangles, 28 by hexagons adjacent to each other, and 29 by the hexagon and rhombus combined. The dotted lines indicate the necessary construction.

FIG. 30.

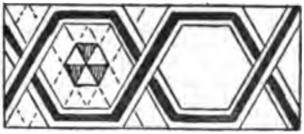


FIG. 31.

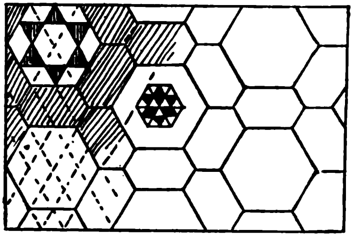


FIG. 32.

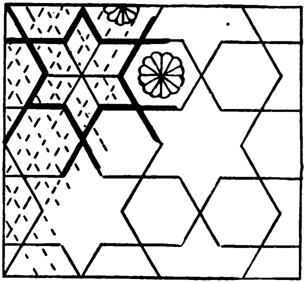


FIG. 33.

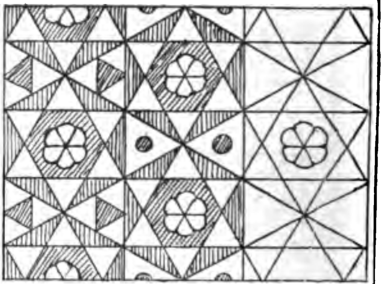


FIG. 34.

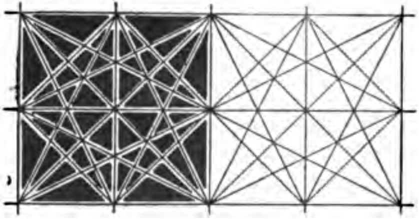
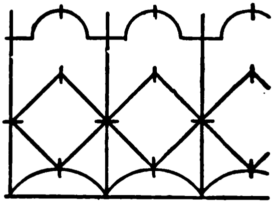


FIG. 35.



Figs. 30–33 are based on the hexagon, and are obtained by drawing lines with the set square intersecting at an angle of 60°, and picking out the pattern. The spaces may be filled in in a great variety of ways. Fig. 33 is taken from an inlaid Indian box. In Fig. 34 the pattern is made by drawing lines as shown, and filling up the spaces. An infinite number of pretty patterns may be created in this way by introducing other lines and varying their direction. The same principle may be used when the squares are placed as in Fig. 21, or when the foundation lines are as in Fig. 27. Fig. 35 shows an arrangement suitable for iron palings.

FIG. 36.

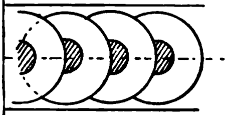


FIG. 37.

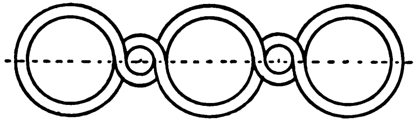


FIG. 38.

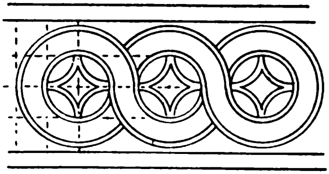


FIG. 39.

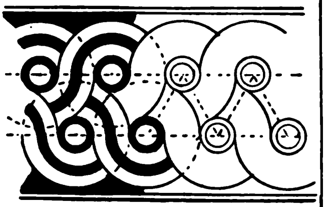


FIG. 40.

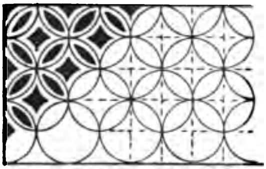


FIG. 41.

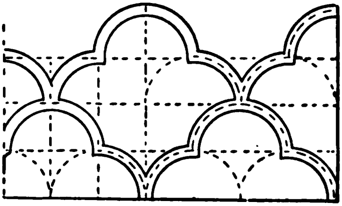
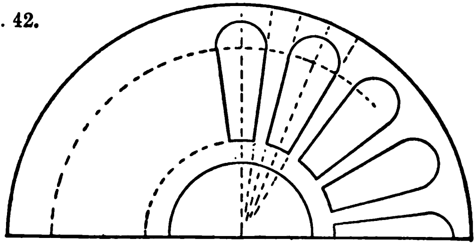


FIG. 42.



The dotted lines indicate the construction for the above Figures, all of which are based on the circle. Figures 37–39 represent forms of the guilloche, so largely used in architecture by the Assyrians and Greeks. Fig. 40 is obtained by intersecting circles; while Fig. 41 is based on semicircles.

FIG. 43.

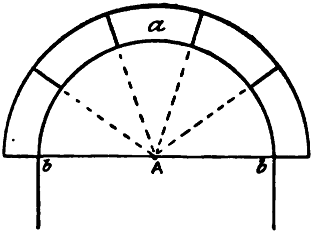


FIG. 44.

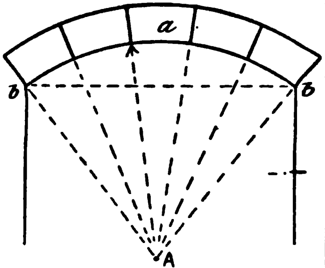


FIG. 46.

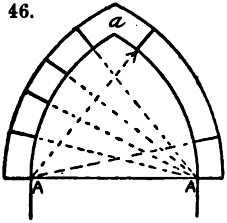


FIG. 45.

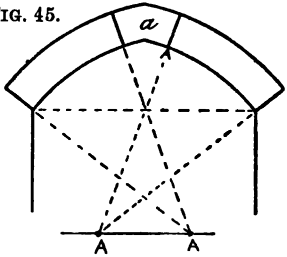


FIG. 47.

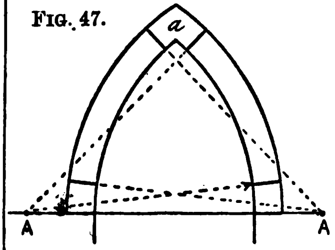
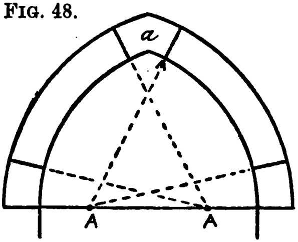


FIG. 48.



ARCHES

The centres from which the curves are described are marked **A**.

The separate stones of the arch are called **voussoirs**; the central one is called the **keystone**, because it is the last stone placed, and fastens the rest together, as *a*. The **span** is the distance between the letters *b b*.

FIG. 49.

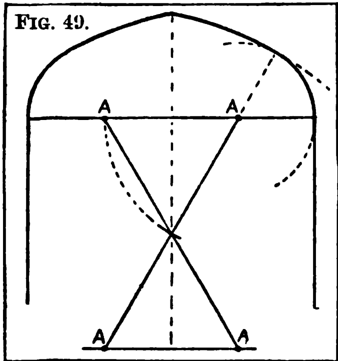


FIG. 50.

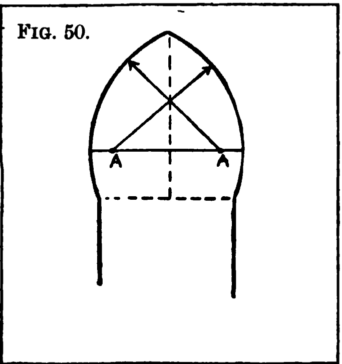


FIG. 51.

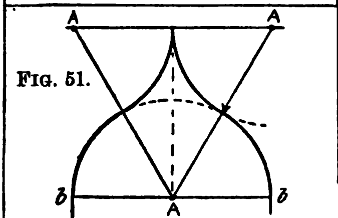


FIG. 52.

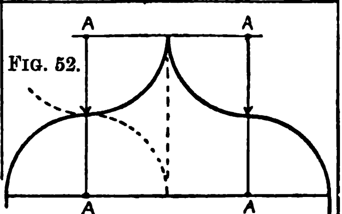


FIG. 53.

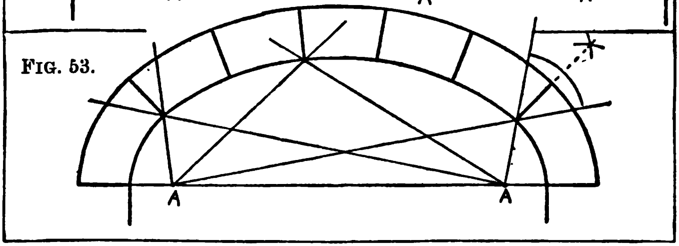


Fig. 43 is a **semicircular** arch; Fig. 44 is a **segmental** arch described from one centre; Fig. 45 is a **segmental** arch described from two centres; Fig. 46 is an **equilateral**, Fig. 47 an **acute or lancet**, and Fig. 48 an **obtuse** arch.

Fig. 49 is a **depressed Tudor** arch described from four centres. Fig. 50 is a **Moorish** arch. Fig. 51 is an **ogee** arch described from three centres. Fig. 52 is an **ogee** arch described from four centres. Fig. 53 is an **elliptic** arch. The joints between the stones are *normals* to the curve, and are obtained as shown by Problem 276. To obtain the outer curve, make the normals equal, and sketch in the curve.

FIG. 54.

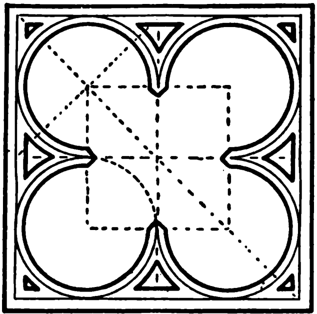


FIG. 55.

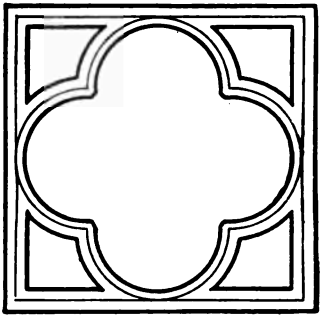


FIG. 56.

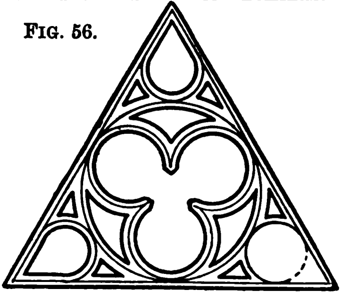


FIG. 57.

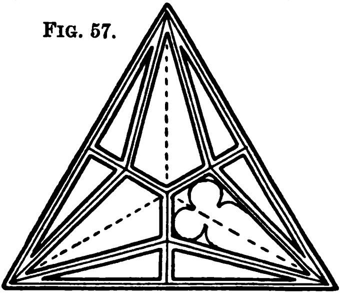


FIG. 58.

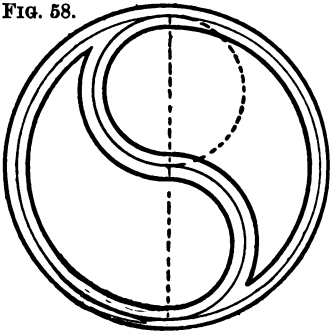
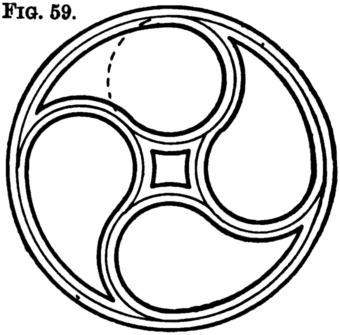


FIG. 59.



GEOMETRICAL TRACERY WINDOWS

In these examples the light centre lines form the foundation. The main construction lines are shown where necessary. For further information, the student is referred to books on architecture. *Draw all the examples at least twice as large.*

Fig. 54 is based upon Problem 135, Fig. 55 on Problem 166, Fig. 56 on oblems 127 and 148. In Fig. 56, first inscribe a circle in the equilateral gle, next three circles in the circle, and then a circle in each corner. In

FIG. 60.

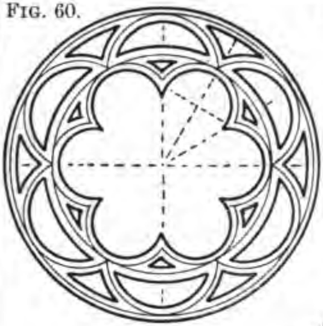


FIG. 61.

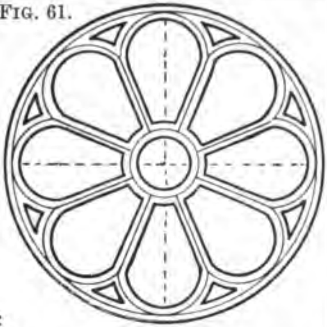


FIG. 62.

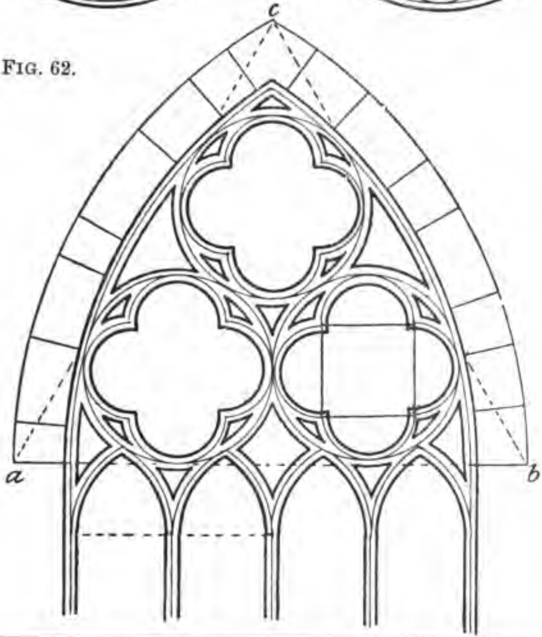


Fig. 57, a slight sketch indicates how the spaces may be further ornamented. Figs. 58 and 59 are based upon the inscription of two and four circles respectively in a circle.

Fig. 60 is based upon Problem 168: six semicircles in a circle, and the same process repeated within the inner circle. Fig. 61 depends upon the same principle. In Fig. 62, first describe the equilateral triangle *a b c*, and inscribe within it three circles; then, from the centres *a* and *b*, describe the inner lines of the arch. Within each circle inscribe four semicircles, and fill in the tracery as shown.

CHAPTER XVI

SOLID GEOMETRY

PLANS, ELEVATIONS, AND SECTIONS OF SOLIDS IN SIMPLE POSITIONS

THE preceding chapters have dealt with *Plane Geometry*—that is, with the representation of figures having *length* and *breadth* only. When we require to represent objects having *length, breadth,* and *thickness*, two drawings are necessary: one to show the length and breadth, and the other the height, or thickness. This representation of length, breadth, and thickness is termed **Solid Geometry**.

To illustrate this, fold a piece of paper at right angles, place it on a board or table adjoining a wall, so that one-half of the paper rests on the table, and the other half against the wall. Take a simple object, such as a box, place it on the paper, and trace its form on the horizontal fold (Fig. 1, *a b c d*); this drawing will show its length and breadth, and will be the **plan** of the box. Next, without moving the box, trace its form on the vertical fold (Fig. 1, A′B′ *a′ b′*); this will show its thickness, or height, and will be the **elevation** of the box.

The surface, or plane, upon which the object stands is called the **horizontal** or **ground plane**, and the drawing, or **plan** on it *shows the space covered by the object on the ground, and its position with regard to the upright plane.* This upright surface is called the **vertical plane**, and the drawing traced on it is termed the **elevation.** The line represented by the crease in the paper where the two planes intersect, or cut each other, is called the **intersecting** or **ground line.**

If the paper be now opened out, the two drawings will be seen upon one surface (Fig. 2), with the line X Y showing the intersection of the two planes. The second drawing (A′B′ *a′ b′*) is termed a **projection,** because each point of the object is projected, or thrown upon the part of the vertical plane exactly opposite to it; thus, *a′* is the projection of *a*, and the line joining *a* and *a′* is called a **projector.**

When the projection is effected by *parallel lines at right angles* to the vertical and horizontal planes, as in Fig. 2, it is called **orthographic projection,** and every part of the object is represented its correct size *by scale,* no matter how distant. In **perspective projection,** the rays of light converge towards one point, and the back lines of the object are represented slightly shorter than the corresponding front ones, as, being farther off, they appear less.

All perpendiculars and parallels must be carefully drawn, and each point should be lettered correctly as it is obtained. The system of lettering is as follows:—The same point, whatever its position, is denoted by the same letter; thus, if A represents an actual point, then *a* shows its plan, and *a′* its elevation. The student, in case of difficulty, should place the object in the required position, remembering, first, that by looking *vertically downwards* the *plan* will be seen; second, that the *elevation* is seen by looking *horizontally forwards*; and third, that *every point in the plan is exactly under the corresponding point in the* *ation.*

FIG. 1.

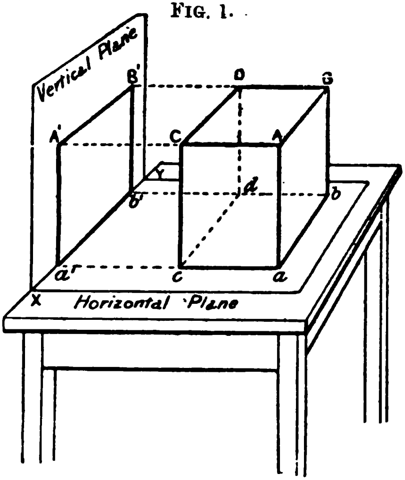
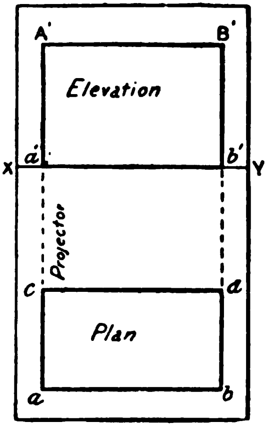


FIG. 2.



The following are the simplest solid forms:—

A **cube** is a solid figure contained by six equal squares.

A **prism** has its two ends formed by equal, similar, and parallel plane figures, and each of its sides parallelograms.

A **pyramid** has a plane figure for its base, and each of its sides are triangles meeting at a point above the base, called the **vertex**, or **apex**.

Notes.—1. Prisms and pyramids are named from the shapes of their ends and bases; thus, a square prism has square ends, an hexagonal pyramid has a hexagon for its base; &c.

2. The line passing through the middle of the solid is termed the **axis**. When the axis is perpendicular to the base, or ends of the solid, it is termed a **right** prism, or pyramid. When the axis is not perpendicular, the prism, or pyramid, is termed **oblique**.

A **sphere** is generated by the revolution of a semicircle about its diameter; every part of its surface is equally distant from the centre.

A **cone** is generated by the revolution of a right-angled triangle about its perpendicular.

A **cylinder** is generated by the revolution of a rectangle about one of its sides.

A **tetrahedron** is a solid contained by four equal equilateral triangles.

An **octahedron** is contained by eight equal equilateral triangles.

A **dodecahedron** is contained by twelve equal and regular pentagons.

An **icosahedron** is contained by twenty equal equilateral triangles.

If the upper part of a pyramid or cone be cut away, the portion left is called the **frustum**, and is said to be **truncated**.

POINTS, LINES, AND PLANE FIGURES

The diagrams in this chapter are drawn to a small scale, and, where dimensions are not given, should be copied about *three times the size.* The *projectors* should be drawn with *very fine lines,* and the *edges* which are *not visible* should always be *dotted.*

PROBLEM 285.—To represent the plan and elevation of a point in the following positions:—1. Touching both planes. 2. One inch in front of the vertical plane, and two inches above the horizontal plane.

1. As the given point touches both planes, its position must be on the intersecting line, **X Y.**

2. Rule a projector of indefinite length. Set off 1″ below **X Y**, and 2″ above it. Then a and a' will be the plan and elevation of the point.

PROBLEM 286.—To represent the plan and elevation of a line in the following positions:—1. Standing vertically, 1″ in front of the vertical plane. 2. Parallel to the horizontal plane, 2″ above it, and at right angles to the vertical plane. 3. Parallel to both planes, and 2″ from them. 4. Lying on the ground, at an angle of 30° to the vertical plane. 5. Parallel to the vertical plane, 1″ in front of it, and making 45° with the horizontal plane.

1. Draw a line standing vertically on **X Y**; this will be the elevation. Its plan will be the point a projected 1″ below **X Y.**

2. Draw the plan $a\ b$ at right angles to **X Y.** The elevation will be the point a' projected 2″ above **X Y.**

3. Draw parallel lines 2″ above and 2″ below **X Y**; both the plan and the elevation in this position will show the full length of the line.

4. In this case the plan will show the full length of the line. Draw $a\ b$ at 30° with **X Y.** As the line is on the ground, its elevation must lie in **X Y.** Project from a and b, giving the elevation $a'\ b'$.

5. Here the elevation must be obtained first, as that will be the true length of the line. Draw $a'\ b'$ at 45°, and project the plan as shown.

Note.—The student should take a pencil, and hold in the positions indicated, if he has any difficulty in realising them.

PROBLEM 287.—The line represented in slightly more difficult cases:—

1. $a'\ b'$ is the elevation of a line inclined at 50° to the vertical plane. Determine its plan.

Project from a' and b'. At a make an angle of 50°, and draw a line to meet the projector from b'. Then $a\ b$ is the plan.

2. $a\ b$ is the plan of a line 2½″ long; to obtain its elevation.

Project from a and b. With a' as centre and 2½″ as radius, cut the projector from b in b'. Then $a'\ b'$ is the elevation.

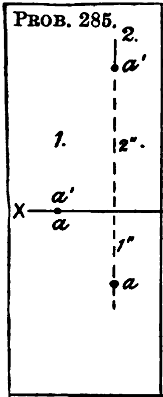
3. $a'\ b'$ is the elevation of a line touching both planes, and making equal angles with them. Draw its plan, and find its true length.

As it makes equal angles with both planes, it will be at an angle of 45° with both, and the plan will equal the elevation. To obtain the true length, the object must be turned at right angles, when a' B′ will represent the actual line.

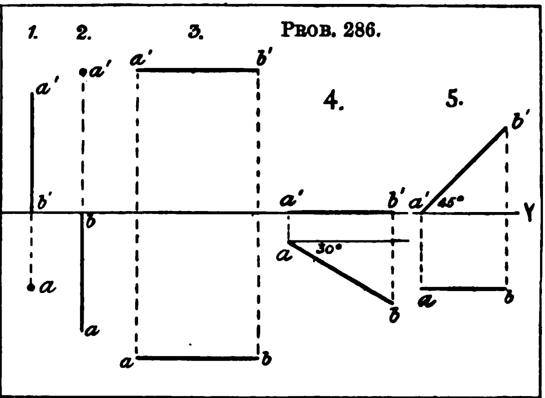
PROBLEM 288.—To represent a square of 2″ sides standing vertically, with its side on the horizontal plane:—1. Parallel to the vertical plane. 2. At right angles to both planes. 3. At an angle of 30° to the vertical plane. 4. Standing with its diagonal vertical, and parallel to the vertical plane. 5. With one diagonal vertical, and the other at 30° to the vertical plane.

In 1, 2, and 3, the figures will explain themselves. In 4, construct the elevation first, and then deduce the plan. In 5, draw the elevation and plan, as if parallel to the vertical plane, as shown in dotted lines; then rotate the plan into position $a\ b\ c$, at 30° to the vertical plan, and project a fresh elevation. Obtain the heights from the first elevation.

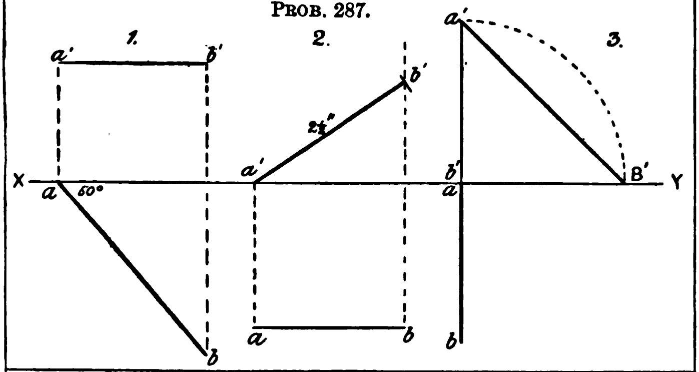
PROB. 285.



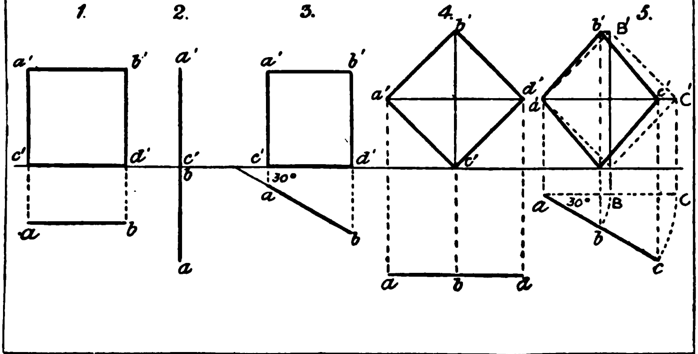
PROB. 286.



PROB. 287.



PROB. 288.



PROBLEM 289.—To draw the plan and elevation of a square of 2″ sides :—

1. Lying horizontally, with one edge parallel to the vertical plane. 2. Lying horizontally, with one edge making 30° with the vertical plane. 3. With its plane inclined at 45° to the ground, and one edge at right angles to the vertical plane. 4. With one diagonal at right angles to the vertical plane, and the other inclined at 45° to the horizontal plane.

In 1 and 2, first draw the plan in the required position, and project the elevations as shown. In 3, draw the elevation *a′ b′* at 45° with X Y, and project for the plan. Make *c a* equal to *a′ b′*.

In 4, first draw the plan *a* B C D and the elevation *a′* B′ D′ of the square when lying horizontally. Next, turn the elevation at an angle of 45° with the horizontal plane, as *a′ b′ d′*. Project from *a′ b′ d′* to meet parallels from C, D, and B, giving the plan *a b c d*.

PROBLEM 290.—To draw the plan and elevation of a square of 2½″ sides :—

1. When one of its diagonals is parallel to both planes, and the other makes 40° with the horizontal plane. 2. With two of its sides parallel to both planes, and its surface making 45° with the horizontal plane.

1. First obtain the plan *a* B C D and the elevation *a′* B′ D′ when placed horizontally, and from this obtain the elevation *a′ b′ d′*, and the plan *a b c d* when one diagonal is at right angles to the vertical plane, and the other is inclined to the horizontal plane at 40°. The position required is at right angles to this. Place the plan *a b c d* so that the diagonal *c b* is parallel to X Y. Project from this to meet parallels from *b′* and *d′*, giving the elevation *a′ b′ c′ d′*.

2. As in the last figure, draw the plan and elevation when the square is horizontal, and obtain from it the elevation and plan when inclined at 45°. Turn the plan at right angles, so that *a c* and *b d* may be parallel to both planes. Project the elevation, obtaining its height as in the previous figure.

EXERCISES

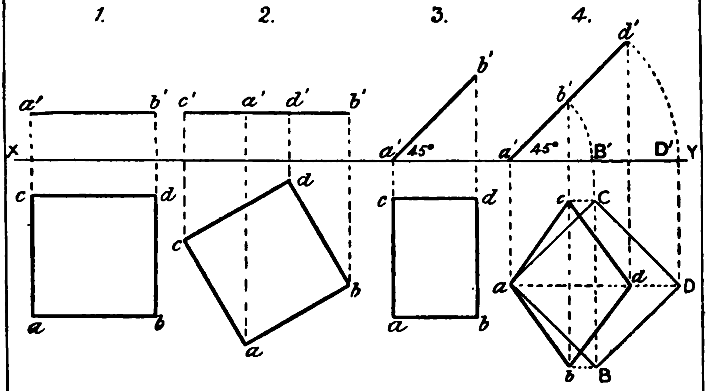
1. Draw the plan and elevation of a line 3″ long inclined at equal angles to both planes.

2. Represent the plan and elevation of an equilateral triangle of 3″ sides standing vertically on one side, with its plane at an angle of 45° to the vertical plane.

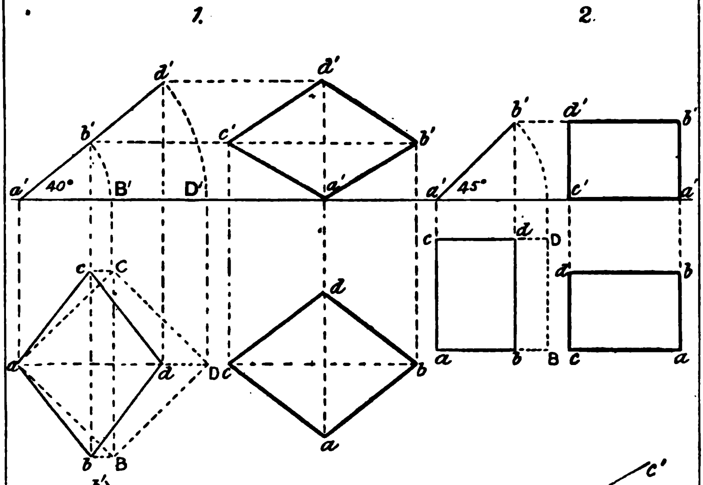
3. *a′ b′* and *a′ c′* (*Ex.* 3) are the elevations of two squares of 3″ and 2″ sides respectively, having their centres in a line parallel to X Y. The nearest edge of the larger square touches the vertical plane. Draw their plans.

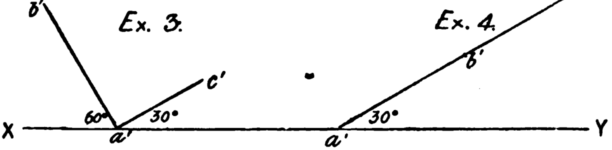
4. *a′ b′ c′* is the elevation of a square with one diagonal parallel to the vertical plane and inclined at 30° to it. Draw the plan. The diagonal *a′ c′* is 3″ long.

PROB. 289.



PROB. 290.





PROBLEM 291.—To draw the plan and elevation of a hexagon of 2″ sides in the following positions:—

1. Lying on the horizontal plane, with one side at an angle of 20° with the vertical plane. 2. With a diagonal parallel to the vertical plane, and inclined at 45° to the horizontal plane. 3. Standing vertically on one edge, with its plane inclined at 50° to the vertical plane.

1. Draw a line making 20° with X Y, and on it construct a hexagon of 2″ sides. This will be the plan. Project each angle to X Y for the elevation.

2. Draw a hexagon of 2″ sides, *a* B C D E F, with one diagonal parallel to X Y. This will be the plan when lying horizontally. Project to X Y, and turn this elevation at an angle of 45°, giving the required elevation, *a′ b′ c′ d′*. From each point in this elevation draw a projector to meet the parallel from the corresponding angle of the first plan. Then *a b c d e f* will be the required plan.

3. Draw elevation *a′* B′ C′ D′ E′ F′ and plan *a* D of the hexagon when its plane is parallel to the vertical plane. Turn the plan at an angle of 50°, and from this project the required elevation, *a′ b′ c′ d′ e′ f′*.

PROBLEM 292.—To draw the plan and elevation of a hexagon of 2″ sides, having two of its edges parallel to both planes, its nearest edge 1″ from each plane, and its surface inclined forwards, at 60° to the horizontal plane.

Obtain the elevation and plan of a hexagon 1″ from each plane when two sides are at right angles to the vertical plane, as in Fig. 2, *Prob.* 291. Now turn the plan at an angle of 90°, and place it so that *a b* is 1″ from X Y; and as the plane of the hexagon slopes forward, *d e* will be farthest from X Y. From this plan project to meet parallels from the first elevation, giving *a′ b′ c′ d′ e′ f′*, the required elevation.

Note.—The plans and elevations of other plane surfaces may be solved in a similar manner, and should be worked as exercises by the student.

PROBLEM 293.—To draw the plan and elevation of a square of 2¼″ sides, one edge being in the horizontal plane, making 45° with the vertical plane, and the surface of the square being inclined at 40° to the horizontal plane.

First draw the elevation, *a′ b′*, and plan, *a b c d*, when one edge is at right angles to the vertical plane and the surface is inclined at 40° to the horizontal plane.

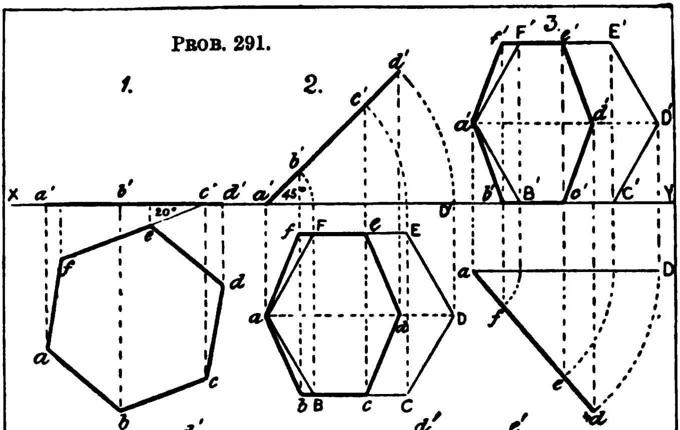
Now place the plan, *a b c d*, at the required angle of 45°, and project the elevation. As the points *a* and *c* are on the horizontal plane, project from them to the intersecting line, X Y. For the elevations of *b* and *d* project to meet the parallel from *b′*. Join the points thus obtained for the required elevation.

EXERCISES

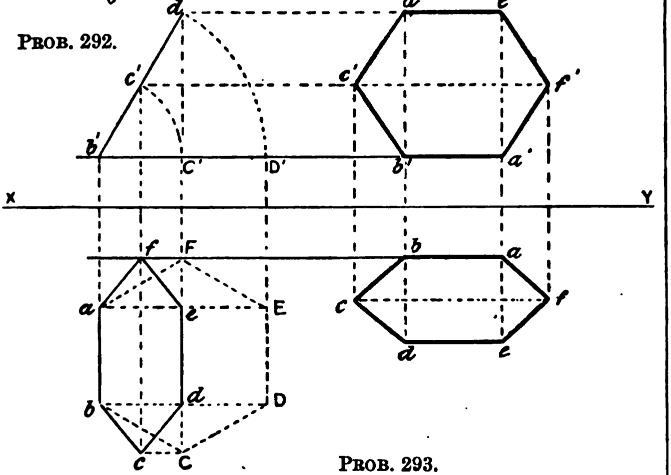
1. Obtain the plan and elevation of an equilateral triangle of 3″ sides when in a similar position to the square in Problem 293.

2. A regular pentagon of 2″ sides stands with one side in the horizontal plane, and inclined to the vertical plane at 45°. Its surface is inclined to the horizontal plane at 30°. Draw its plan and elevation.

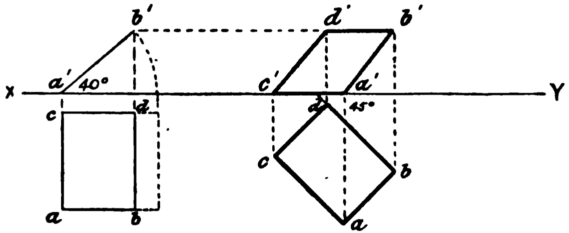
PROB. 291.



PROB. 292.



PROB. 293.



THE CUBE

PROBLEM 294.—To draw the plan and elevation of a cube of 3″ sides:—

1. Standing on the horizontal plane, with one face parallel to the vertical plane. 2. With one face on the horizontal plane, and with a vertical face inclined at 30° to the vertical plane. 3. Standing on one edge with one face, making an angle of 30° with the horizontal plane, and its vertical faces parallel to the vertical plane.

1. The plan will be the square *a b c d*, having two of its sides parallel to X Y. The elevation will be a square of the same size.

2. Place the square *a b c d* at an angle of 30° to X Y for the plan. From each angle of the plan draw projectors. Make *d′ h′* equal to *a b*, and draw *d′ b′* parallel to X Y, completing the elevation.

3. As the vertical faces are parallel to the vertical plane, draw *e′ f′* at an angle of 30° with X Y, and on it construct a square. This will be the elevation. For the plan, draw projectors from each angle. Take a point, *d*, at any given distance in front of the vertical plane, and draw *d g* parallel to X Y. Make *a d* equal to *e′ f′*, and draw *a f* parallel to *d g*.

Note.—The line *e h* will be dotted, because, when the cube is viewed from above, it would not be seen. In Figure 2 the line *c′ g′* is dotted for a similar reason.

PROBLEM 295.—To draw the plan and elevation of a cube of 3″ sides when standing with its vertical faces at right angles to both of the planes, and with one face inclined to the horizontal plane at an angle of 30°.

Draw the elevation and plan when the vertical faces are parallel to the vertical plane, as in Problem 294, Fig. 3. The required elevation and plan will be then seen when the projections already obtained are viewed at right angles.

Turn the plan already found so that *g f* is parallel to X Y, and project to meet the parallels from *a′ b′ f′* for the elevation.

Note.—The student should carefully follow the lettering, and note that the same point keeps the same letter all through.

PROBLEM 296—To draw the plan and elevation of a cube of 3″ sides standing on one edge, with its vertical faces inclined to the vertical plane at 40°, and its sloping faces inclined to the horizontal plane at equal angles.

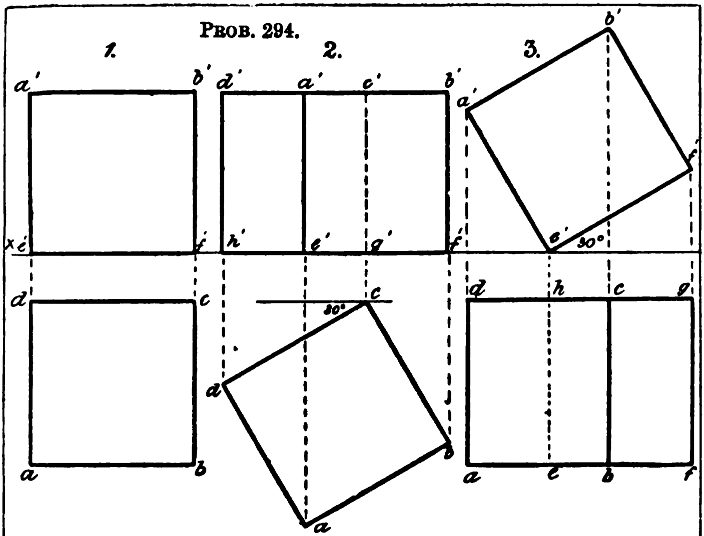
Draw the elevation, *a′* B′ C′ D′, and the plan, E C, of the cube when its vertical faces are *parallel* to the vertical plane. Turn the plan E C so that *a c* is at an angle of 40° with X Y. For the elevation, first obtain the front face by drawing projectors from *d* and *c* to meet parallels from D′ and C′. Obtain the back square in a similar manner, and join the angles of the two squares.

Note.—The new plan may be placed at the side of the first one, if thought desirable, as in the previous problem.

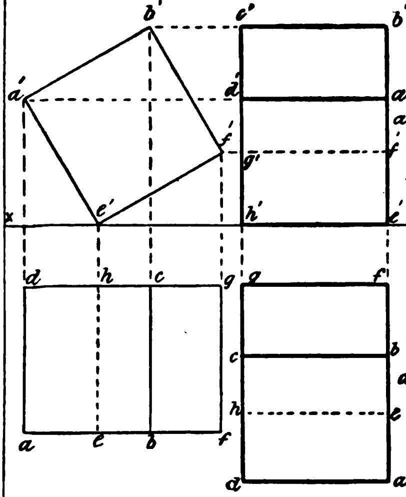
EXERCISE

Draw the plan and elevation of a cube standing on one edge when its vertical face is inclined to the vertical plane at 60°, and one of its sloping faces is inclined to the horizontal plane at 30°.

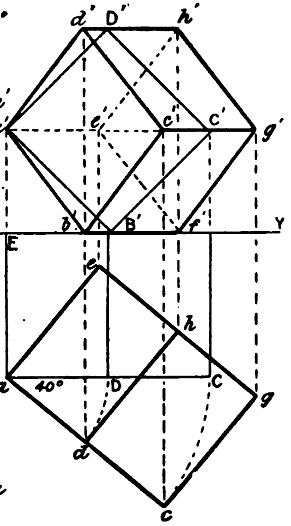
PROB. 294.



PROB. 295.



PROB. 296.



Note.—Very often the plan and elevation only of an object will not sufficiently represent it, and different views from other standpoints are needed. These may be more readily obtained by altering the position of the *intersecting line*, and assuming fresh vertical or horizontal planes, instead of moving the plan or elevation. If the student be only taking Section I. of the Science Syllabus, the problems dealing with the alteration of the intersecting or ground line may be omitted.

PROBLEM 297.—To draw the plan and elevation of a cube of 3″ sides, standing on the horizontal plane, with one of its faces parallel to the vertical plane. Also draw a second elevation when one of its vertical faces makes an angle of 50° with the vertical plane.

Draw the plan and elevation as in Problem 294, Fig. 1. Now, instead of placing the plan at the given angle, turn the intersecting line, **X Y**, until it makes 50° with the vertical plane, and project at right angles from each point of the plan, making the height of the cube the same as in the original elevation.

PROBLEM 298.—To draw the plan and elevation of a cube of 3″ sides standing with its edge on the horizontal plane, its vertical faces inclined at 40° to the vertical plane, and one of its sloping faces making 30° with the horizontal plane; also another view of the object when its vertical faces are at right angles to the vertical plane.

Draw the plan and elevation when the vertical faces are parallel to the vertical plane. Now, instead of turning the plan into a position so that the vertical faces would be at the required angle, as in Problem 296, imagine a fresh vertical plane whose intersection, **X′ Y′**, with the horizontal plane makes 40° with the vertical faces of the cube. If the paper be now turned so that **X′ Y′** be horizontal, the plan will be found inclined at 40° to the new intersecting line. From each point of the front face, *d a c b*, project at right angles to **X′ Y′**, and make *a″ b″ c″ d″* the same heights above **X′ Y′** as *a′ b′ c′ d′* are above **X Y**. Proceed in a similar manner for the back face, and complete the elevation, dotting the lines which are not visible.

For the **second elevation**, draw a new intersecting line, $\mathbf{X^2\ Y^2}$, at right angles to the vertical face, *d a c b*. Project at right angles for the elevation, and obtain the heights as before, *b‴* being the same distance above $\mathbf{X^2\ Y^2}$ that *b′* is above **X Y**; &c.

Notes.—1. Compare the first projections of this figure with that obtained in Problem 296, and notice that one plan answers for both elevations, thus securing greater clearness and saving of time. For the second position, compare Problem 295.

2. This important principle may be applied to any solid.

PROBLEM 299.—To draw the projections of a cube when the plane of one of its faces is inclined to the horizontal plane, and at right angles to the vertical plane.

Note.—To understand this problem let the student place a book on the table, and on the cover place a small cube, or any similar object. The plan of the cube in this position will be a square; but if we open the cover of the book, still keeping the cube fixed on it, we shall notice that the shape of the plan changes, although the object still remains the same size.

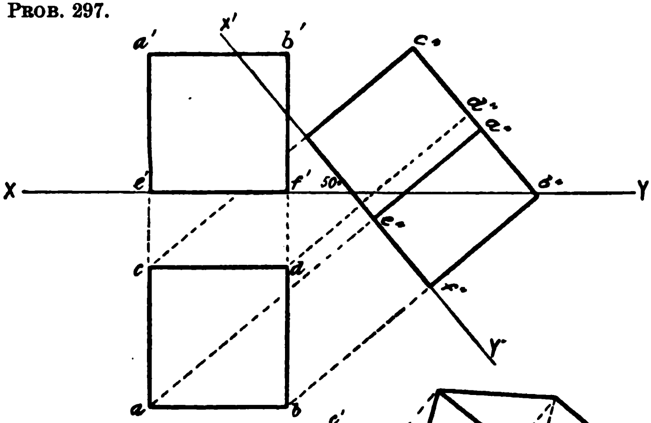
Draw the plan, *a* **B C D**, of the cube when standing with its face in the horizontal plane, and project the points of this face to **X Y**. Draw *a′ v′* at the required inclination (say 45°) to the horizontal plane, and rotate **B′ C′** into it, giving *b′ c′*. Draw perpendiculars to *a′ v′* at *a′*, *b′*, and *c′*, and make them equal to **C D**. Then *e′ c′* will be the elevation of the cube. To obtain the plan, draw projectors from the top face of the cube to meet parallels from **B, C**, and **D**. Project the bottom face in a similar manner, and complete the plan, dotting the invisible edges.

Notes—1. Any solid in a similar position may be projected in a similar manner by first drawing its true shape on the horizontal plane.

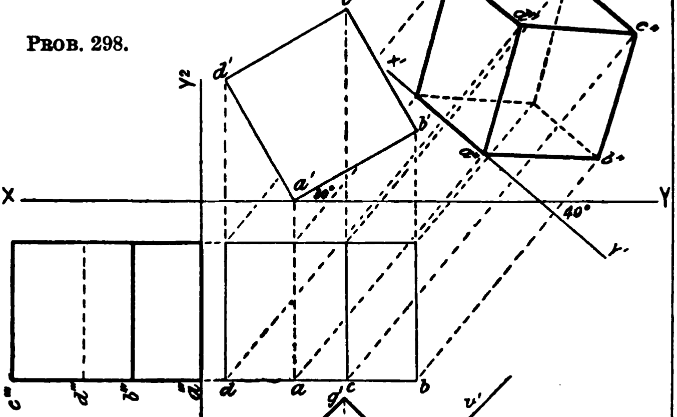
2. All lines that are parallel to the vertical plane are seen their true length in the elevation, and all lines that are parallel to the horizontal plane are seen in their true length in the plan.

3. The line *a′ v′*, which represents where the oblique plane (upon which the cube is supposed to stand) cuts the vertical plane, is called the **vertical trace**; and *a′* **H**, which represents where this oblique plane would cut through the horizontal plane, is called the **horizontal trace**.

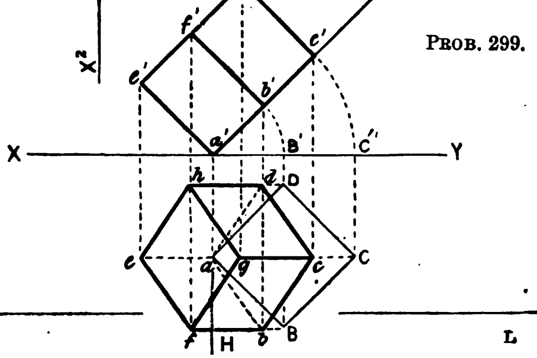
PROB. 297.



PROB. 298.



PROB. 299.



PROBLEM 300.—The elevation of a cube with its vertical faces at right angles to the vertical plane is given. Find the plan.

Let $a'g'$ be the elevation. Draw parallels to X Y from e', f', h'. Take any point H, and with $a'e'$ (the true length of the side of the cube) as radius, obtain points G and E. Draw E F parallel to H G, and join F G. Then E F G H will be the true shape of one of the faces of the cube. Project from c' and g', and obtain the width of the plan as shown.

Note.—The student will notice that when the cube is in this position the plan and elevation are of the same shape.

PROBLEM 301.—The plan of a cube with its vertical faces inclined to the vertical plane is given. Find the elevation.

Let $a g$ be the plan. We must first obtain the true shape of vertical face, as in Problem 300. Turn the plan so that its vertical faces would be parallel to the vertical plane as c D B A. Project from c, D, B, and A. Point d is on the horizontal plane, therefore mark D′ on X Y. With D′ as centre and $c g$ as radius, cut the projectors in c' and A′. Draw A′B′ parallel to D′c', and join B′c', thus obtaining the elevation of the cube when its vertical faces are parallel to the vertical plane. Project from a, b, and d to meet parallels from A′ and B′. Join the points thus obtained, giving $a' b' c' d'$. Obtain the back face of the figure similarly, and complete the cube.

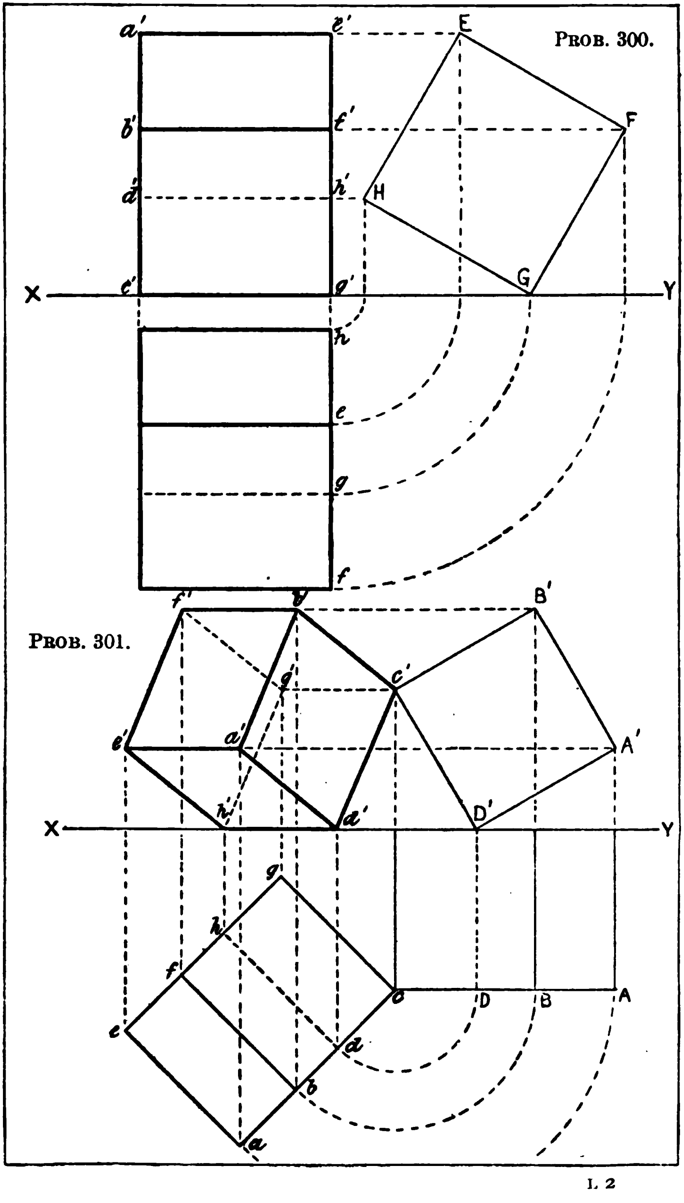
PROBLEM 302.—The elevation of a cube with its vertical faces inclined to the vertical plane at a given angle (say 45°) is given. To determine its plan.

Draw a line $a c$ (*Prob.* 301) at 45° to X Y, and project each point of the elevation to it, giving the points a, b, c, and d. Draw $a e$ at right angles to $a c$, to meet the projector from e', and complete the figure.

EXERCISES

1. Draw a square of 3″ sides standing on the horizontal plane with one side inclined at 60° to it. This square is the elevation of a cube. Obtain its plan and two fresh elevations, one on a plane inclined to the vertical plane at 30° to the right of the figure, and the other on a plane inclined to the vertical plane at 45° to the left.

2. Draw the plan and elevation of a square prism 4″ long and 2″ wide lying on one side. The length of the prism is inclined to the vertical plane at an angle of 30°. Also obtain a fresh elevation on a vertical plane inclined to one of the ends at 30°.



PROB. 300.

PROB. 301.

PRISMS.

The **square prism**, or parallelopiped, is exactly similar to the cube in its projections, and presents no fresh points of difficulty.

PROBLEM 303.—To draw the plan and elevation of an equilateral triangular prism 3″ long, side of triangle 2″, in the following positions:—1. Standing on its end with one side inclined to the vertical plane at 20°. 2. Lying on its side with its ends parallel to the vertical plane. 3. Lying on its side with its ends at right angles to the vertical plane. 4. Standing on one edge with its axis inclined to the right at 30° with the vertical plane, and one of its rectangular faces inclined to the horizontal plane at 45°.

1. The plan will be an equilateral triangle with its side *a c* inclined at 20° to X Y. For the elevation, project as shown.

2. Draw the elevation, which will be the equilateral triangle, *a′ b′ c′*, and project the plan.

3. First draw the elevation A B C, when the end is parallel to the vertical plane. Draw a line at right angles to the vertical plane, and transfer the widths to this line, giving the points *a*, *b*, and *c*. From these points draw parallels to X Y, make them 3″ long and complete the plan. Project to meet the parallel from B for the elevation.

4. Draw the triangle A′ *b′* C′ with its side A′ *b′* inclined to X Y at 45°. This will be the elevation of the prism when its axis is at right angles to the vertical plane. From this obtain the plan. Turn this plan so that *b e* makes an angle of 30° to the right with the vertical plane. Draw projectors from each point of the ends to meet parallels from C′ and *b′*, and complete the elevation.

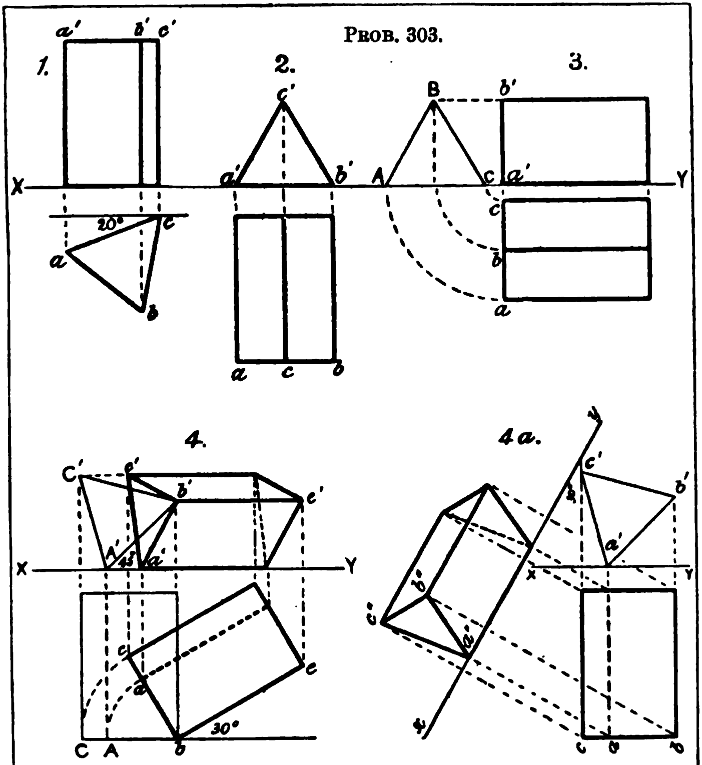
Note.—This may be solved by shifting the ground line instead of the plan, as in Problems 297 and 228. (*See* 4*a*.) Draw the elevation and plan when the axis is at right angles to X Y. Now as the prism is inclined to the right, draw the fresh ground line, *x y*, on the left side of the plan, and at an angle of 30° with its length. Project at right angles to *x y* and make *c″* and *b″* the same height above *x y* that *c′* and *b′* are above X Y.

PROBLEM 304.—To draw the plan and elevation of the same prism as above, when resting on one of the edges of its triangular ends, with one of its rectangular faces inclined at 30° to the horizontal plane, and its axis parallel to the vertical plane. Also a second elevation when its axis is inclined at 50° to the vertical plane.

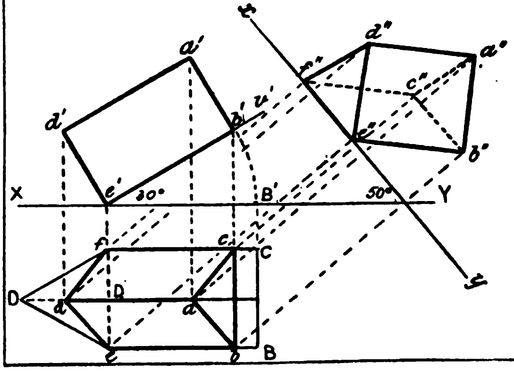
Draw the plan of the prism *e* B C *f* when lying on the horizontal plane as in Problem 303, Fig. 3, and project to X Y. At *e′* draw the vertical trace *e′ v′* of the oblique plane on which the prism is supposed to stand. Make *e′ b′* equal to *e′* B′. At *e′* and *b′* draw perpendiculars, make them equal to D D, the height of the prism, and join *a′ d′* to complete the elevation. Project from each point of the elevation for the plan.

To obtain the **second elevation**, draw *x y* at an angle of 50° with X Y. Project from *d*, *e*, and *f* at right angles to *x y*. As *e* and *f* are on the horizontal plane, *e″* and *f″* will lie on *x y*. Make *d″* the same height above *x y* that *d′* is above X Y. Project from *a*, *b*, and *c*. Make *b″*, *c″*, and *a″* the same distance above *x y* that *b′* and *a′* are above X Y. Join the ends and complete the elevation.

Prob. 303.



Prob. 304.



PROBLEM 305.—To draw the plan and elevation of any right prism. In this case an hexagonal prism 3″ long, sides of the hexagon 1½″, in the following positions:—1. Standing with one end in the horizontal plane and one of its sides inclined to the vertical plane at an angle of 50°. 2. Lying on one of its sides at right angles to the vertical plane. 3. Lying on one side with its axis parallel to the vertical plane.

1. Draw a line making 50° with X Y, and on it construct a regular hexagon of 1½″ sides. This will be the plan. The elevation may be found as shown.

2. On X Y construct a regular hexagon, this will be the elevation, and project the plan.

3. Proceed as in Problem 303, Fig. 3, by drawing the elevation A D when the end is parallel to the vertical plane. The plan will be the same as that in Fig. 2 turned at right angles, and the widths will be obtained in the same manner from the hexagon first constructed. Project from the plan to meet the parallels from C and D for the elevation.

Notes.—1. This method may be applied to all prisms. If a pentagonal prism be given, then first construct a regular pentagon, &c.

2. In the case of the hexagon it should be noted that A B is half F C, and A *f* is half A B. In this particular case the width of the prism could have been determined without the end elevation; for if the side be 1½″, then the full width *f c* would be 3″.

PROBLEM 306.—To draw the plan and elevation of any right prism (say octagonal), with its axis parallel to the horizontal plane, and inclined at 45° to the vertical plane.

1st method.—Proceed as in Problems 296 or 303.

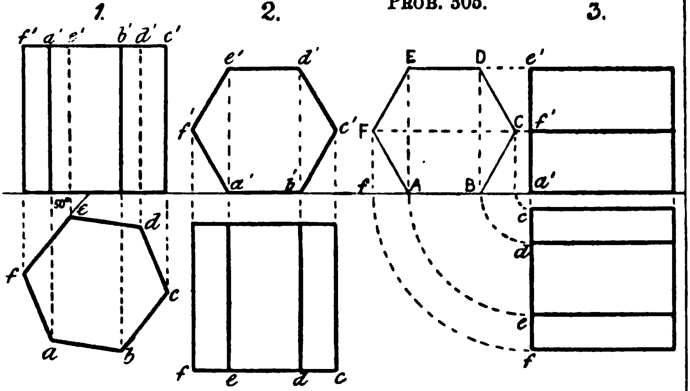
2nd method.—Let the sides of the octagon be 1″, and the length of the prism 3″.

Draw the end elevation A E. Project the plan *g l*. Draw *x y* at an angle of 45° to X Y. Imagine the paper turned so that *x y* is horizontal, then the plan *g l* will be seen to be at an angle of 45° to *x y*. Project at right angles to *x y* from *g*, *f*, *e*, and *d*. Make *c′*, *d′*, and *e′* the same height above *x y* that C, D, and E are above X Y, and from each point draw lines parallel to *x y*. Obtain the back end of the octagon by projecting to meet the parallels already drawn, and complete the figure.

PROBLEM 307.—To draw the projections of a prism (say pentagonal) when its axis is parallel to the vertical, but inclined to the horizontal plane.

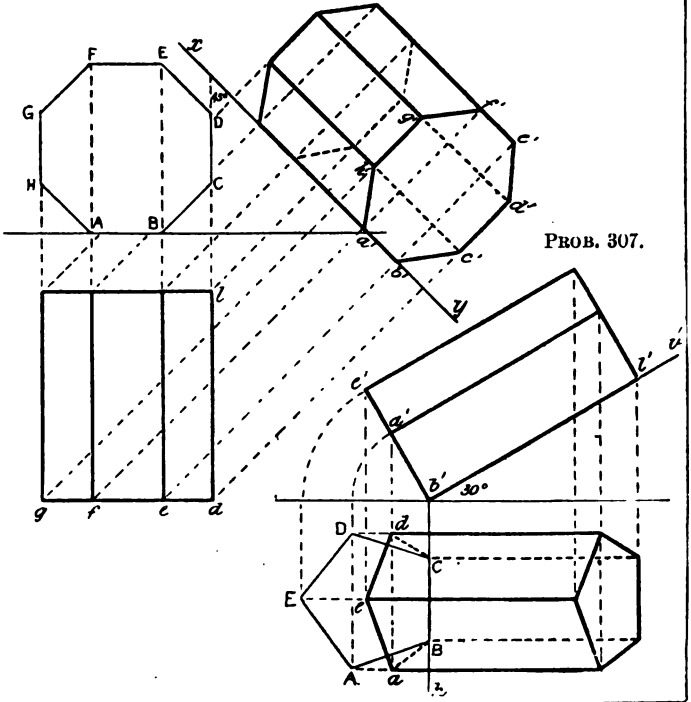
Let the solid rest on one of the sides of the pentagon, and be inclined at 30° to the horizontal plane. Draw the traces *b′ h* and *b′ v′* of the plane on which the solid stands. On *b′ h* construct a regular pentagon A D, and from each angle draw lines parallel to X Y. Mark off *b′ l′* the required length of the prism, and erect perpendiculars at *b′* and *l′*. Project from E and D to X Y, and rotate these points to the perpendicular at *b′*. From *a′* and *e′* draw parallels to *b′ l′*, giving the elevation. For the plan, project from each point of the elevation to the parallels already drawn, as shown.

Prob. 305.



Prob. 306.

Prob. 307.



PYRAMIDS

PROBLEM 308.—To draw the plan and elevation of a square pyramid 3″ high in the following positions:—1. With its base in the horizontal plane and one of its edges making 60° with X Y. 2. With one of its triangular faces in the horizontal plane, and its base at right angles to the vertical plane.

1. Draw *d c* making 60° with X Y. On it construct a square, and draw the diagonals. This will be the plan. For the elevation project from each angle of the square to X Y, and from the centre *e* project the altitude, making *e′* three inches above X Y. Join *e′* with each of the points on X Y.

2. First obtain the plan and elevation, when the pyramid is standing on the horizontal plane with one edge parallel to X Y. Now turn the elevation so that one side rests on X Y. Then *a′ b′ e′* will be the required elevation. For the plan project from *a′*, *b′*, and *e′* to meet parallels from *c*, *b*, and E.

PROBLEM 309.—To draw the plan and elevation of a pentagonal pyramid with its axis parallel to the vertical plane, and its base inclined to the horizontal plane at 45°. One of its edges to rest on the ground. Also an elevation of the pyramid when its axis is inclined to the vertical plane at 45°.

Draw the horizontal trace *a′ h* perpendicular to X Y, and the vertical trace *a′ v′* making 45° with it. Let the dimensions of the pyramid be as in the preceding problem. On *a′ h* construct the regular pentagon *a b* C D E, and find the centre F. Project from D, C, and F, to X Y, and turn these points into the vertical trace. At F′ draw F′*f′* perpendicular to *a′ v′*, and join *f′* with *a′*, *e′*, and *d′*. This will be the elevation. For the plan, project from each point of the elevation to meet the parallels from C, D, E, and F. Join *f* with *a*, *b*, *c*, *d*, and *e*.

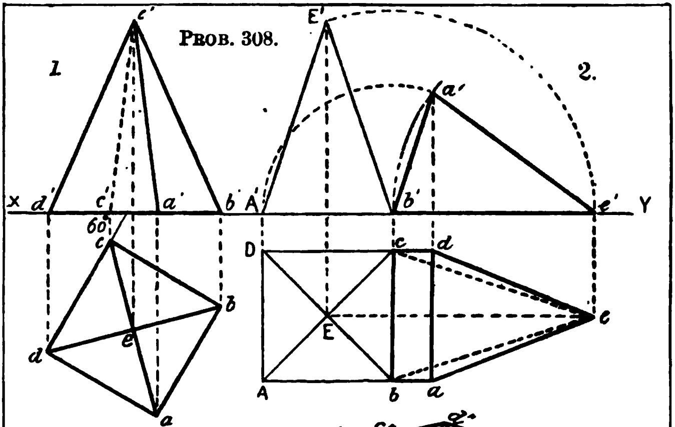
For the **elevation**, when the axis is inclined to the vertical plane, draw *x y* at 45° to the axis, and draw projectors from *a*, *b*, *c*, *d*, *e*, and *f*. Points *a* and *b* are on the ground, therefore their elevations *a″* and *b″* will be on *x y*. Make *e″* and *c″* the same distance above *x y* that *e′* is above X Y, and *d″* of corresponding height to *d′*. Join these points. Project *f″* as much above *x y* as *f′* is above X Y, and join with each point of the base.

PROBLEM 310.—To draw the plan and elevation of a tetrahedron of 2″ edge:—1. Standing with one face on the horizontal plane and one edge at right angles to the vertical plane. 2. Standing on one edge with its axis horizontal and parallel to the vertical plane.

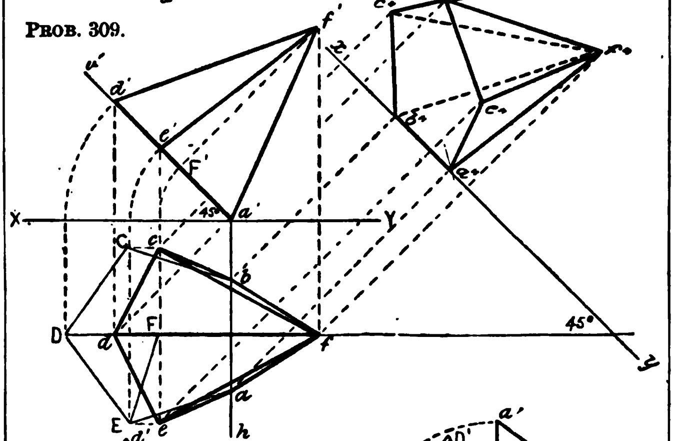
1. Draw an equilateral triangle *a b c* with one side at right angles to X Y. Find the centre *d* and join with each angle of the triangle. This will be the plan. For the elevation project from each point of the plan as shown. To obtain the height, draw from *d* a line at right angles to *b d*. With centre *b* and radius *b c* cut the perpendicular in D. Then *d* D is the required height. *b d* D is the true size of a section from the axis of the figure to one angle.

2. Draw the plan A *b c* as before, and project to X Y. Turn the elevation so that the axis is horizontal, obtaining the altitude as before. For the plan project as shown.

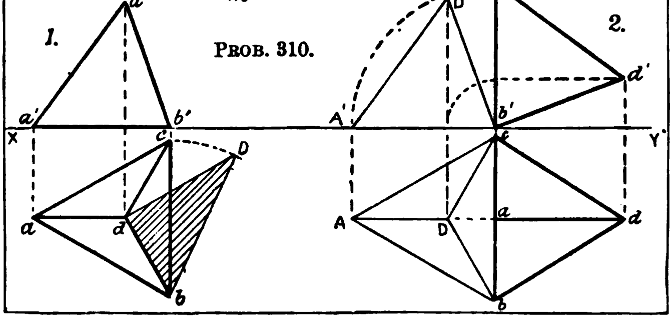
PROB. 308.



PROB. 309.



PROB. 310.



PROBLEM 311.—To construct the plan and elevation of an octahedron:—1. When one of its axes is vertical. 2. When one of its triangular faces is in the horizontal plane.

Note.—The octahedron has eight faces, each of which is an equilateral triangle. It is formed of two square pyramids placed base to base, and all its edges are equal. It has three diagonals, all of equal length.

1. Draw *a d* at any given angle to X Y. On it construct a square and draw the diagonals. This will be the plan. For the elevation draw a projector from *e*, and make the axis *e′ e′* equal to *a c*. Bisect *e′ e′* by *a′ c′* and draw projectors from each angle of the plan. Join the points *a′*, *b′*, *c′*, and *d′*, with the points *e′ e′*.

2. As each face is an equilateral triangle, draw *a b c*; this will be the plan of the bottom face. Describe a circle about the triangle, and draw the hexagon *a e*. Draw *d e*, *e f*, *f d*.

Project the elevation of the face *a d b*. To obtain the height, with centre *b′* and radius equal to an edge of the solid, as *a b*, cut the perpendicular from *d* in *d′*. Join *d′* with *a′* and *b′*. From *d′* draw *d′ e′* parallel to X Y to meet the projector from *e*. Join *e′* and *b′*.

Note.—The student may easily make an octahedron to illustrate these positions. If *a b* be the edge of the solid, develop eight equilateral triangles as shown, on cardboard or thick paper. Cut out the shape of the development, and by folding the edges of the triangles and gumming over with thin paper, the solid may be made.

THE CIRCLE

PROBLEM 312.—To draw the plan and elevation of a circle of 3″ diameter:—1. With its plane vertical and parallel to the vertical plane. 2. With its plane vertical and inclined to the vertical plane at 40°. 3. With its plane inclined to the horizontal plane at 45° and at right angles to the vertical plane.

1. The elevation will be the circle *a′ b′*, and its plan the straight line *a b*.

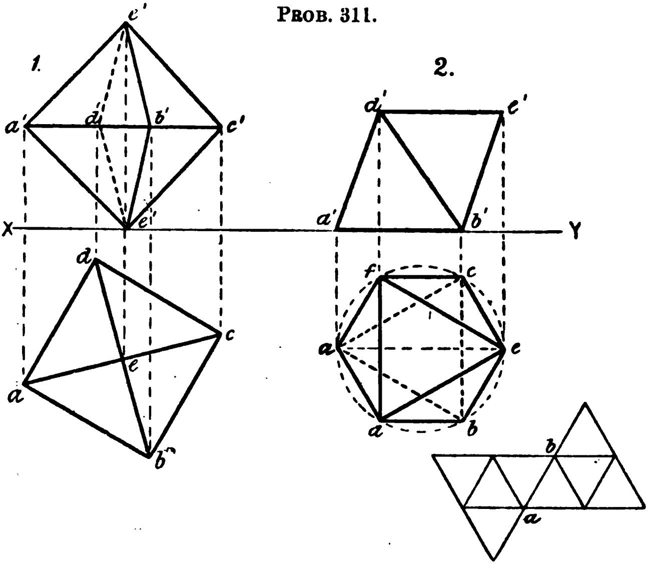
2. The plan will be the straight line *a b* inclined at 40° to X Y. For the elevation, which will be an ellipse, describe a semicircle on *a b*. Draw the semi-diameter *c* C. Take any number of points in the curve as E and D equidistant from C, and draw perpendiculars E *e*, D *d*. From *a*, *b*, *c*, *d*, and *e*, draw perpendiculars to X Y. Make *c′ c′* equal to *a b*, and bisect it by a horizontal line for the points *a′* and *b′*. Set off *e* E on each side of *a′ b′* for the points *e′ e′* and *d′ d′*. Draw the curve of the ellipse through the points thus obtained.

3. The elevation will be the straight line *a′ b′* at an angle of 45° to X Y. For the plan describe a semicircle on *a′ b′*, and divide it as in the preceding figure. Project from *a′*, *e′*, *c′*, *d′*, *b′*. Draw *a b*, and on each side of it set off the widths as in Fig. 2 and draw the ellipse.

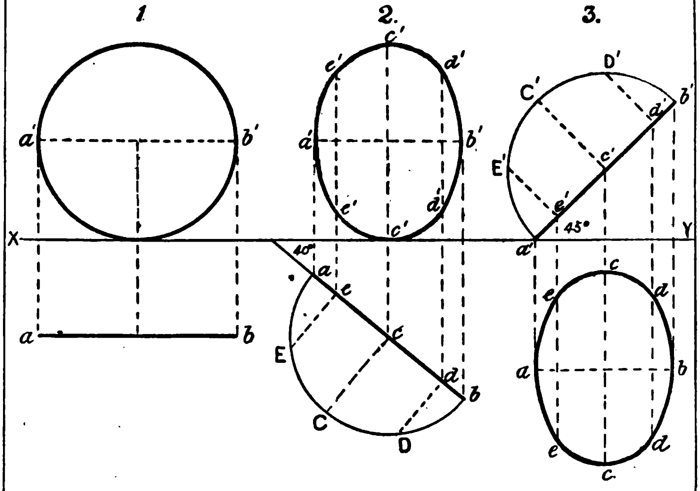
Notes.—1. It will be found convenient to divide the semicircle into 4 or 6 equal parts, thus obtaining either 8 or 12 points in the curve.

2. In large figures the French curve may be used to trace the ellipse.

PROB. 311.



PROB. 312.



PROBLEM 313.—To draw the plan and elevation of a circle of 3″ diameter when its plane is inclined at 60° to the horizontal plane, and a diameter is parallel to both planes. Also a second elevation, when the same diameter is inclined to the vertical plane at 45°.

First draw the elevation A B, when the plane of the circle is at right angles to the vertical plane and inclined to the horizontal plane at 60°.

Describe a semicircle on A B, and obtain the points D and E. From D, C, E, and B draw parallels to X Y. Draw a centre line *a′ b′*, and from this line set off on each side the distances C C, E E, &c., giving the points *c′ c′*, *e′ e′*, and *d′ d′*. The ellipse drawn through these points will be the elevation.

For the plan project vertically from each of the points *c′*, *d′*, *a′*. Then draw perpendiculars from D, C, E, and B, obtain these points on the projector from *c′*, and from each of the points draw parallels. The intersection of these lines with the vertical projectors will give the points *a*, *b*, *c*, *d*, and *e* of the plan. Through these points draw the ellipse.

For the **second elevation**, draw *x y*, making 45° with the diameter *c c*. Draw four lines parallel to *x y*, and the same distances above it as the lines from D, C, E, and B are above X Y. From *a* and *b* draw projectors to meet the lines A and B, giving the points *a″* and *b″*; from *d* and *d* draw projectors to D, giving the points *d″*, *d″*; from *c* and *c* draw projectors to C for the points *c″*, *c″*; and from *e* and *e* draw projectors to E for the points *e″*, *e″*. Through the points thus obtained draw the ellipse.

THE SPHERE

PROBLEM 314.—To draw the plan and elevation of two spheres of 3″ and 1″ diameters respectively, standing in contact with each other on the horizontal plane, the line joining their centres being parallel to the vertical plane.

As the line joining their centres is parallel to the vertical plane, the elevation will be two circles touching each other. Describe a circle of ½″ radius on X Y. Produce its diameter and make *a′* B equal to the sum of the radii of the two circles, that is 2″. Draw a line parallel to X Y and 1½″ distant from it. With centre *a′* and radius *a′* B intersect the parallel in *b′*. With *b′* as centre describe the larger circle.

For the plan, draw projectors from *a′* and *b′*, and with radii equal to 1½″ and ½″ describe two circles. Keep *a b* parallel to X Y.

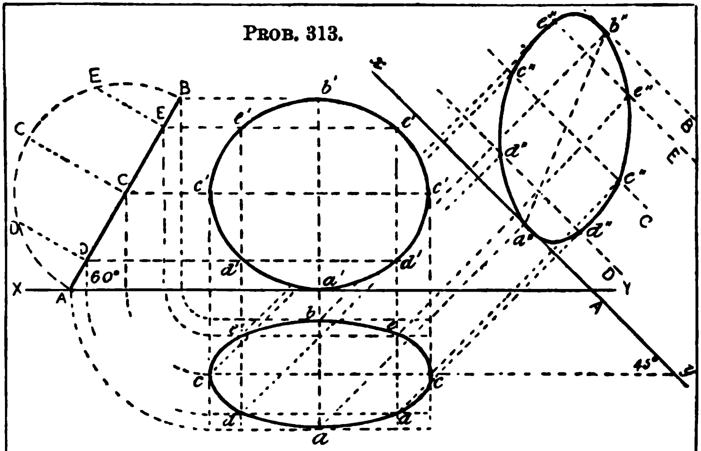
PROBLEM 315.—Four equal spheres of ¾″ radius, each touching the other three, rest on the ground plane; the lines joining the centres of the three spheres resting on the ground form an equilateral triangle, one side of which is at right angles to the vertical plane. Obtain the plan and elevation.

First draw the plan. Construct an equilateral triangle of 1½″ sides, having its side *a c* perpendicular to X Y. With each angle of the triangle as centre, and a radius of ¾″, describe three circles. The centre *d* of the triangle will be the centre for the fourth sphere.

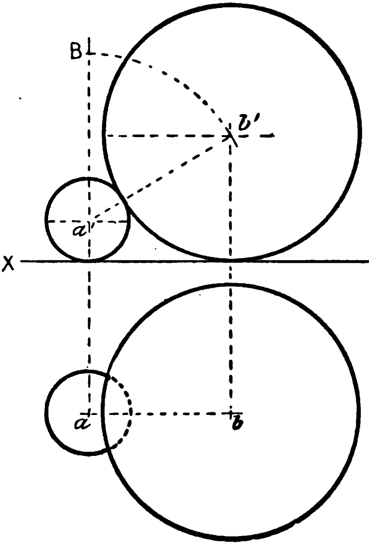
For the elevation project from *a* and *b*, and describe two circles from *a′* and *b′*. The circle described from *c* lies exactly behind the circle described from *a*. For the centre of the top circle draw a projector from *d*, and with centre *b′* and a radius of 1½″ cut this projector in *d′*.

Note.—If *b d* had been inclined to the vertical plane then the height of *d′* e the line *a′ b′* would be found by obtaining the height of the tetrahedron by uniting the centres of the four circles. (*Problem* 310.)

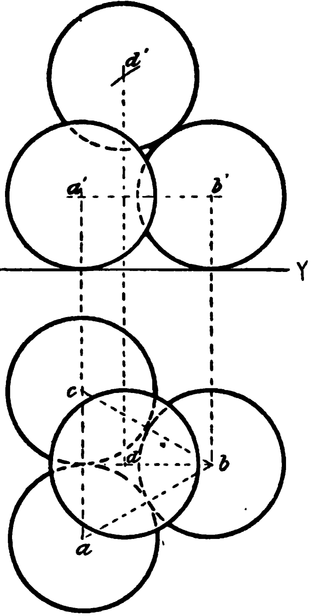
Prob. 313.



Prob. 314.



Prob. 315.



THE CYLINDER AND CONE

PROBLEM 316.—To draw the plan and elevation of a cylinder:— 1. Standing with its end in the horizontal plane. 2. Lying with its axis at right angles to the vertical plane. 3. With its axis parallel to both planes.

The projections in this case explain themselves.

PROBLEM 317.—To draw the plan and elevation of a cone:—1. Standing on the horizontal plane. 2. With its axis horizontal and at right angles to the vertical plane. 3. With its axis parallel to both planes.

These projections, like the cylinder, require no comment.

PROBLEM 318.—To draw the plan and elevation of a cylinder, diameter 3″, length 4″, lying on the ground with its axis inclined to the vertical plane at 30°.

Draw *d c* making 30° with X Y, and on it construct a rectangle 4″ long and 3″ wide. This will be the plan of the cylinder.

On *b c* describe a semicircle. Divide it into 4 equal parts, and draw lines through each point of division parallel to *a b* and *c d.* From each point in *b c* and *a d* draw perpendiculars to X Y, and obtain the ellipses as in Problem 312, Fig. 2.

PROBLEM 319.—To draw the plan and elevation of a cylinder with its axis inclined to the horizontal plane at an angle of 30°, and parallel to the vertical plane. Also a side elevation of the cylinder when viewed from the left at right angles to its first elevation.

The elevation will be a rectangle with its side *a′ b′* inclined at 30° to the horizontal plane.

For the plan describe a semicircle on *a′ c′*, divide it, and draw parallels to *a′ b′* as in the previous problem. From each point in *a′ c′* and *b′ d′* draw perpendiculars. Draw *b c* parallel to X Y at any convenient distance, and on each side of it set off the ordinates as in the previous problems.

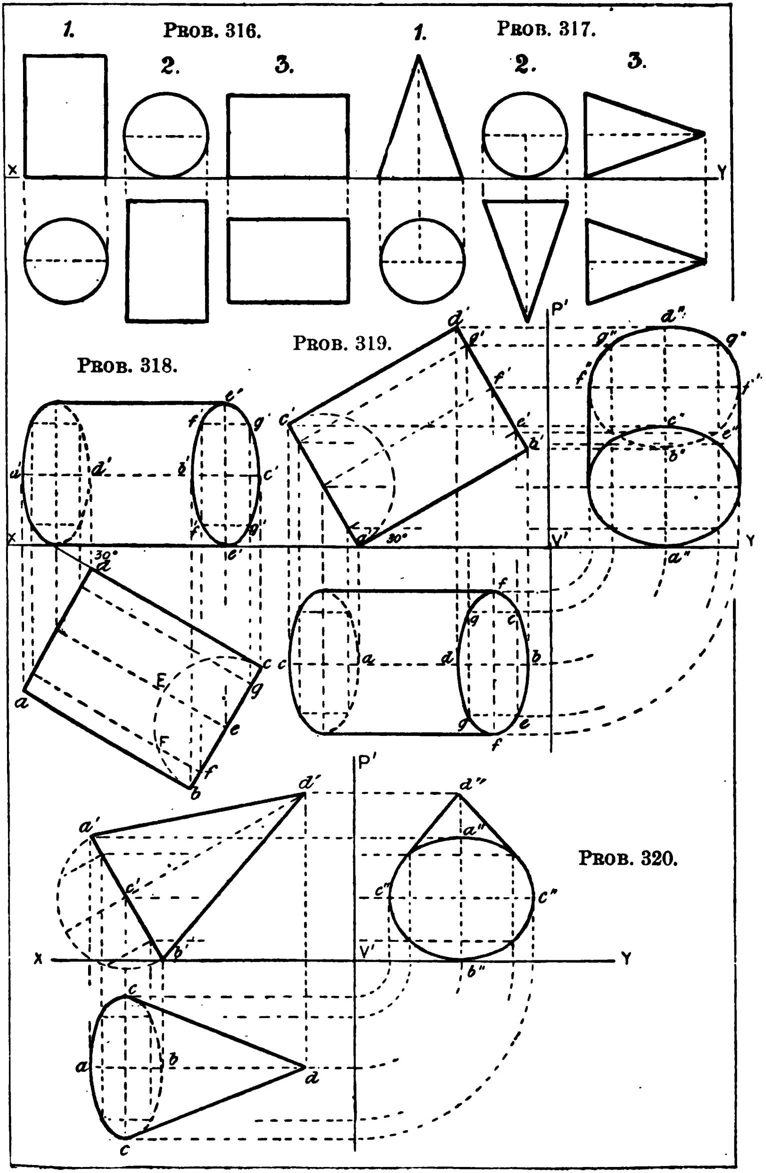
To obtain the **side elevation** first draw V′ P′, the side elevation of the vertical plane. Next draw the centre line *a″ d″*, and place it as far from V′ P′ as the centre line, *b c*, of the plan is from X Y. The cylinder will then be shown in elevation its correct distance from X Y; whatever distance *f* is from X Y in the plan, *f″* should be the same distance from V′ P′ in the side elevation. Project horizontally from each point of the elevation, and set off the ordinates on each side of the centre line as before; or turn the widths from the plan into X Y as shown.

PROBLEM 320.—To draw the plan and elevation of a cone, having its base inclined at 60° to the horizontal plane, and its axis parallel to the vertical plane. Also a side view at right angles to the first elevation.

Draw *a′ b′* making 60° with X Y. Bisect it and set up the height of the cone *c′ d′*. Join *a′* and *b′* with *d′*. This will be the elevation.

For the plan describe a semicircle on *a′ b′*, and divide it as in the previous problems. Draw a centre line *a d* parallel to X Y. Set off the ordinates, draw the ellipse and the sides of the cone.

For the **side elevation** draw V′ P′, and set off the centre line, *b″ d″*, as shown. The rest of the projection is similar to that of the cylinder. For the sides of the cone draw tangents to the ellipse from *d″*.


PROB. 316.
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PROB. 317.
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PROB. 318.
PROB. 319.
PROB. 320.
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PROBLEM 321.—To draw the plan and elevation of a cone lying with its side in the horizontal plane and its axis parallel to the vertical plane.

Let the diameter of its base be 2½″, and its altitude 3″.

Draw the elevation A *b′* F when the cone has its axis vertical. Turn this elevation so that *b′* F will be horizontal, then *a′ b′ f′* will be the required elevation.

For the plan describe a semicircle on *a′ b′*, and draw perpendiculars from each point. Draw the centre line *b f* parallel to X Y, and complete as in the previous problems.

PROBLEM 322.—To draw the plan and elevation of a cone lying with its side in the horizontal plane, and its axis inclined to the vertical plane at 45°.

Obtain the elevation and plan when the axis is parallel to the vertical plane as in Problem 321. This plan may now be placed so that its axis makes 45° with X Y, and a new elevation projected having the same height as *a′ b′ f′*. The better method, however, is to move the intersecting line to the required angle (45°). If the paper be now turned so that *x y* is horizontal, then *b a f* will represent the ***plan*** of the cone in the required position. In projecting the elevation, care must be taken to make each point in the new elevation the same distance above *x y* as the corresponding point is above X Y. First project from *b* to *x y*, obtaining *b″*, next draw from *a* and make *a″* as far above *x y* as *a′* is above X Y. Then *a″ b″* is the elevation of *a b*. Now project *c c*, *d d*, *e e*, and *f*. It is only necessary to set off the height of *c″*, *d″*, and *e″* once, as a parallel to *x y* will give the opposite point. To complete the elevation, draw a tangent to the curve from *f″* for the side of the cone.

Notes.—1. A projection of the cone at any other angle may be obtained in a similar way, whether it be on its side, or as in Problem 320.

2. The projections of a cylinder having its axis inclined to the vertical plane may be obtained in the same manner.

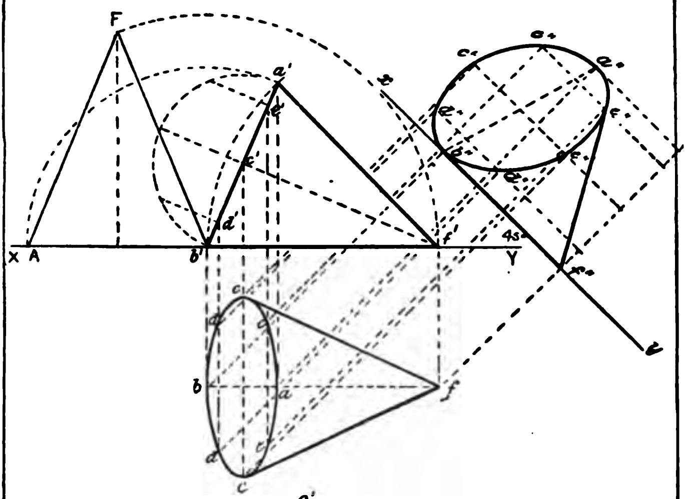
PROBLEM 323.—A pentagonal right prism, side of base 1·5″, height 1″, rests on a horizontal plane. On it is placed a right cone, whose base circle touches the sides of the top of the prism. The height of the cone is 2″. Draw the plan and an elevation on a plane making 40° with one side of the base of the prism. (*Sc.*, 1889.)

Draw *a b* making 40° with X Y, and on it construct a regular pentagon of 1·5″ sides. Find its centre and inscribe a circle. This will be the plan.

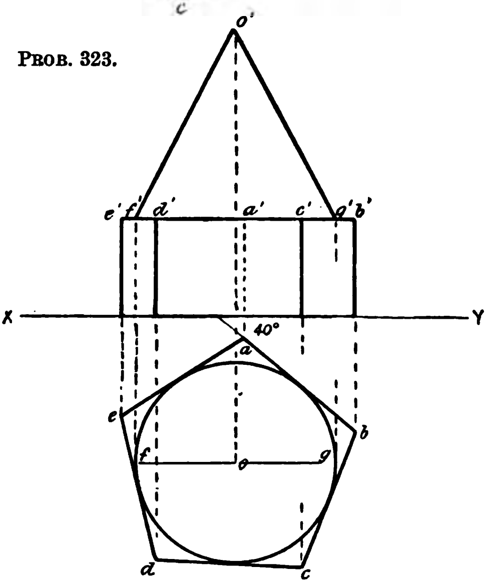
For the elevation, project from each point of the pentagon to a distance of 1″ above X Y, and draw *e′ b′* parallel to X Y, giving the elevation of the prism. Draw a diameter through *o* parallel to X Y. Project from each extremity of the diameter to *e′ b′*, and from *o* to a distance of 2″ above *e′ b′*. Join *o′* with *f′* and *g′*.

PROB. 321.

PROB. 322.



PROB. 323.



SECTIONS

The plan and elevation of an object frequently do not furnish all the information that is necessary to construct it. In the construction of a building, for instance, not only is it necessary to show its height, and the space covered by it on the ground, but also its internal structure. To represent this we must imagine the building to be cut through by a vertical plane, and a drawing made of the cut or **section.** This drawing would show the position, arrangement, and thickness of the floors, &c.

The portion of the object that is cut through is usually indicated by drawing a series of equidistant parallel lines, making an angle of 45° with the intersecting line.

The difference should be noticed between the **sectional plan and elevation,** and the **true shape** of the section. The *sectional plan* is the appearance of the object when viewed vertically from above; the *sectional elevation* is its appearance when viewed horizontally forwards; the *true shape of the section* is seen when the section is looked at perpendicularly to the plane of section.

The sectional plan will only show the true shape of the section when the section is made by a *horizontal* plane passing through the object. The sectional elevation shows the true shape when the section is made by a plane at *right angles to the horizontal plane* and *parallel to the vertical plane.* In all other sections the true shape should be projected at right angles to the plane of section.

PROBLEM 324.—The elevation of a cube is given. Find the sectional plan and the true shape of the section made by a plane at right angles to the vertical plane, passing through $a'\,b'$.

Find the plan of the cube. From a' and b' draw perpendiculars to X Y. Then $a\,b$, $a\,b$ will represent the plan of the section. Draw the parallel section lines and complete.

Note.—The portion of the cube above $a'\,b'$ must be considered to be removed. The part of the plan in thick line would then be the only portion visible.

For the **true shape** project perpendicularly from a' and b', and set off the width $a\,a$ equal to $a\,a$ in the plan.

PROBLEM 325.—The plan of a cube cut by a vertical section through $a\,b$ **is given. Obtain its elevation and the true shape of the section.**

Project from c and d. Make $c'\,c'$, $d'\,d'$ equal to the side of the plan, and join. From a and b project to the elevation for the section.

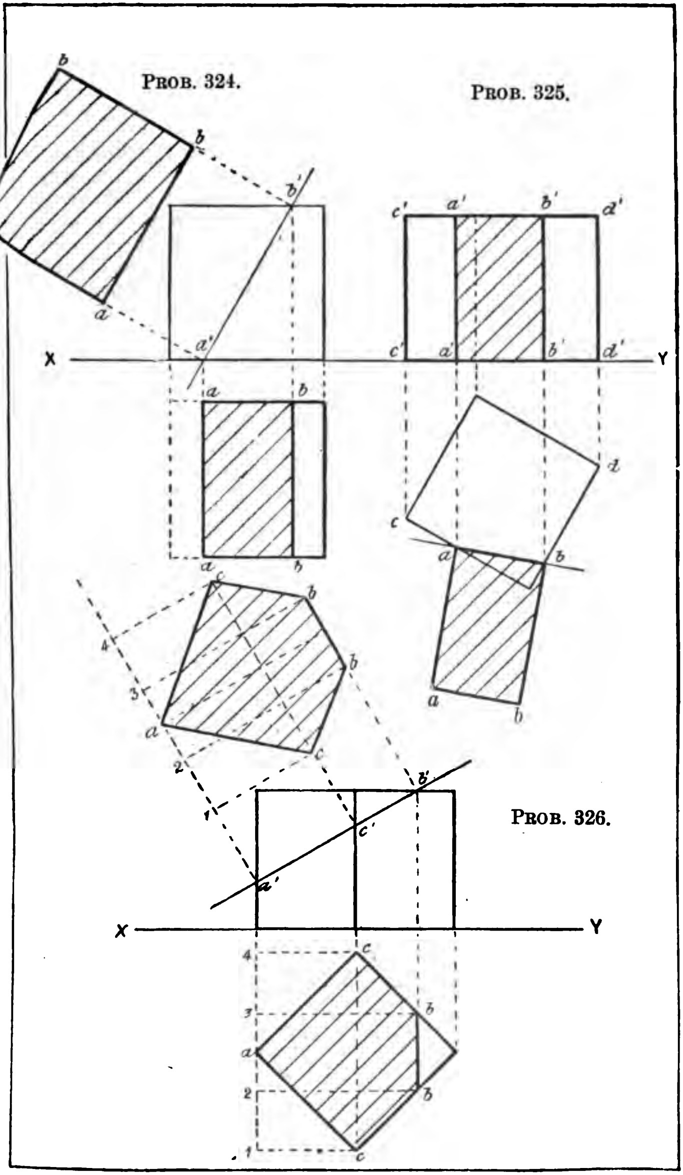
Note.—It is not necessary to project from the front corner, as the portion of the cube below $a\,b$ is supposed to be removed.

The **true shape** is the rectangle $aa\;bb$ having its side aa equal to $a'\,a'$.

PROBLEM 326.—A cube is cut by a plane passing through $a'\,b'$. **Obtain the plan and the true shape of the section.**

Draw the plan of the cube. From b' project to the plan. The whole of the top of the cube to the left of bb is cut away, and must be shaded with sectional lines.

For the **true shape,** project at right angles from each point where the section plane cuts the cube. Mark the point a on the projector from a', and draw a centre line perpendicular to $a'\,a$. Make $c\,c$ and bb equal to cc and bb in the plan.


PROB. 324.
PROB. 325.
PROB. 326.
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PROBLEM 327.—A cube of 2″ sides stands on the horizontal plane, two of its vertical faces are inclined at 30° to the vertical plane. The cube is cut by a plane which is perpendicular to the vertical plane, makes an angle of 60° with the horizontal plane, and passes through the centre of the top face of the cube. Draw the plan, elevation, and section. (*Art*, 1887.)

Draw the plan of the cube with two of its sides inclined at 30° to X Y, and project the elevation. The position of the section plane must now be ascertained. Find the centre *o* of the plan. As the section plane is perpendicular to the vertical plane, the line *e e* will indicate where the plane cuts the top of the cube. Project this line to the elevation, and from *e′* draw *e′ f′* to meet X Y at an angle of 60°. From *f′* project to the plan. If we imagine the portion of the cube above *e′ f′* to be removed, then *f a b c f* will be the sectional plan.

To obtain the **true shape** of the section, we may project at right angles from each point where the section plane cuts the elevation, and transfer the widths of the section from the plan as in Problem 326, or we may project from the plan and transfer the distances *f′* 1 and *f′ e′* from the elevation. *Both methods* are shown in this problem. The student must be guided by the space at his disposal, and the general arrangement of the figure, as to which is the better course to adopt. From *e*, *e*, *f*, *f*, draw lines parallel to X Y. Set off *f′* 1 and *f′ e′* equal to *f′* 1 and *f′ e′* in the elevation. From *e′* draw a perpendicular to meet the parallels from *e e*. From 1 draw a perpendicular to meet the parallel from *a*, and from *f′* draw a perpendicular to meet the parallels from *f f*. Join the points thus obtained.

Note.—The student will notice that the *verticals* in the sectional plan and the true shape are equal.

PROBLEM 328.—The plan of a cube cut by a vertical section plane, 1 4, is given. Draw the elevation and the true shape of the section.

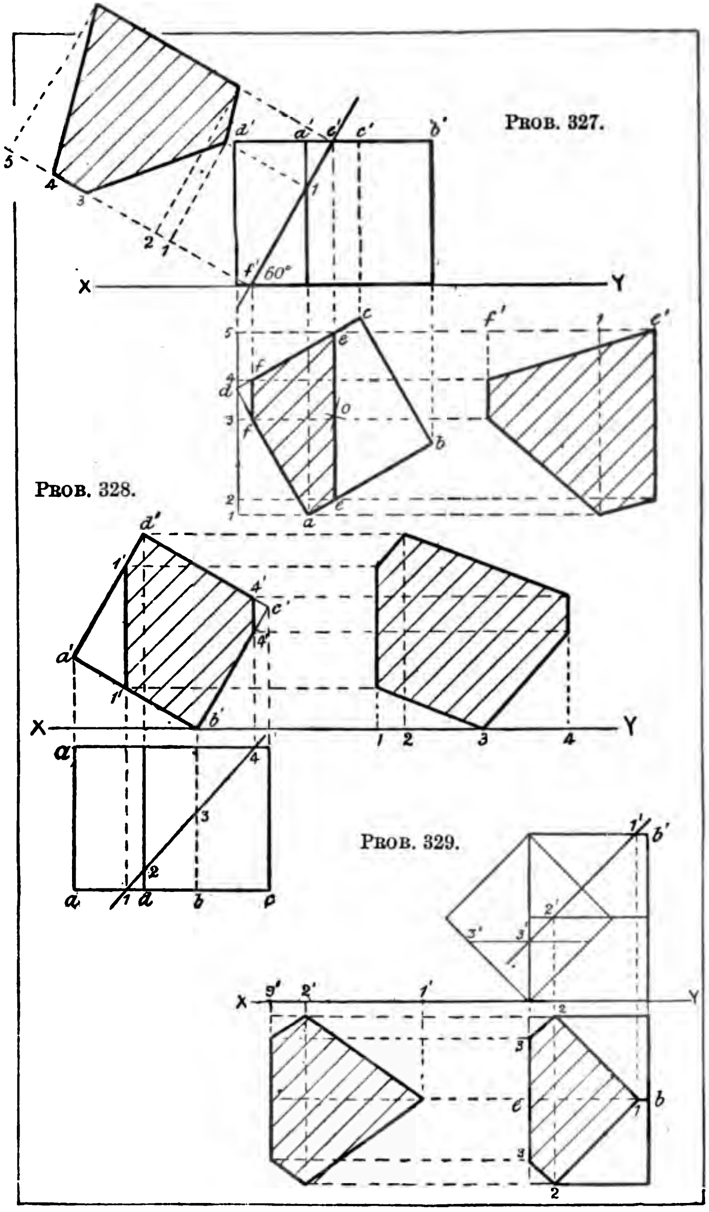
Draw perpendiculars from *a*, *b*, *c*, *d*. From *b′* with radius equal to *a a*, cut the projectors from *a* and *c* in *a′* and *c′* and complete the square. Project from 1 and 4 and complete the section.

For the **true shape** project from each point of the section parallel to X Y. On X Y set off 1, 2, 3, 4, equal to the same distances as in the plan. From 1 project to the parallels from 1′ 1′ from 2 to the top line, the point 3 is on X Y, and from 4 project to the parallels from 4′ 4′. Join the points thus obtained.

PROBLEM 329.—The elevation of a cube cut by a section plane, 1′ 3′, is given. Obtain its plan and the true shape of the section.

The plan will be the same shape as the elevation. From 1′ project to the corresponding line in the plan, from 2′ obtain 2 2. The point 3′ falls on the face of the square which is represented by a line in both elevation and plan; hence to obtain the width of the section draw the true shape of the end of the figure. The horizontal line drawn through 3′ will be the required width. Set off *e* 3 on each side of *e b* equal to 3′ 3′. Draw 3 2, 2 *b*.

For the **true shape** project as shown, setting off the distances 1′, 2′, 3′ on X Y equal to 1′, 2′, 3′, in the elevation.



PROB. 327.

PROB. 328.

PROB. 329.

PROBLEM 330.—The plan of a cube cut by a vertical plane, 1 4, is given. Obtain its sectional elevation.

The elevation of the cube will be of exactly the same shape as the plan. The points 2 and 3 of the section may be projected to the corresponding lines of the elevation, giving the two points 2′ and 3′. As the vertical faces of the cube are represented by one straight line in both plan and elevation, it will be necessary to draw the true shape of the vertical face as in the previous problem.

Draw parallels from each point of the elevation. Set off *b* 1, *b d*, *b f*, *b* 4, *b h* on X Y, and from *b*, *d*, *f*, and *h* draw perpendiculars to meet the parallels from the corresponding point of the elevation, giving the elevation of the cube when turned with its vertical face *b h* in front and parallel to the vertical plane. From 1 and 4 draw perpendiculars, giving the widths of the section on the vertical faces. From 1′ 1′ draw parallels to meet *e′ c′*, and from 4′ 4′ draw parallels to meet *f′ d′*. Join the points thus obtained and complete the section. Notice that the angles *d* and *f* are cut away, and are therefore not shown in the elevation.

PROBLEM 331.—The plan of a cube cut by a plane parallel to the vertical plane is given. Obtain its elevation.

Draw the elevation of the cube as in Problem 301. For the section find each point where the section plane cuts the lines of the plan, on the corresponding line of the elevation. Point 1 is on line *a e*, then project from 1 to *a′ e′*, point 2 is in *d h*, then 2′ will be in *d′ h′*, 3 is in *b f*, then 3′ will be in *b′ f′*, 4 cuts *b c* and *c d*, then its projections will be in *b′ c′* and *c′ d′*.

Join the five points and complete the section. The **true shape** will be the same as the section obtained, because the section plane is parallel to the vertical plane.

PROBLEM 332.—The elevation of a cube cut by an oblique plane at right angles to the vertical plane is given. Obtain its sectional plan, and true shape of the section.

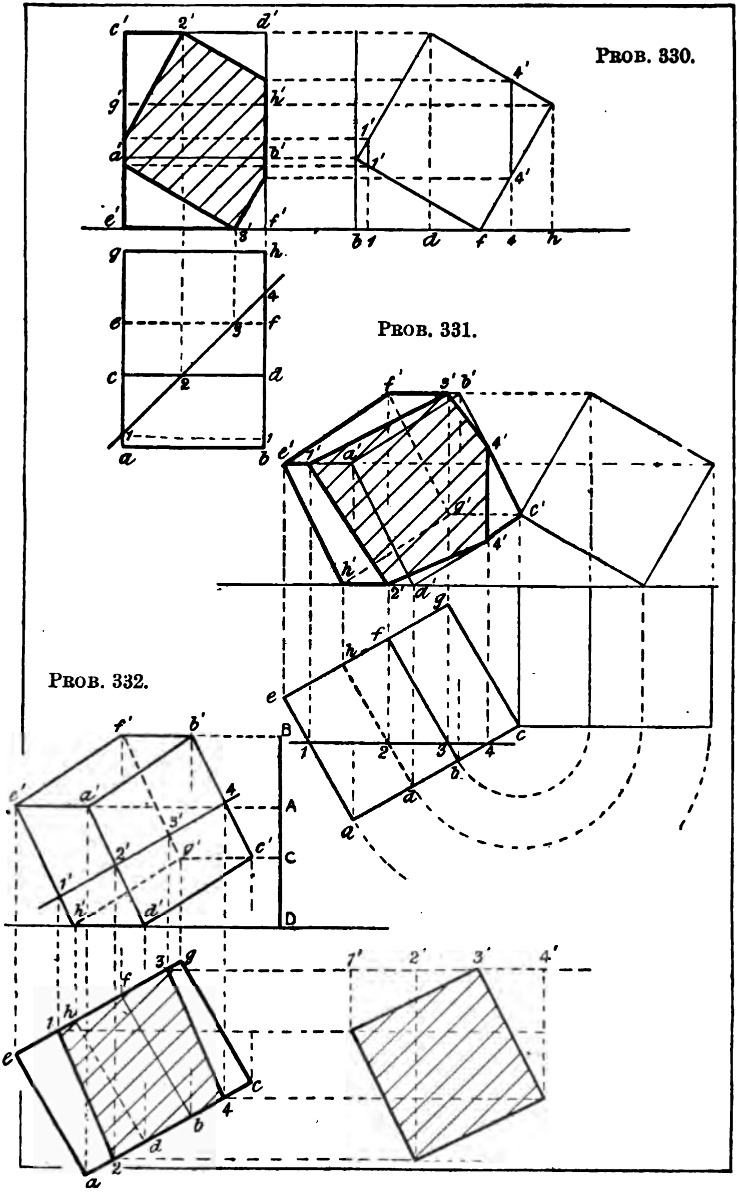
To obtain the plan, first draw the elevation D C A B of one face of the cube when at right angles to the vertical plane. The plan of this vertical face would be a line of exactly the same length. Project from each of the points *a′*, *b′*, *c′*, *d′*, and mark *c*. With *c* as centre and D B as radius, intersect the projector from *a′* giving *c a* the plan of the front vertical face. On *c a* mark off *c b* and *c d* equal to D C and D A.

Draw projectors from each point of the elevation of the back face, and from *a* draw a perpendicular *a e* to meet the projector from *e′*. Draw *e g* parallel to *a c*. Join *g c*, *b f*, and *d h*.

For the **section** project from 1′ to *e h*, 2′ to *a d*, 3′ to *f g*, and 4′ to *b c*. Join 1 and 2, 3 and 4, and complete the section.

The **true shape** of the section will be found as in the previous problems.

Note.—The cube has been treated in a variety of positions, and the student should apply similar methods of treatment to the square prism.



PROB. 330.

PROB. 331.

PROB. 332.

PROBLEM 333.—The plan of a square prism cut by a vertical plane is given. Find the elevation.

In drawing the elevation, *b′ d′* will equal *a c*. The section may be readily understood by following the lines.

PROBLEM 334.—The plan of an equilateral triangular prism is given. Obtain the sectional elevation and the true shape of the section when cut by a vertical plane passing through 13.

Project the elevation, obtaining its height by finding the height of the equilateral triangle described upon *a b*. Draw perpendiculars from 1, 2, and 3, to the corresponding lines of the elevation. Join the points thus obtained, and complete the elevation.

For the **true shape**, draw as shown, obtaining the heights from the elevation, or project from the elevation as in Problem 327.

PROBLEM 335.—The elevation of an hexagonal prism having one of the diagonals of its ends parallel to X Y is given. Draw the plan and the true shape of the section when cut by a plane, 1′ 4′, at right angles to the vertical plane.

Draw the plan, and project from 4′. For the **true shape** project horizontally from each point of the section. Transfer 1′, 2′, 3′, 4′, from the elevation, and complete as shown.

PROBLEM 336.—The elevation of an hexagonal prism is given. Obtain the sectional plan when cut at right angles to the vertical plane by the plane, 1′ 3′.

Draw the plan as in Problem 305. For the width of the section on the vertical face make 3 3 equal to *a b*. The problem may be easily followed from the construction lines.

PROBLEM 337.—The plan of an hexagonal prism with its axis inclined to the vertical plane is given. Obtain the sectional elevation made by a vertical plane passing through 1 4.

Project from each point of the plan. On *a d* construct half a regular hexagon as shown, and set off twice *c c* for the height of the elevation. Project from 1, 2, 3, and 4 for the section as shown.

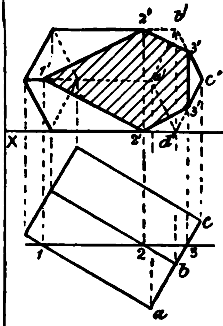
PROBLEM 338.—The elevation of an hexagonal prism with its axis inclined to the vertical plane is given. Obtain the sectional plan when cut by a plane, 1′ 4′, at right angles to the vertical plane.

First obtain the true shape of the end. Project horizontally from *e′* and *f′*. Take any point A and draw the diagonal A D at an angle of 60° with X Y. On it describe a circle and inscribe the hexagon in it. Project from each angle of the elevation. In the projector from *f′* take any convenient point *f*.

With centre *f* and radius F C, intersect the projector from *c′*. Draw *f c* and mark *e* and *d*. Now draw perpendiculars to *f c* from *f*, *e*, *d*, and *c*, meeting the projectors from the back face in *n*, *m*, *l*, and *k*. Join *n k*.

For the **section** draw perpendiculars from 1′, 2′, 3′, and 4′. Join the points thus obtained, and complete the figure.

PROB. 333.



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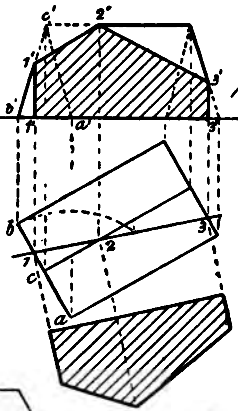

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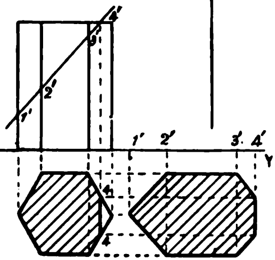

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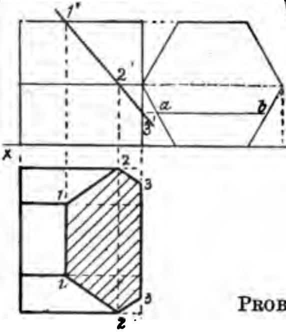
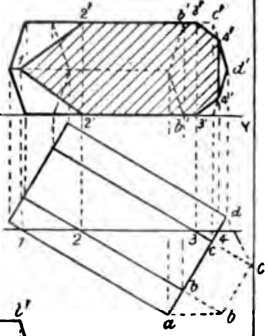


PROB. 337



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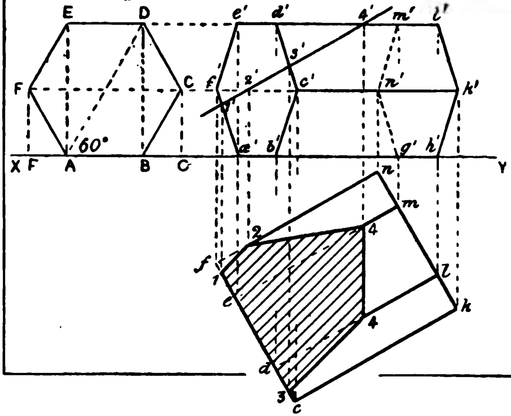


PROBLEM 339.—The elevation of a square pyramid cut by an oblique plane is given. Draw the plan, side elevation when viewed at right angles from the left, and true shape of the section.

Draw the plan. Project 1′ and 3′ to it. It is evident that 2′ cannot be projected to the corresponding line in the plan, because the line is vertical.

To obtain the plan of 2′ draw a horizontal line *a a* through 2′. Make 2 2 in the plan equal to *a a*, and complete the section.

For the **side elevation** draw V′ P′, project the angles of the plan on to it, transfer the points to X Y, and complete the second elevation. From 1′ 2′, and 3′ project horizontally to meet the edges of the pyramid. Join the points thus obtained, and complete the figure.

Note.—Finding the side elevation is the *general method* for ascertaining the width of the section of a pyramid when the point cannot be projected directly from the elevation to the plan. 2′ 2′ in the elevation is the required width for 2 2 in the plan. In a pyramid having the diagonals of its base at right angles, as in the square and octagon, a horizontal line drawn through the centre of the section as *a a* will give the width of the section at that point directly.

For the **true shape** project as shown, making 2 2 equal to 2 2 in the plan.

PROBLEM 340.—The elevation of an hexagonal pyramid cut by a plane, 1′ 3′, is given. Draw the sectional plan, and the true shape of the section.

Draw the plan of the pyramid. Project 1′ and 3′, giving 1 1 and 3 3.

As in the preceding problem, 2′ cannot be directly projected. Find the side elevation as in the last problem. 2′ 2′ will then represent the true width of the section. Transfer this to the plan, and complete the section.

The **true shape** may be easily followed from the lines.

PROBLEM 341.—The plan of an octagonal pyramid cut by a vertical plane is given. Draw the elevation and true shape of the section.

Draw the elevation, and project points 1, 2, 4, and 5. To obtain the elevation of 3, draw a side elevation of the pyramid, and project from 3 to V′ P′. Turn 3 into X Y, and erect a perpendicular to meet the outside edge of the pyramid at 3′, giving the highest point of the section.

From 3′ draw 3′ 3′ parallel to X Y, giving the required point.

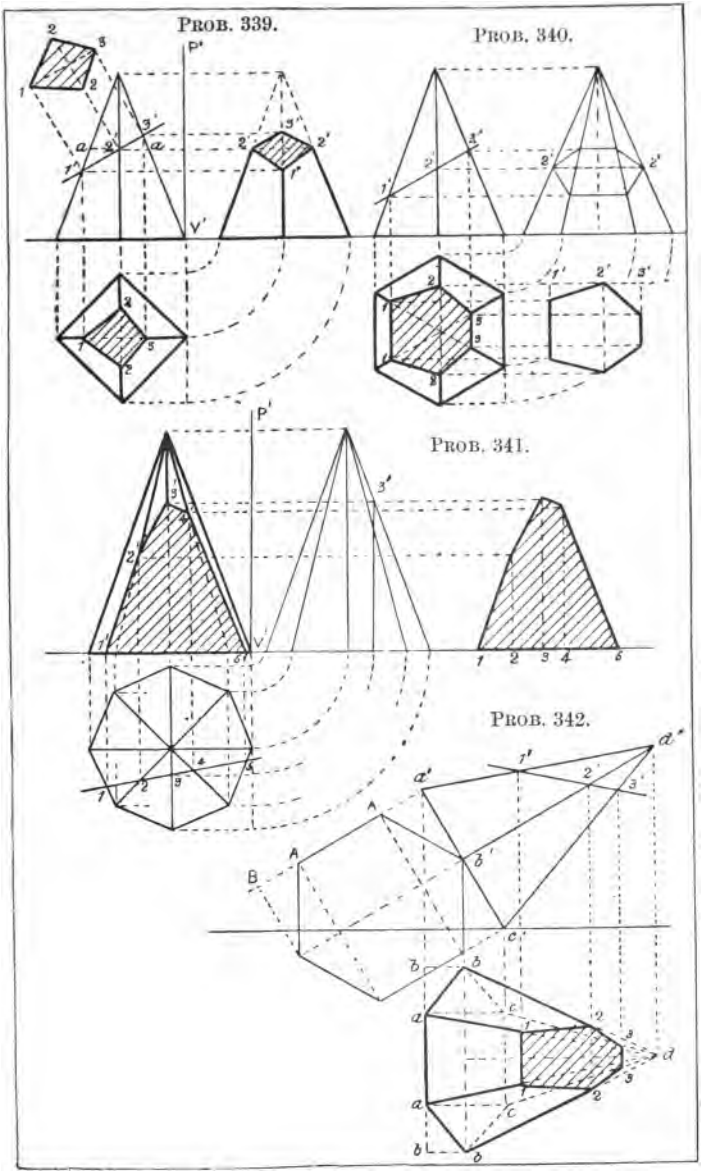
Join the points, and complete the section.

For the **true shape**, set off 1, 2, 3, 4, 5 on X Y and from 2′, 3′, 4′, project horizontally to meet the perpendiculars from 2, 3, and 4.

PROBLEM 342.—The elevation of an hexagonal pyramid cut by an oblique plane perpendicular to the vertical plane is given. Obtain its sectional plan.

First draw the true shape of the base of the pyramid.

Project from each point of the elevation. Make *b a*, *a a*, *a b* equal to B A, A A, A *a′*, and complete the plan. For the section, project from 1′ to *d a*, *d a*, from 2′ to *d b*, *d b*, and from 3′ to the remaining two edges. Join the points and complete the section.



PROB. 339.

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PROB. 341.

PROB. 342.

PROBLEM 343.—The elevation of an hexagonal pyramid lying on one of its sides with its axis parallel to the vertical plane is given. Find the plan when cut by a plane passing through 1′ 3′.

Draw the true shape of the base and obtain the plan as in the previous problem. From 1′, 2′, and 3′ project to the corresponding lines of the plan, and complete the section. Notice that *b b* will not be seen in the plan, because *b*′ would be cut away by the section plane.

PROBLEM 344.—The plan of the same pyramid with its axis parallel to the vertical plane is given. Obtain its sectional elevation when cut by a vertical plane 1 5.

First draw a regular hexagon on *a b*. This will be the plan of the pyramid when standing on its base. Project to X Y and obtain the elevation. For the height, with centre E′ and radius *a g*, cut the projector from G in G′.

Turn this elevation so that A′ G′ lies on X Y.

For the section project from 3, 4, and 5 to the corresponding lines in the elevation. As the section plane cuts the lines *a b* and *d e* whose elevations are the points *a*′ and *e*′, these points will represent points of the section. Join the points and complete the section.

PROBLEM 345.—The plan of the same pyramid has its axis inclined to the vertical plane. Obtain the elevation when cut by a vertical plane passing through 15.

The elevation *a*′ *e*′ *g*′ must be first obtained as in the previous problem. Project from *a* and *b* to X Y, from *c* and *f* to meet the parallel from *f*′, and from *d* and *e* to meet the parallel from *e*′. Join the points thus obtained. Project *g* to X Y, and join *g*′ with the angles of the hexagon.

For the section project as shown from 1, 2, 3, 4, and 5.

PROBLEM 346.—The elevation of a sphere cut by a horizontal plane is given. Obtain its plan.

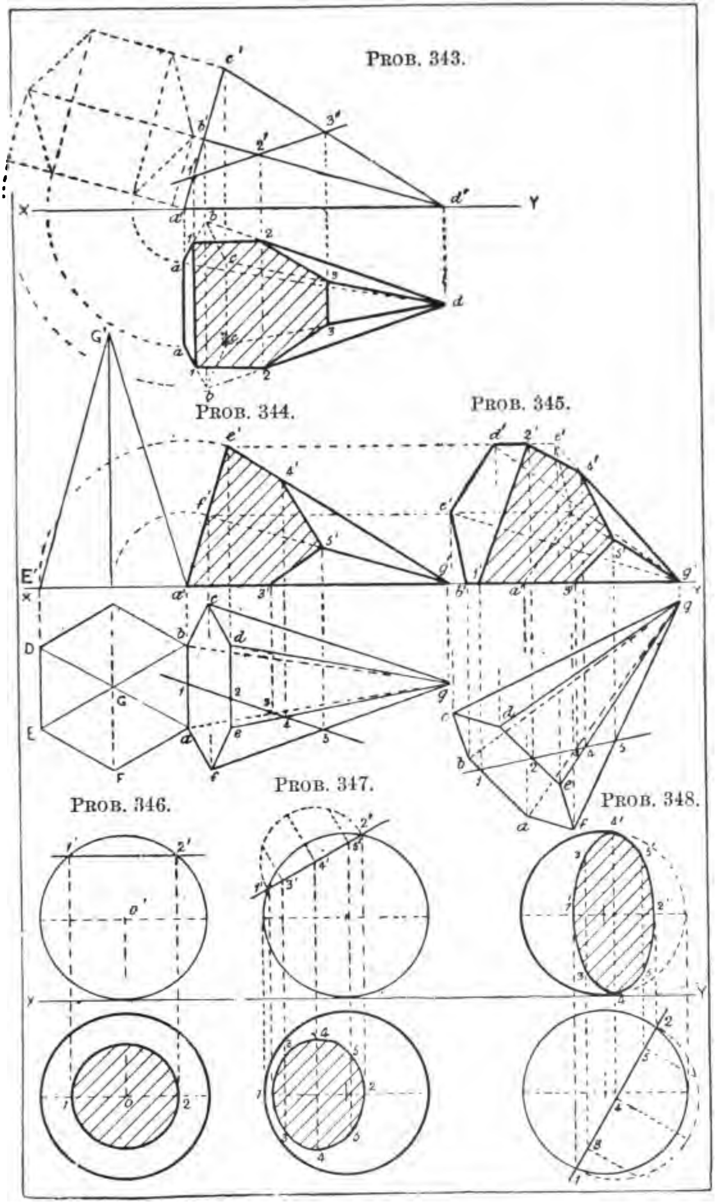
Project the centre *o*′ of the sphere, and describe a circle having the same radius as the given circle. From 1′ and 2′ project to the diameter of the plan, and describe the circle.

PROBLEM 347.—To draw the sectional plan of a sphere, when its elevation is cut by an oblique plane.

Project the circle and points 1′ and 2′ as before. We have now to obtain the plan of the circle of which 1′ 2′ is the elevation. Describe a semicircle on 1′ 2′, divide it, and draw perpendiculars. Project from 3′, 4′, and 5′, and on each side of the diameter set off distances equal to the perpendiculars at 3′, 4′, and 5′. Sketch in the curve, and complete the section.

PROBLEM 348.—The plan of a sphere cut by a vertical plane is given. Obtain its elevation.

This is solved in an exactly similar manner to the preceding problem.



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PROB. 345.

PROB. 346.

PROB. 347.

PROB. 348.

PROBLEM 349.—The elevation of a cylinder cut by a plane passing through 1′ 2′ is given. Find its plan, and a side elevation when viewed at right angles to its first position.

The plan will be a rectangle. Project from 1′ and 2′ as shown.

For the **side elevation**, turn the plan at right angles as shown, and project to meet the parallels from 1′ and 2′.

PROBLEM 350.—The elevation of a cylinder when its axis is parallel to both planes is given. Find the sectional plan made by a plane passing through 1′ 2′.

The plan of the cylinder will be a rectangle equal to the elevation. To obtain the width of the section draw half the true shape of the end. Then 2′ 2′ will be half the width of the section. Set off 2′ 2′ on each side of the centre line of the plan.

PROBLEM 351.—A cylinder standing on one end is cut by a plane, 1′ 2′. Draw its sectional plan, also the true shape of the section.

The plan of the cylinder will be a circle. Project from 2′ and complete the section.

The **true shape** of the section will be a segment of an ellipse. Project at right angles to 1′ and 2′, and draw 1 *a* at right angles to the projectors. Make 2 2 in the true shape equal to 2 2 in the sectional plan. To obtain other points in the curve, draw 3 3 through the centre of the plan, project to the section line 1′ 2′, from thence project at right angles, making 3 3 equal to 3 3 in the plan. Other points, as 4 4, may be obtained in a similar fashion. Through the points thus obtained draw the curve.

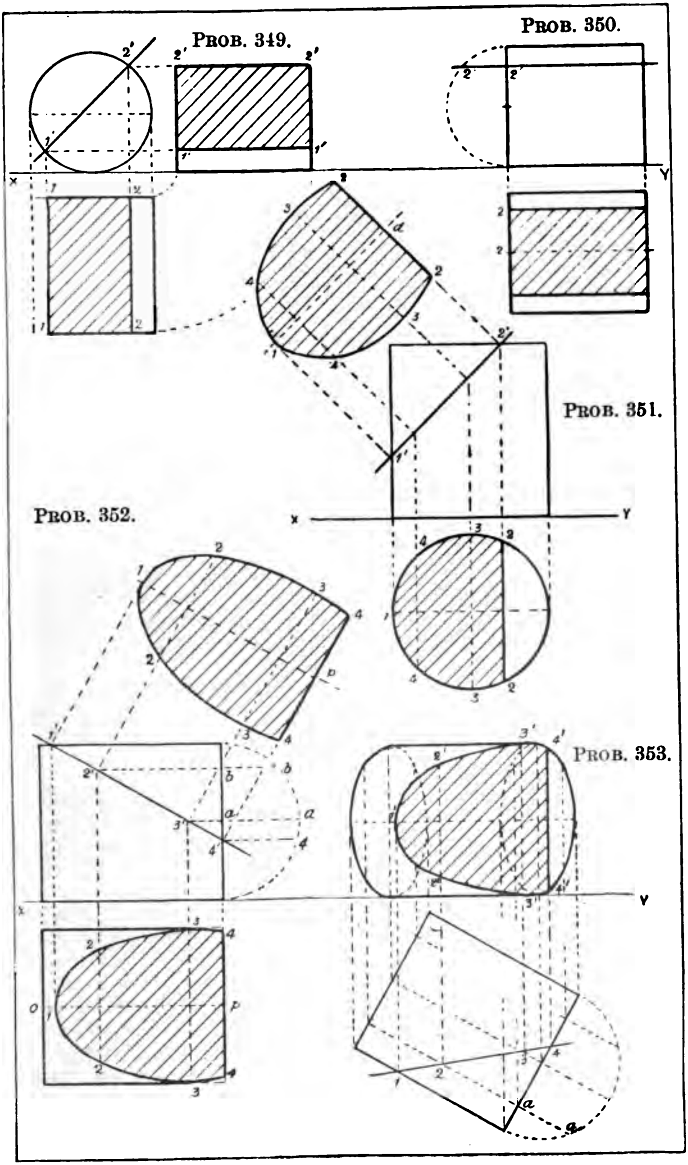
PROBLEM 352.—Given the elevation of a cylinder cut by a plane, 1′ 4′. Draw the sectional plan and true shape of section.

Draw the plan, which will be a rectangle, and mark the centre line *o p*. On the elevation describe a semicircle which will be half the true shape of the end. Project from 1′ to *o p*, then 1 will be the extremity of the axis of the ellipse. 4′ 4′ will be the width of the section on the end of the cylinder, then set off the distance 4′ 4′ on each side of *p*, giving the points 4 4. Through the centre *a* of the semicircle draw *a a*, and produce it to meet the section line at 3′. Project from 3′ to the plan, giving the points where the section is widest. Any number of points in the curve may be obtained. Take 2′ a point in the section line, and project horizontally and vertically. Set off the distance *b b* on each side of *o p* for the points 2 2. Through the points sketch the curve of the section.

For the **true shape** project at right angles as shown, and make the widths 2 2, 3 3, and 4 4 equal to the widths 2 2, &c. in the plan.

PROBLEM 353.—The plan of a cylinder cut by a vertical plane is given. Obtain its elevation.

First obtain the elevation of the cylinder as in Problem 318. Project from 1 to the centre line of the elevation for 1′, from 4 to the face giving 4′ 4′. From 3, the point where the section plane cuts the axis, project to the elevation for 3′ 3′. If the ordinate *a a* of the semicircle be produced to cut the section line in 2, two more points in the curve may be obtained. Project from 2, and set off the ordinate *a a*, on each side of the centre line for the points 2′ 2′. Through the points obtained sketch the curve and complete.



PROB. 349.

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PROB. 352.

PROB. 353.

PROBLEM 354.—The elevation of a cylinder cut by a horizontal section plane 1′ 3′ is given. Obtain its plan. Also obtain an elevation when cut by a vertical plane, 3′ 5′, and viewed from the right.

First draw the plan of the cylinder as in Problem 319. Project from 1′ and 3′ for points 1 and 3 3. Produce *b′ b′* to cut the section line in 2′. Project from 2′ to the plan, and set off the distance *b′ b′* on each side of the centre line for the points 2 2, or draw parallels from *b b* in the plan as shown. Sketch in the curve and complete the section.

For the **second elevation** draw V′P′ the edge of the vertical plane, project the points to it from the plan, turn them into X Y, and erect perpendiculars to meet the parallels from the elevation as in Problem 319. Project horizontally from 3′ and 5′, thus obtaining the top and bottom of the section. From 4′ draw a parallel and set off *a′ a′* on each side of the centre line, thus obtaining the points 4′ 4′. Sketch in the curve and complete.

Note.—If another point *c′* between 1′ and 2′ be taken and projected to the plan, the ordinate *c′ c′*, set off on each side of the centre line, will give *c c*, two more points in the curve.

PROBLEM 355.—A cone stands with its base in the horizontal plane. Show:—1. A section made by a horizontal plane. 2. A section made by an oblique plane passing through the apex. 3. The true shape of a section made by a vertical plane passing through the apex.

These sections can be followed from the figures without difficulty.

PROBLEM 356.—The elevation of a cone cut by a plane 1′ 5′ is given. Draw its plan, the true shape of the section, and a side elevation viewed from the left. (See page 116 for the sections of the cone.)

There are two convenient methods for finding the sectional plan.

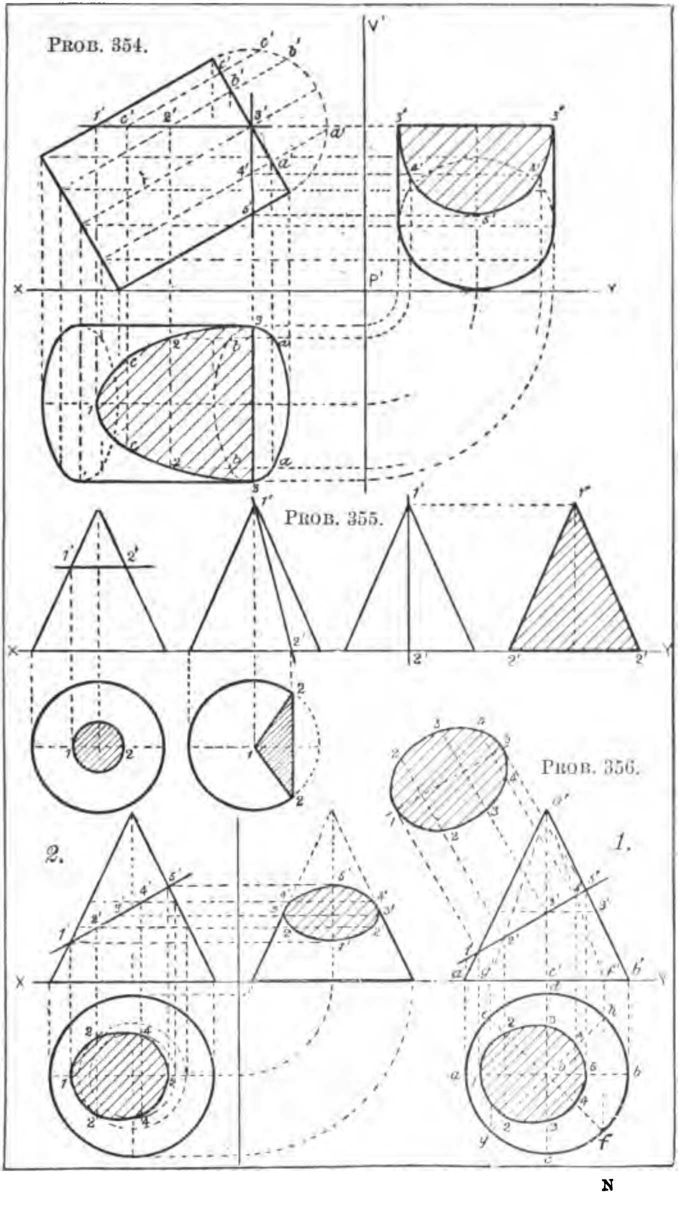
1. Draw the plan of the cone and divide it into any number of equal parts (in this case eight) by the lines *a b*, *c d*, *e f*, and *g h*. Obtain these lines on the elevation. The plane of section cuts these lines in the points 1′, 2′, 3′, &c.

Point 1′ is on line *a′ o′*, therefore its plan will be on *a o*, 2′ is on line *g′ o′*, and its plan will be on *g o*, and the corresponding point on the opposite side of the cone will be on *e o*. Project 4′ to *f o* and *h o*, and 5′ to *b o*. As 3′ cannot be projected vertically, set off 3′ 3′ on each side of line *a b*, as in Problem 329. Sketch in the curve and complete the section.

2. Draw the plan, and find points 1 and 5 as before. Take any number of points as 2′ and 4′, and imagine horizontal sections passing through these points. The plans of these sections will be circles. Draw two circles having diameters equal to the horizontal lines passing through 2′ and 4′. From 2′ and 4′, the points where the section plane cuts the circles, project to the plans of the corresponding circles. Draw the curve through the points obtained.

For the **true shape** of the section, project as shown, making 2 2, 3 3, and 4 4 equal to 2 2, 3 3, 4 4 in the plan.

For the **side elevation.** Turn the plan at right angles. From 1′, 2′, 4′, and 5′ project horizontally, and make the widths 2′2′ and 4′4′ equal to 2 2 and 4 4 in the plan.


PROB. 354.
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PROBLEM 357.—The elevation of a cone cut by a section plane parallel to its side is given. Obtain the sectional plan and the true shape of the section.

Draw the plan of the cone, and find points 1 1 and 4 of the section. To find other points in the curve, take any number of points, as 2′ and 3′, and draw horizontal section lines through each point. Obtain the circles on the plan of which these lines are the elevations. Project from 2′ for points 2 2, and from 3′ for 3 3. Through the points obtained draw the curve.

For the **true shape**, project at right angles to the section line from 1′, 2′, 3′, and 4′. Draw a centre line, and make 1′ 1′, 2′ 2′, 3′ 3′ equal to 1 1, 2 2, 3 3 of the plan. The curve drawn through these points will be a *parabola*.

PROBLEM 358.—The plan of a cone cut by a vertical section plane is given. Find the elevation and true shape of the section.

Draw the elevation, and project points 1 and 7 of the section to XY. Next obtain the height of the section. Draw *O* 4 perpendicular to the section line, and with centre *O* describe a circle touching the section line. From *a*, the point where the circle cuts *O d*, project to the elevation for *a′*. Through *a′* draw a horizontal line representing the elevation of the circle, and project from 4 to meet this line at 4′, giving the highest point of the section. To obtain other points in the curve, take 2 and 3, and describe circles through them from *O*. Project from *b* and *c*, where these circles cut *O d*, and draw the elevations at *b′* and *c′*. Projectors from 2, 3, 5, and 6 to meet these lines will give the required points of the curve. Sketch the curve, and complete.

For the **true shape**, make the perpendicular 4 4_1 equal to the height that 4′ is above X Y, and the perpendiculars at 2, 3, 5, and 6 equal to the heights of the corresponding points above X Y.

PROBLEM 359.—The elevation of a cone cut by a plane parallel to the axis is given. Draw the sectional plan.

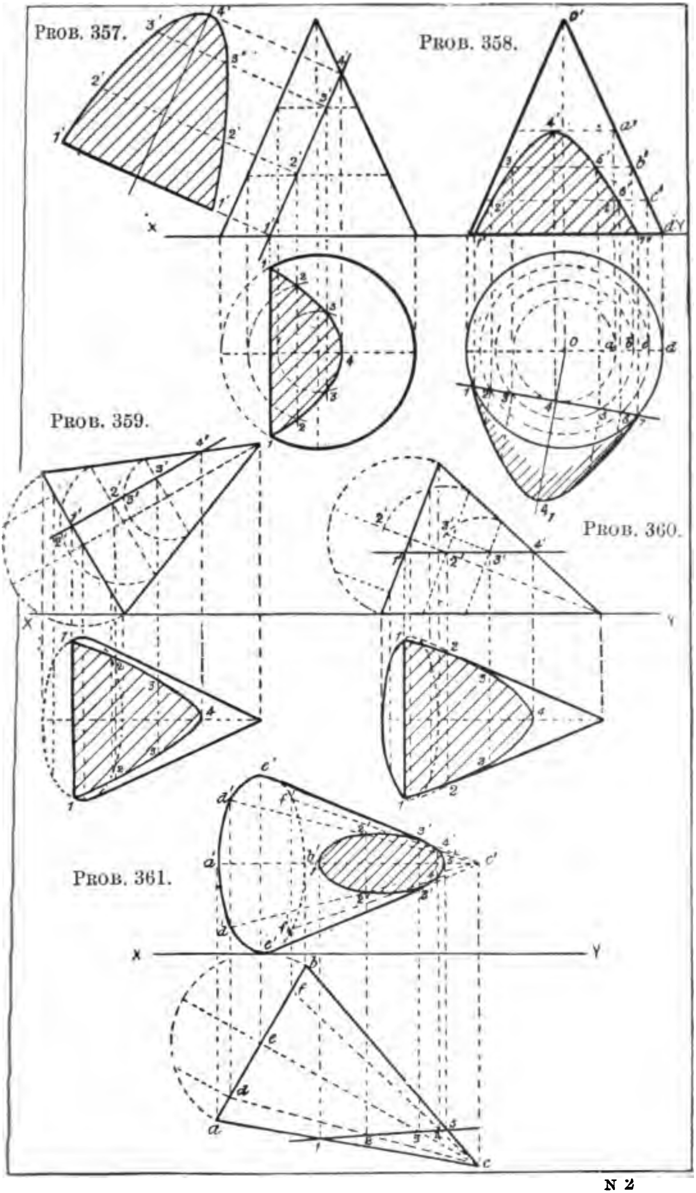
Obtain the plan of the cone by Problem 320. Project 1′ and 4′ for points 1 1 and 4. For other points, take 2′ and 3′, and draw sections through them parallel to the base. On each of these lines describe semicircles giving half the true shape of the sections at these points. Project from 2′ and 3′, and on each side of the centre line of the plan set off a distance equal to 2′ 2′ and 3′ 3′ for the points 2 2 and 3 3. Sketch in the curve through the points obtained.

PROBLEM 360.—A cone lying on its side is cut by a plane parallel to the ground-line. Draw the plan.

Draw the plan by Problem 321, and project for points 1 1 and 4. Assume other points in the section, as 2′ and 3′, and draw sections through them parallel to the base, as before. Draw half the true shape, and erect perpendiculars to the sections at 2′ and 3′. Project from 2′ and 3′, and set off distances equal to 2′ 2′ and 3′ 3′, on each side of the centre line of the plan, for points 2 2 and 3 3. Draw the curve through the points thus found.

PROBLEM 361.—The plan of a cone, its axis being horizontal, is given. Draw a sectional elevation when cut by a vertical plane, 1 5.

Project the elevation. The section may be obtained either as in the previous problem, or by assuming the cone to be a pyramid, as in Problem 356, Fig. 1. If treated as a pyramid, join *d*, *e*, and *f* with the vertex, *c*, and draw the elevations *d′ c′*, *f′ c′*, &c., of these lines. From the point 1 on *a c* project to *a′ c′*, from 2 on *d c* to *d′ c′*, from 3 on *e c* to *e′ c′*, from 4 on *f c* to *f′ c′*, and from 5 on *b c* to *b′ c′*. Through these points draw the curve of the section.



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CHAPTER XVI

EXERCISES

Notes.—1. Any Figures relating to these Exercises will be found on the opposite page.

Copy the given Figures at least twice the given size.

1. The elevation of a right hexagonal prism is given. Determine its plan.

2. Draw the side elevation and plan of a pentagonal prism, 3" long, of which the given Figure is the end view.

3. The lines *a b*, *a c* are the elevations of two circles. Obtain their plans.

4. The elevation of a square pyramid cut by a plane *a b* is given. Draw its plan.

5. The end elevation of an hexagonal prism, 1½" long, cut by a plane *a b*, is given. Determine the sectional elevation made [illegible] Draw the sectional plan.

6. The plan of a sphere cut by a horizontal plane is given. Draw its [illegible] a vertical plane passing through *a b*.

7. The elevation of a cylinder cut by a [illegible] plan.

8. The lines *a b*, *a c* are the plans of two unequal equilateral triangles. Draw the elevation.

9. The elevation of a square pyramid, lying on one of its triangular faces, [illegible] parallel to the vertical plane, is given. Find the sectional plan [illegible] pyramid through *a b*.

10. The plan of a [illegible] is given. Find an elevation on *a b*.

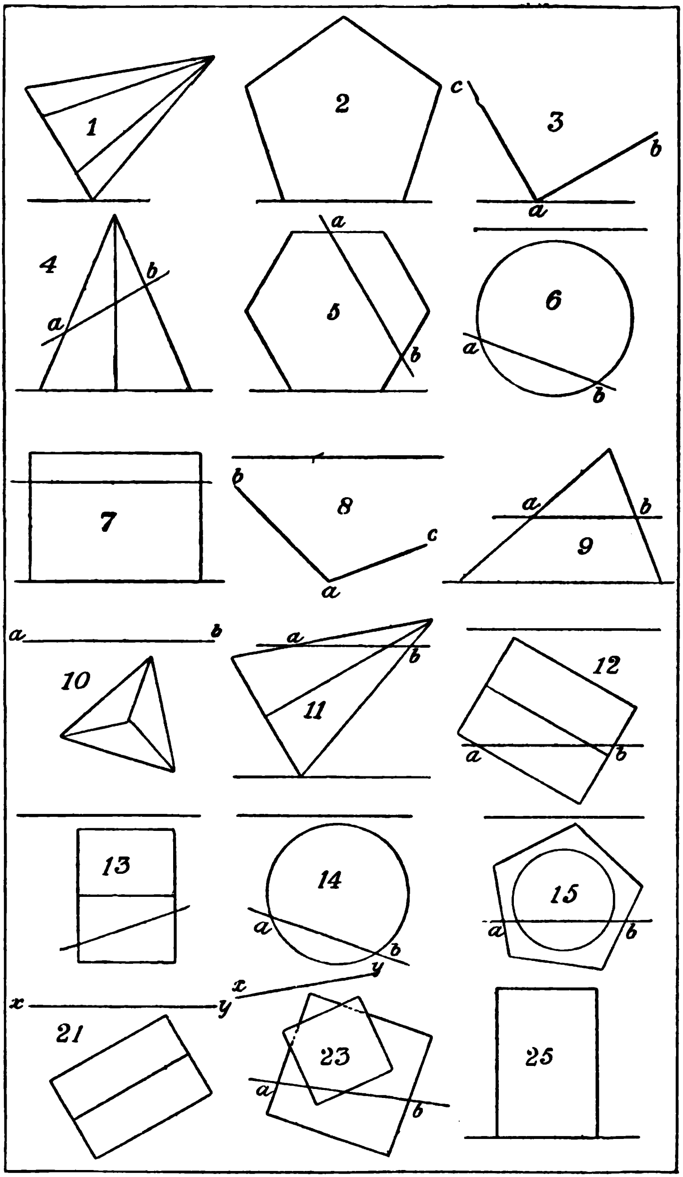
11. The elevation of a right square pyramid is given. Determine its [illegible] plane *a b*.

12. [illegible] elevation of the square prism when cut by a vertical plane is given. Find its elevation. [illegible] Find the elevation of the section made [illegible]

[illegible] plan of a pentagonal prism, 1" in height, [illegible] Draw the elevation when cut by a [illegible]

[illegible] a square of 3·5" sides, and its axis 3·25" [illegible] Draw its plan, and a sectional [illegible] line bisecting the plan of the axis [illegible] when its base is inclined at 47° [illegible]

[illegible] base is a hexagon of 1·25" side, [illegible] horizontal plane. Give the [illegible] edge and a quarter of [illegible] as long as no edge [illegible] upon the line joining A, [illegible] inclined to the vertical [illegible] Draw its elevation [illegible] square. Determine [illegible] *Prob. 369*, [illegible] the larger [illegible] side edges [illegible] in a vertical [illegible] Determine [illegible]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EXERCISES

CHAPTER XVI

Notes.—1. Any Figures relating to these Exercises will be found on the opposite page. 2. Copy the given Figures at least twice the given size.

1. The elevation of a right hexagonal prism is given. Determine its plan.

2. Draw the side elevation and plan of a pentagonal prism, 3″ long, of which the given Figure is the end view.

3. The lines *a b*, *a c* are the elevations of two circles. Obtain their plans.

4. The elevation of a square pyramid cut by a plane *a b* is given. Draw its plan.

5. The end elevation of an hexagonal prism, 1½″ long, cut by a plane *a b*, is given. Draw the sectional plan.

6. The plan of a sphere is given. Determine the sectional elevation made by a vertical plane passing through *a b*.

7. The elevation of a cylinder cut by a horizontal plane is given. Draw its plan.

8. The lines *a b*, *a c* are the plans of two unequal equilateral triangles. Draw the elevation.

9. The elevation of a square pyramid, lying on one of its triangular faces, with its axis parallel to the vertical plane, is given. Find the sectional plan made by a plane cutting the pyramid through *a b*.

10. The plan of a tetrahedron is given. Find an elevation on *a b*.

11. The elevation of a right square pyramid is given. Determine its sectional plan when cut by the plane *a b*.

12. Draw the sectional elevation of the square prism when cut by a vertical plane, *a b*.

13. The plan of a cube cut by a vertical plane is given. Find its elevation.

14. The plan of a cone is given. Find the elevation of the section made by the vertical plane, *a b*. (*Prob.* 358.)

15. The given Figure represents the plan of a pentagonal prism, 1″ in height, surmounted by a cylinder 2″ in height. Draw the elevation when cut by a vertical plane through *a b*.

16. A pyramid having for its *base* a square of 2·5″ sides, and its axis 3·25″ long, rests with one *face* on the horizontal plane. Draw its plan, and a sectional elevation on a vertical plane, represented by a line bisecting the plan of the axis and making 60° with it. (*Sc.*, 1870.) (*Prob.* 308.)

17. Draw the plan of the above pyramid when its base is inclined at 47° and one edge at 27°. (*Sc.*, 1870.)

18. Draw the plan of a right pyramid whose base is a hexagon of 1·25″ side, and its axis 3·25″, when it stands upright on a horizontal plane. Give the section made by a vertical plane which cuts off half one edge and a quarter of the next, measuring downwards from the vertex. (*Sc.*, 1871.)

19. Draw the plan of a cube of 3″ edge, in any position, as long as no edge is horizontal. (*Sc.*, 1872.) (*Prob.* 299.)

20. A pentagon, **A B C D E**, side 1·75″, revolves upon the line joining **A**, with the centre of the opposite side, till its plane becomes inclined to the vertical plane at 50°. Draw its plan. (*Prob.* 291.)

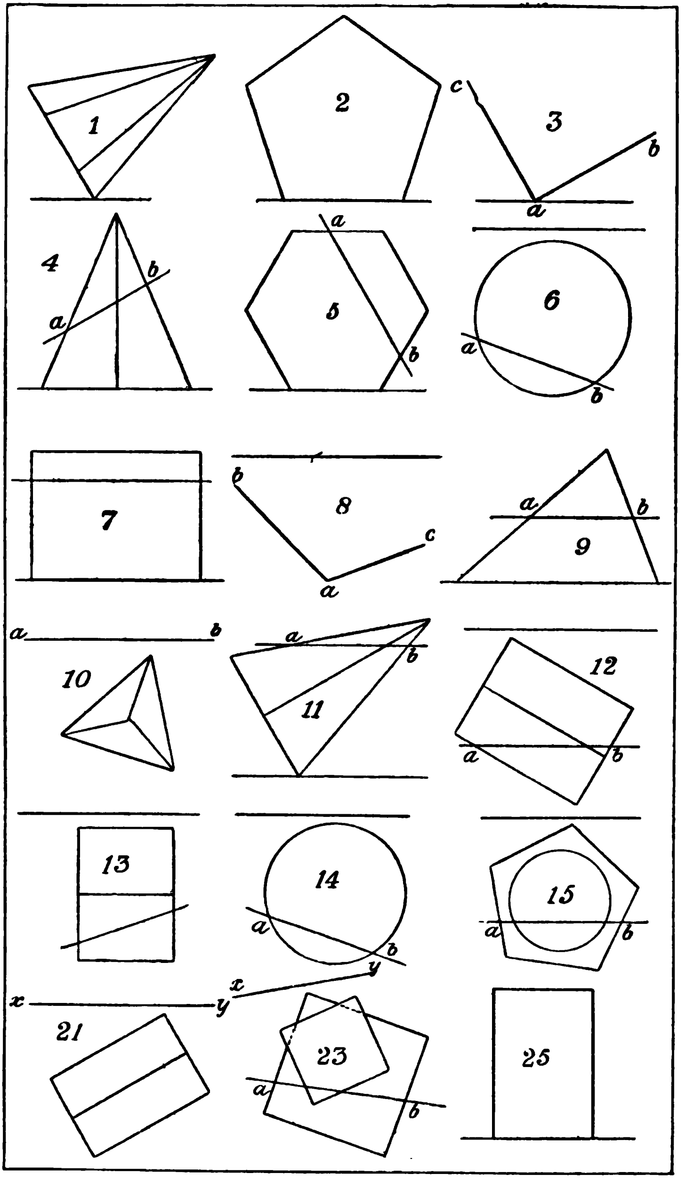
21. The plan of an equilateral triangular prism is given. Draw its elevation on **X Y**. (*Sc.*, 1876.)

22. A rectangle, length 2·5″, breadth 1·75″, is the plan of a square. Determine the inclination of the plane of the square. (*Sc.*, 1877.) (Converse of *Prob.* 289, *case* 3.)

23. The plans of two cubes are given, the smaller resting on the larger. Draw an elevation on *x y*, and a section on *a b*. (*Sc.*, 1878.)

24. A square prism, 3″ long and 1·5″ square, lies with one of its side edges horizontal, and one of its sides inclined at 30°. Make an elevation on a vertical plane which is at 50° to the axis. (*Sc.*, 1882.) (*Prob.* 298.)

25. The given rectangle represents the elevation of a square. Determine the inclination of its plane to the vertical plane. (*Sc.*, 1884.)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SECTION II

SOLID OR DESCRIPTIVE GEOMETRY

CHAPTER XVII

INTRODUCTION, PROJECTION OF POINTS AND LINES

It is advisable for the student to have made himself thoroughly familiar with the preceding chapter on the Plans and Elevations of Solids in simple positions. The common terms used are explained at the commencement of that chapter. The explanation of fresh terms will be given in connection with the chapters or problems in which they occur.

Solid or Descriptive Geometry deals with the representation of objects having three dimensions—*length, breadth, and thickness*—upon a plane surface.

A **plane surface** is perfectly level, and is defined by Euclid as 'a surface in which, any two points being taken, the straight line between them lies wholly in that surface.' Henrici calls it 'the path of a moving line.'

These representations are effected by means of lines drawn perpendicularly from each point of the object to two given plane surfaces, already known as the Vertical and Horizontal Planes. These planes are called, from their position with regard to each other, the **Co-ordinate Planes**; and, from the fact that upon them the objects are drawn or projected, the **Planes of Projection**; they are supposed to be of indefinite extent, and they intersect each other at right angles in the straight line known as the Ground Line or Intersecting Line, forming four angles, which are known as the 1st, 2nd, 3rd, and 4th **Dihedral Angles** (angles formed by two intersecting planes) (Fig. 1).

The lines drawn from the object to the co-ordinate planes are termed **Projectors**, and the representations on those planes are **Projections**. When the projectors are perpendicular to the planes of projection, the projection is **Orthographic.** The projection on the horizontal plane is the **Plan**; that on the vertical plane is the **Elevation.**

The foot of the perpendicular drawn from a given point to a plane is the **Projection of that point.** The line which passes through the projections of all the points of a given line on a plane is the **Projection of that line,** and the plane which contains all the projectors of the points in that line is called its **Projecting Plane.**

FIG. 1.

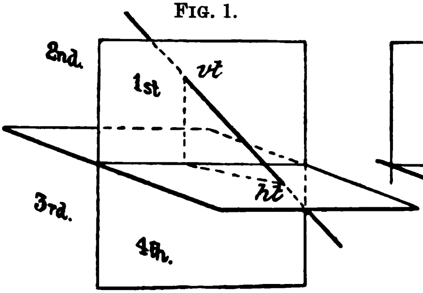


FIG. 2.

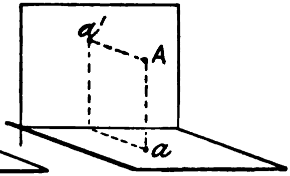


FIG. 3.

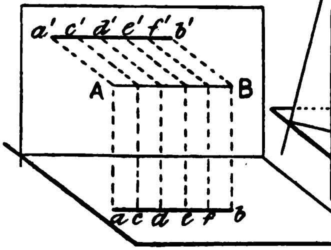
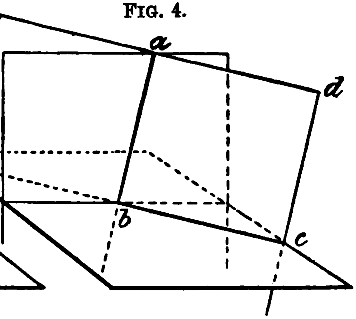


FIG. 4.



For example, in Fig. 2, *a* and *a′* are the projections of the point A. In Fig. 3, *a b* and *a′ b′* are the projections of the line A B, because they pass through the projections *a*, *c*, *d*, *e*, *f*, *b*, and *a′*, *c′*, *d′*, *e′*, *f′*, *b′*, of its several points on the co-ordinate planes; and the planes A B *a b* and A B *a′ b′* are its projecting planes.

The point where a line meets a plane is called the **Trace of that line.** Where it meets the vertical plane is the **Vertical Trace**, as *v t*, Fig. 1; and where it meets the horizontal plane is the **Horizontal Trace,** *h t*, Fig. 1. In the same manner the lines where a plane intersects the co-ordinate planes are the **Traces of that plane.** In Fig. 4, *a b* and *b c* are the **Vertical** and **Horizontal Traces** respectively of the plane *a b c d*,

PROJECTION OF POINTS

Note.—The vertical plane is denoted by the letters V.P., the horizontal plane by H.P., and the line of intersection by **X Y**.

It has already been pointed out that the co-ordinate planes intersect each other, forming four dihedral angles. These angles may be easily shown by slitting two pieces of cardboard as far as their centres, as shown by the lines **A** and **B**, **Fig.** 1, and then fitting them together in the form of a cross, as in Fig. 2.

The angle in *front* of the V.P. and *above* the H.P. is known as the **1st dihedral angle**; the angle *behind* the V.P. and *above* the H.P. is the **2nd dihedral angle**; the angle *behind* the V.P. and *below* the H.P. is the **3rd dihedral angle**; and the angle in *front* of the V.P. and *below* the H.P. is the **4th dihedral angle.**

To show the position of a point with respect to the co-ordinate planes, take a pencil, say 2″ long, and assume A, one end of it, to represent the point. Place the pencil in the first dihedral angle perpendicular to the H.P. and 1″ in front of the V.P. as A *a*, Fig. 3; then the foot of the pencil, *a*, will be the plan of A, and *a′*, the end of the projector drawn from A to the V.P., will be its elevation. If the pencil be now moved into the second dihedral angle and placed 1½″ behind the V.P. as B *b*, then *b* and *b′* will represent the plan and elevation of the point. Now place the pencil in the third dihedral angle ½″ behind the V.P. as C *c*; then *c* will be the plan, and *c′* the elevation. When placed in the fourth dihedral angle ¾″ in front of the V.P., then *d* and *d′* will show the plan and elevation respectively.

As drawings are made on one flat surface we must imagine the vertical plane turned in the direction of the arrows until it coincides with the horizontal plane. The elevations *a′*, *b′*, *c′*, *d′*, already marked, will then assume the positions shown in Fig. 4, the plans *a*, *b*, *c*, *d* remaining fixed. We thus find that a point in the *first dihedral angle* has its *plan below* and its *elevation above* X Y; in the *second*, its *plan and elevation* are *both above* X Y; in the *third*, the *plan* is *above* and the *elevation below* X Y; in the *fourth*, *both plan and elevation* are *below* X Y. These positions must be thoroughly understood and carefully verified by the student before proceeding to the following problems.

PROBLEM 1.—To determine the projections of the following points:—A, 1″ in front of the V.P. and 1½″ above the H.P.—B, ¾″ behind the V.P. and 1¼″ above the H.P.—C, 1½″ behind the V.P. and 2″ below the H.P.—D, 1¾″ in front of the V.P. and 1·25″ below the H.P.—E, in the H.P. and ·75″ in front of the V.P.—F, in the V.P. and 1″ below the H.P.—G, in both planes.

The point A will have its plan 1″ below, and its elevation 1½″ above X Y. B is in the second dihedral angle; its plan will be ¾″ above X Y, and its elevation 1¼″ above X Y. C is in the third angle, and its plan will be 1½″ above, and its elevation 2″ below X Y. D is in the fourth angle, and its plan will be 1¾″ below X Y, and its elevation 1·25″ below X Y. E will have its plan ·75″ below, and its elevation on X Y. F will have its plan on, and its elevation 1″ below, X Y. G, being in both planes, will have both its plan and elevation on X Y, as that is the line common to both planes.

FIG. 1.

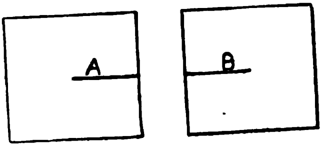


FIG. 2.

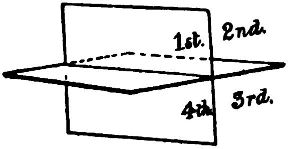


FIG. 3.

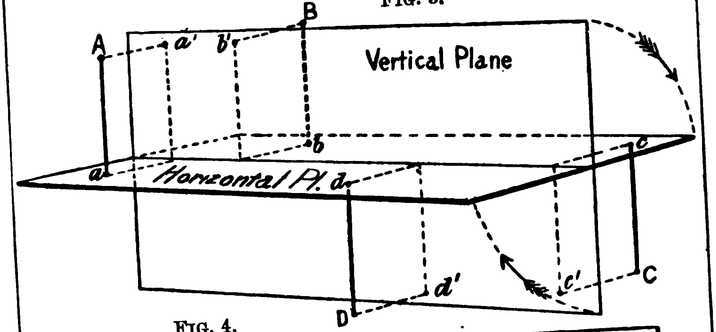
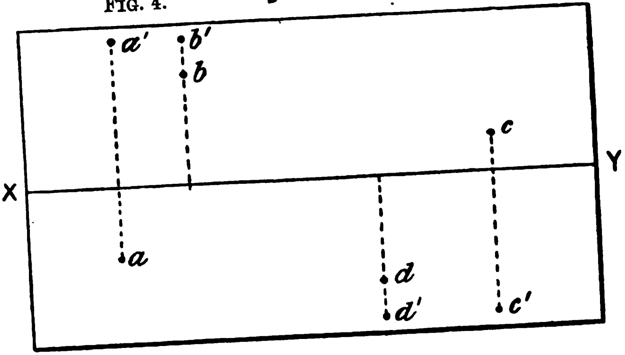
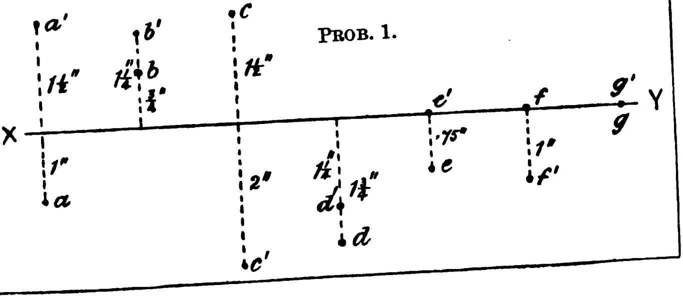


FIG. 4.



PROB. 1.



PROBLEM 2.—Given the projections of the points A, B, C, D, and E, to determine their position with respect to the Co-ordinate Planes.

Note.—The figures on page 187 are drawn half size.

This problem is the converse of Problem 1.

By using the cardboard model the position of each point may be readily fixed. First fix the position of the point with reference to the V.P. If the plan is *below* X Y, the point is *in front* of the V.P., and if the plan is *above* X Y, the point is *behind* the V.P.

A is 1¼″ behind the V.P. and 1″ below the H.P., and in the third dihedral angle. B is ½″ in front of the V.P. and 1½″ below the H.P., and in the fourth dihedral angle. C is in the V.P. and 2″ above it, and is between the first and second dihedral angles. D is 2″ behind the V.P. and ½″ above the H.P., and is in the second dihedral angle. E is 2″ in front of the V.P. and on the H.P., and is between the first and fourth dihedral angles.

PROBLEM 3.—Given the projections of a point, to determine its actual distance from X Y.

Let a and a' be the projections of the point. a' represents the actual height of the point above the H.P., so that if a line be drawn through a' parallel to X Y the point must be in this line. a represents the distance the point is in front of the V.P. If the planes be viewed in profile a would be seen in position a''. The perpendicular from a'' to meet the parallel from a' will give A, the actual position of the point, and A O, the diagonal of the rectangle thus formed, will be the actual distance of the point from X Y.

PROBLEM 4.—A point A is 1½″ in front of the V.P. and above the H.P., and its distance from X Y is 2″; determine its plan and elevation.

This is the converse of Problem 3.

Draw two lines to represent the profile view of the planes. Set off O a'' 1½″ from the V.P., and through a'' draw a line parallel to the V.P. With centre O and radius 2″ cut the vertical line drawn through a'' in the point A above X Y. From A draw A a' parallel to X Y. Then a' will be the elevation of the point. Turn a'' into the projector from a', giving a the plan of the point.

EXERCISES

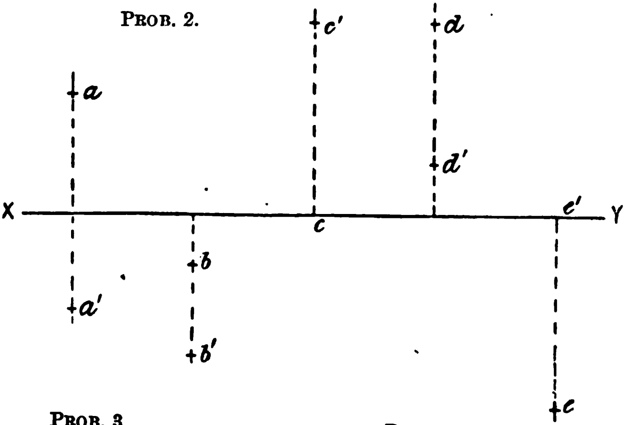
1. Show the plans and elevations of the following points.

A, 3″ above the H.P. and 1·8″ before the V.P.
B, 2″ above the H.P. and 3·5″ behind the V.P.
C, 2·5″ below the H.P. and 4″ before the V.P.
D, in the H.P. and 2·5″ before the V.P. (*Sc.*, 1871.)

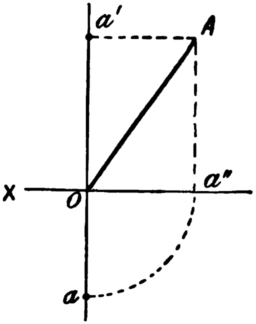
2. State the exact position with regard to the two planes of projection, of the points whose plans and elevations are given. Fig. 1. (*Sc.*, 1872.)

3. A point A has its plan 1½″ below X Y and its elevation 1″ above the H.P. What is its true distance from X Y?

PROB. 2.



PROB. 3.



PROB. 4.

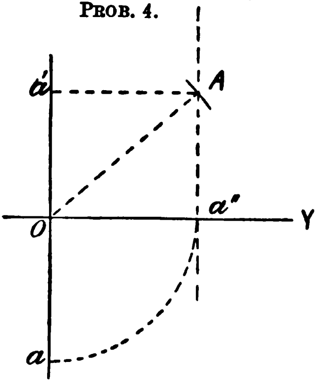
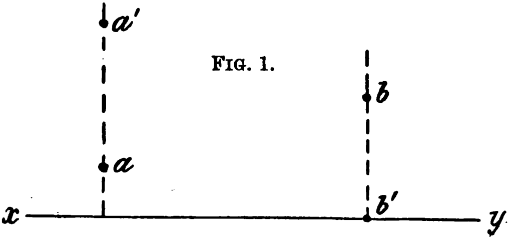


FIG. 1.



PROJECTION OF LINES

A line, like a point, may be situated in any of the four dihedral angles, or it may be in two, or even three, of the angles by passing through the planes, as the line *a d*, Fig. 1; here the portion *a b* is in the first dihedral angle, *b c* is in the second, and *c d* is in the third. The points *b* and *c* where the line passes through the planes are the **traces** of the line on those planes. The elementary geometry dealt with in this book will be chiefly confined to the representations of objects situated in front of the V.P. and above the H.P., that is, in the first dihedral angle.

Note.—The projections of lines in various positions with respect to the co-ordinate planes have already been dealt with in Problems 286 and 287, Chapter XVI., and will therefore require no further explanation. It will also be assumed that the principle of *Changing the Intersecting or Ground Line*, so that fresh views of the object may be obtained, has been thoroughly mastered. (See Chapter XVI., page 144.)

PROBLEM 5.—To determine the traces of a line, A B, the projections being given.

1. As has been already explained, the traces of a line are the points where it meets a plane. As both the plan and elevation of the line A B are inclined to the co-ordinate planes, it will have both a vertical and a horizontal trace. The horizontal trace will be in the H.P.; to determine it, produce *a′ b′* to meet X Y in *h*. From *h* draw a perpendicular to meet *a b* produced. H T will be the horizontal trace.

To find the vertical trace, produce *a b* to meet X Y in *v*. From *v* draw a perpendicular to meet *a′ b′* produced. V T will be the vertical trace.

Note.—To verify this solution, let the student take a piece of wire, A B, and place it between the cardboard co-ordinate planes as shown in Fig. 2. The points A and B would then be the traces of A B. From A and B draw A *a* and B *b* perpendicular to the Intersecting Line, and join *a* B and *b* A. Then *a* B will be found to be the elevation of A B, that is, the space it covers on the V.P. when looked at horizontally forwards, and A *b* will be the plan of A B, or the space it covers on the H.P. when looked at vertically downwards.

2. In this position there will be only a horizontal trace, as A B is parallel to the V.P. and cannot therefore meet it. Produce *a′ b′* to meet X Y in *h*, and from *h* draw a projector to meet *a b* produced. H T will be the horizontal trace.

3. In this case there can be no traces, as the line is parallel to both the co-ordinate planes.

4. Here the line, although oblique, has only one trace, its projections both passing through X Y.

5. As the line is vertical, it has only a horizontal trace.

6. The line being horizontal, it has only a vertical trace.

From these cases we may deduce the following facts.

(1.) When a line is *parallel to both* of the *planes of projection*, it can have *no traces*.

(2.) When *parallel to either* of the *planes*, it has only *one trace*.

(3.) When *parallel to neither* of the *planes*, it may have *one trace*, as in Case 4, or *two*, as in Case 1.

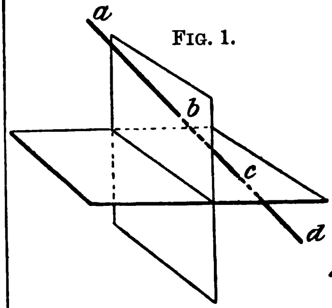
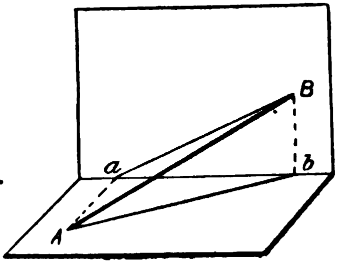
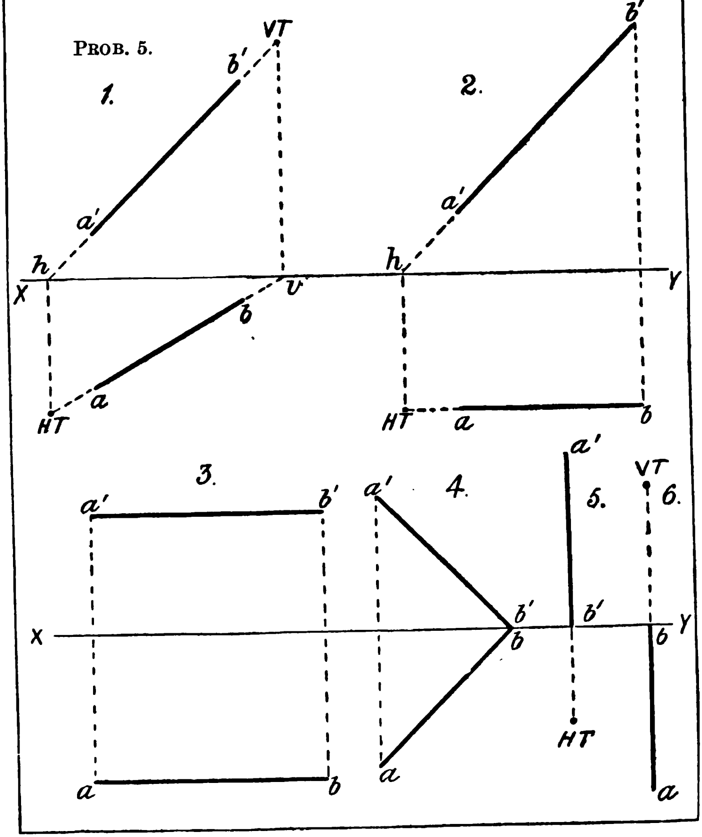


FIG. 1.

FIG. 2.



PROB. 5.



PROBLEM 6.—Given the traces of a line, to find its projections.

Let V and H represent the vertical and horizontal traces. From V and H draw V *v* and H *h* perpendicular to X Y. Join V and *h*, and H and *v*, giving the vertical and horizontal projections respectively.

PROBLEM 7.—The projections of a line, A B, being given, to determine its traces. (Two positions are shown.)

Here the traces will not both fall in front of the vertical and above the horizontal plane. The same rule must be followed. For the *horizontal trace*, produce *a′ b′*, the *vertical projection*, to meet X Y in *h*, and from *h* draw a perpendicular to meet *a b* produced. H T is the required trace. For the *vertical trace* produce the *horizontal projection* to meet X Y in *v*, and from *v* draw a perpendicular to meet *a′ b′* produced. V T will be the vertical trace.

Note.—To illustrate this, take a piece of wire and place it with its extremity A turned from the vertical plane (Fig. 1). B will evidently be its trace on the H.P. It is also evident that its trace on the V.P. cannot be found by producing it towards A. If the wire be forced through the card at B and pushed forward until it meets the V.P. at C, then C will be its vertical trace. If the dotted construction lines be now drawn, the reasons for the above solution will be readily understood. Traces of lines falling within the other dihedral angles may be solved in a similar manner.

PROBLEM 8.—Given the projections *a b*, *a′ b′* of a line, to find its true length, and its inclinations to the planes of projection.

The *inclination of a line to a plane* is the *angle* between the *line* and its *projection* on that plane.

Notes.—1. In Fig. 2 the angle A B *a* is the inclination of A B to the H.P., because it is the angle between the line and its plan; and the angle B A *b* is the inclination to the V.P. because it is the angle between the line and its elevation. These angles are usually indicated by the Greek letters θ and ϕ. It will assist the memory to note that θ, the inclination to the H.P., has the horizontal stroke, and ϕ, the inclination to the V.P., the vertical stroke.

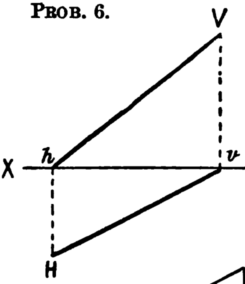
2. If we imagine the line A B (Prob. 8) in its actual position, over *a b*, and supported by vertical projectors equal in height to *m a′* and *n b′*, we shall have a trapezoid standing on *a b*, and having two of its sides perpendicular to *a b*. Suppose this trapezoid to be turned down into the H.P., we shall then have the figure A *a b* B, of which the side A B will represent the true length of *a b*.

To find then the true length of the line, draw *a* A and *b* B at right angles to *a b*, and make them respectively equal to *m a′* and *n b′*, the heights which the points A and B are above the H.P. Join A B. Then A B will be the true length of the line. If the top edge of the trapezoid were produced to meet the H.P., the angle made by this line and the base, *a b*, produced would evidently be the angle at which the line is inclined to the H.P. Produce then A B to meet *a b* produced. The angle thus obtained is the inclination of A B to the H.P., and H, the point where A B meets the ground, must necessarily be the horizontal trace of A B.

If *a′* A′ and *b′* B′ be drawn at right angles to *a′ b′*, and made equal to *n b* and *m a* respectively, then A′ B′ will also represent the true length of A B, and if A′ B′ and *a′ b′* be produced to meet, the inclination of A B to the V.P. will be found, and V, the point where the two lines meet, will represent the vertical trace of A B.

Note.—If the angle of inclination found by producing *a b* and A B should fall beyond the paper, then a parallel to *a b* from B will give the angle θ.

PROB. 6.



PROB. 7.

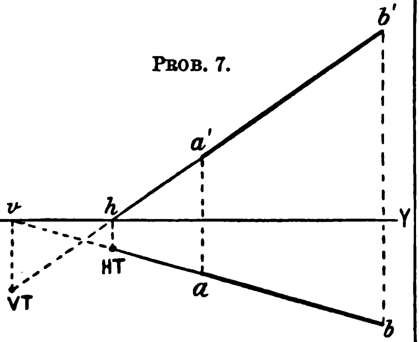
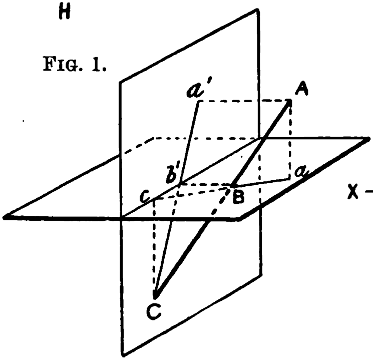


FIG. 1.



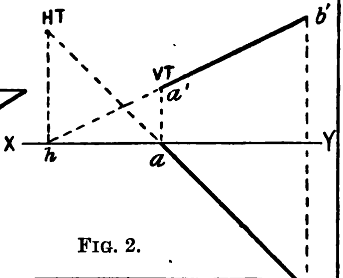
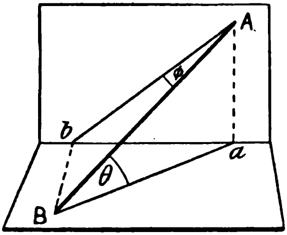
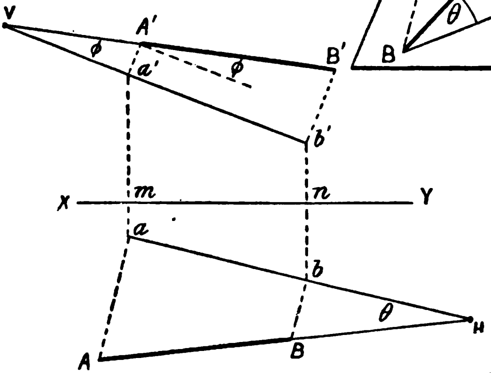


FIG. 2.



PROB. 8.



PROBLEM 9.—Another solution for Problem 8.

Let *a b* and *a′ b′* be the given projections. When a line is parallel to either of the co-ordinate planes, its projection on that plane represents the true length of the line. If we then cause *a b* to revolve on an axis at *a* until it is parallel to the V.P., as *a* B, then its elevation will be the true length of A B. Through *a* draw *a* B parallel to X Y, and turn *a b* into this line. Through *b′* draw a parallel to X Y. Project B to this line, giving B′. Then *a′* B′ will be the true length, and the angle θ will be inclination of A B to the H.P.

In the same way, if the elevation, *a′ b′*, be revolved upon *b* until it is parallel to X Y, and A′ be projected to the parallel through *a*, then *b* A will also represent the true length of A B, and the angle ϕ will be the inclination of A B to the V.P.

Note.—The student should verify these results by taking the projections given in Problems 8 and 9 in exactly the same position, and of the same length. The true lengths in all four cases should then be found to correspond.

PROBLEM 10.—To find the inclinations and true length of a line when the projections are at right angles to X Y.

This depends on the same principle as the previous problem. Let the plan, *a b*, be rotated into X Y. *a′* A′ will show the distance *a* is from the V.P. A′ B′ will be the true length, the angle θ the inclination to the H.P., and ϕ the inclination to the V.P.

The actual distance of A B from X Y will be shown by the perpendicular *b′ c′*.

PROBLEM 11.—To find the true length when the line passes through the plane.

Let *a b* and *a′ b′* be the projections. The portions of the line not lying in the first dihedral angle are dotted. The only difference is that the perpendiculars to *a* and *b* must be set out in *opposite* directions; *b* B equal to *b′ m*, and *a* A equal to *a′ n*. Then A B will represent the true length, and will pass through *c*, the horizontal trace of A B.

PROBLEM 12.—To mark off a given length on a line, the projections of which are given.

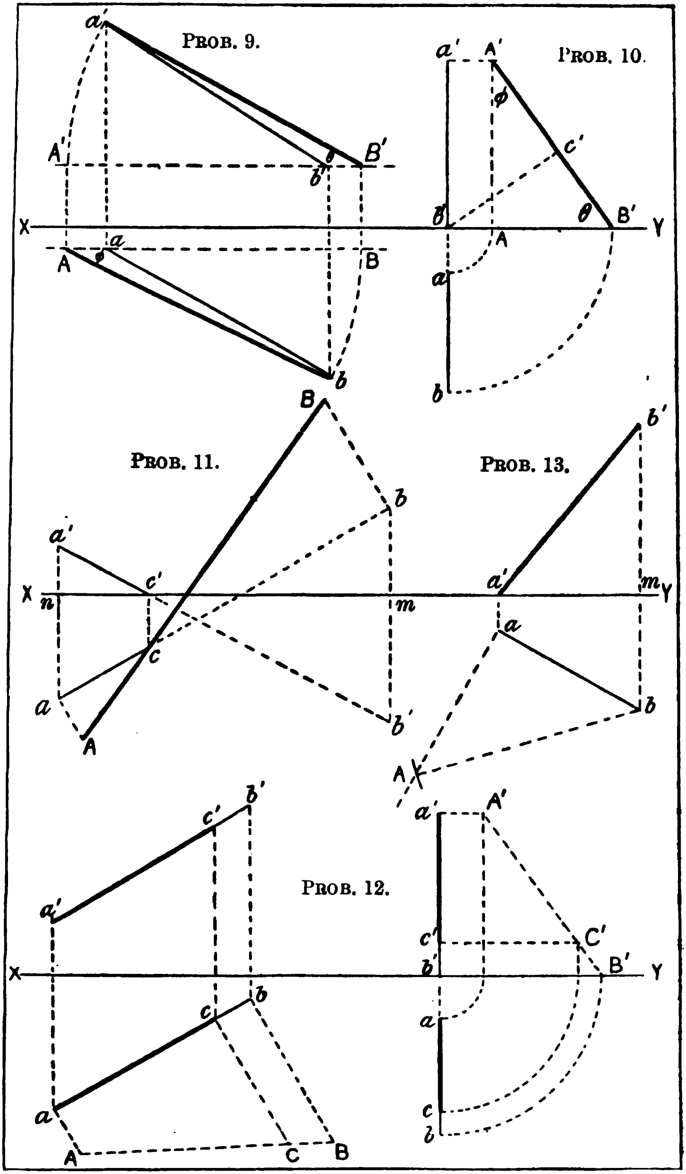
Find the true length of the line by Problem 8. On it set off the required distance, say A C. Draw C *c* parallel to B *b* and project. *a′ c′* is the elevation and *a c* the plan of the required length of line. Two cases are shown; the second is based upon Problem 10.

PROBLEM 13.—Given the plan and the true length of a line, to construct its elevation.

Let *a b* be the plan, and *a* the point on the ground. At *a* draw *a* A perpendicular to *a b*. With centre *b*, and the true length of the line as radius, intersect the perpendicular in A. Project from *a* and *b*. Make *m b′* equal to *a* A. Then *a′ b′* will be the required elevation.

Notes.—1. Proceed in a similar way if the plan were required.

2. If the plan of the line and its inclination to the H.P. were given, a similar construction would give the elevation. The only difference would be that the angle at *b* would be given instead of the length *b* A.


PROB. 9.
a′
A′
B′
b′
X
a
A
B
b
PROB. 10.
a′
A′
φ
c′
b′
θ
B′
Y
A
a
b
PROB. 11.
B
b
a′
c′
X
n
m
c
a
A
b′
PROB. 13.
b′
a′
m
Y
a
b
A
PROB. 12.
b′
c′
a′
X
b
c
a
A
C
B
a′
A′
c′
C′
b′
B′
Y
a
c
b

PROBLEM 14.—Given the plan and elevation of any plane figure, to determine its true form.

Let *a b c* and *a′ b′ c′* be the projections of a triangle.

Find the true lengths of each of the sides by Problem 8, as shown.

With A C, A B, and B C as sides, construct the triangle A B C.

Note.—If the projections of a quadrilateral figure or any polygon were given, it would be necessary to divide it into triangles, and then ascertain the dimensions of the lines.

PROBLEM 15.—Given the elevation of a line and its inclination to the H.P., to determine its plan.

1. Let *a′ b′* be the elevation, and 40° its inclination to the H.P.

Assume this line to lie on the surface of a cone, and to have its extremity *a′* at the apex, and its other extremity *b′* in the circumference of the base. Let the slope of the cone be 40°, the required inclination of the line to the H.P. Draw *a a′*, and from *a′* draw *a′* B to meet X Y at an angle of 40° (make the angle at *a′* 50°); this will represent a side of the cone. If a circle be described from centre *a* with radius *a* B, it will represent the base of the cone, and any line drawn from *a* to the circumference will be the plan of a line inclined at 40° to the H.P. From *b′* draw *b′ b*. Then *a b* will be the required plan.

2. When the line passes through X Y proceed as shown in the second figure. Through *a′* draw *a′* A′ parallel to X Y, and draw the projector from *a′*. At *b′* draw a line making 40° with X Y to meet the parallel from *a′* in A′. Draw A′ A, and, with centre *b′* and radius *b′* A, describe an arc cutting the projector from *a′* in *a*. Join *a b′*.

Note.—In this case consider the cone mentioned in Case 1 to be inverted and to have its apex on X Y.

PROBLEM 16.—Given the plan of a line and its inclination to the V.P., to find its elevation.

1. Let *a b* be the plan, and the inclination to the V.P. 35°.

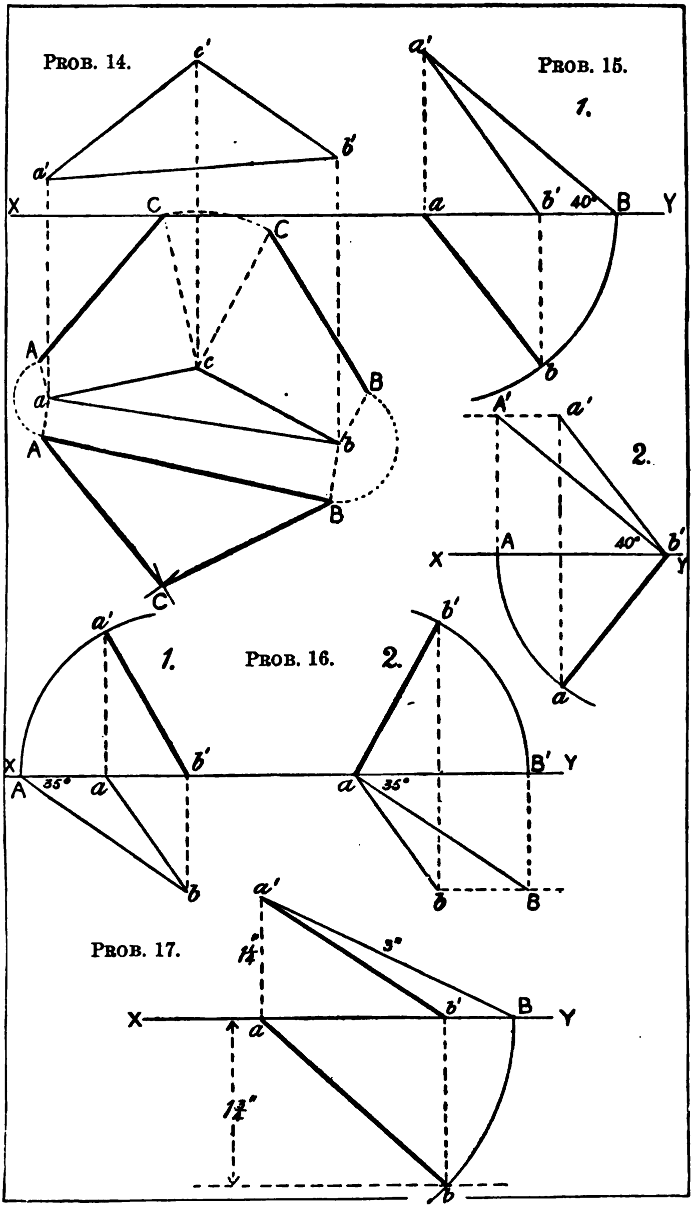
This is a similar process to that used in the previous problem.

Draw *b b′*, and at *b* draw *b* A to meet X Y at an angle of 35°. With centre *b′* and radius *b′* A describe an arc. Draw *a a′* to meet this arc. Join *a′ b′*.

2. When the trace of the line is in X Y. Draw *b* B parallel to X Y. At *a* make an angle of 35°, and produce the line to cut *b* B in B. At B draw B B′. With centre *a* and radius *a* B′ describe an arc to meet the projector from *b* in *b′*. Join *a b′*.

PROBLEM 17.—A is a point in the V.P. 1¼″ above the H.P.; B is a point in the H.P. 1¾″ from the V.P. The real distance from A to B is 3″. Draw the plan and elevation of the line, joining A and B. (*Sc.*, 1887.)

Draw X Y, and mark the elevation, *a′*, of the given point 1¼″ above it. Get the plan of *a′*. Draw a line 1¾″ below X Y and parallel to it; the plan of B must lie in this line. Draw *a′* B, 3″ long. With centre *a* and radius *a* B describe an arc to cut the parallel line in *b*. Then *b* will be the plan of B when 1¾″ in front of the V.P. Join *a b*. Project *b* to *b′*. Join *a′ b′*. Then *a b* and *a′ b′* will be the required plan and elevation.


PROB. 14.
c′
a′
b′
X
C
C
A
c
a
B
b
A
B
C
PROB. 15.
1.
a′
a
b′
40°
B
Y
b
2.
A′
a′
A
40°
b′
X
Y
a
PROB. 16.
1.
2.
a′
b′
X
A
35°
a
b′
b
a
35°
B′
Y
b
B
PROB. 17.
a′
1¼″
3″
X
a
b′
B
Y
1¾″
b

PROBLEM 18.—Given the inclinations of a line (of any given length) to both of the co-ordinate planes, to determine its projections.

Let the line be inclined to the H.P. at 40° and to the V.P. at 35°.

1. At the point B in X Y draw a line inclined to the H.P. at 40°, and make B a' equal to the given length. Draw a' a. At a' make the angle B a' C equal to the inclination to the V.P., 35°. From B draw B C perpendicular to a' C. With centre a' and radius a' C describe an arc cutting X Y in b'.

Join a' b'. This will be the elevation of the required line. To obtain its plan, describe an arc from centre a with radius a B. Project from b' to b. Join a b for the plan.

Note.—This is a more difficult problem to follow. It will assist the student if he cuts out in paper the trapezium a B C a' (Fig. 1), making a' B equal to the given line, the angle a' B a=40°, and B a' C=35°. Now fold the paper along the line a' B. Place it on the cardboard planes so that it rests on the sides a B and B C. The reason for the construction will then be seen.

2. *When the line passes through* X Y. At any point, a, draw a B equal to the given line and inclined to the H.P. at 40°. Through B draw a line B b' parallel to X Y. Make the angle B a C equal to 35°, the inclination of the line to the V.P., and from B draw B C perpendicular to a C. With centre a and radius a C, describe an arc cutting the parallel through B in b'. Join a b' for the elevation of the line. To obtain the plan, draw B d perpendicular to X Y. With centre a and radius a d describe an arc. Project from b' to meet this arc at b. Join a b for the plan.

Notes.—1. The line in this case is in a similar position to the line in Prob. 15, Case 2.

2. The sum of the inclinations to the vertical and horizontal planes can never exceed 90°. When they equal 90° then the plan and elevation are in one straight line.

PROBLEM 19.—Given the projections of two intersecting straight lines, to determine the angle between them, and obtain another elevation on a ground line inclined at 75° to the V.P.

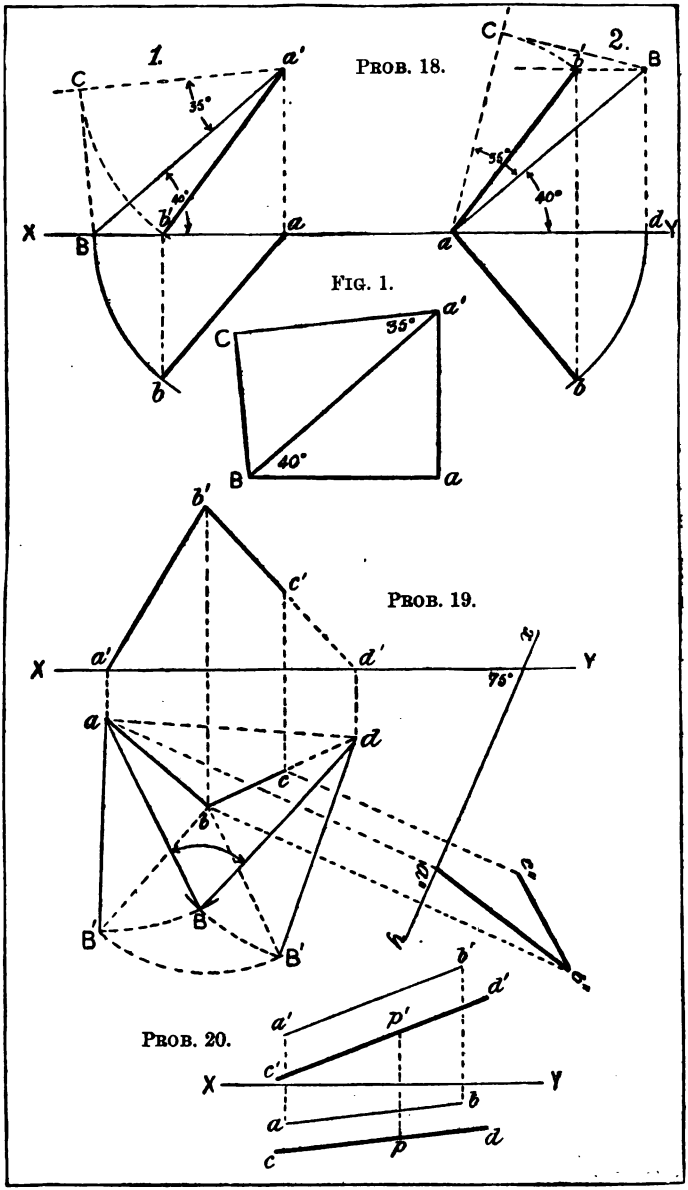
Let a' b', b' c' and a b, b c be the given projections. Produce b' c' to X Y, and obtain the horizontal trace, d, of the line B C. Join a d. The problem now resolves itself into Problem 14. Find the true lengths of a b and b d, and construct the triangle a d B. The angle at B will be the required angle.

To find a *fresh elevation*, proceed as has been already explained in Chapter XVI. Draw x y, making 75° with X Y. Imagine the figure to be turned so that x y is horizontal. Project from a, b, and c. The point, a, is on the H.P.; therefore its elevation will be a''. Make b'' and c'' the same distance above x y that b' and c' are above X Y. Draw a'' b'' and b'' c''.

PROBLEM 20.—Through a given point to draw the projections of a line parallel to a given line.

If the projections of lines are parallel to one another the lines themselves will be parallel.

Through p and p', the projections of the given point, draw c' d' parallel to a' b' and c d parallel to a b.



PROB. 18.

FIG. 1.

PROB. 19.

PROB. 20.

PROBLEM 21.—Given the plans of two intersecting lines, the elevation of one, and a point in the elevation of the second, to determine the elevation of the second line.

Let $a\,b$ and $c\,d$ be the plans, $a'\,b'$ the elevation of $a\,b$, and d' a point in the elevation of $c\,d$. The point, f, is common to both lines, because they intersect. Project f to f'. Then f' is a point in $c'\,d'$ as well as in $a'\,b'$. Through f' draw $d'\,f'$ to meet the projector from c. Then $c'\,d'$ is the elevation of $c\,d$.

Note.—The condition, therefore, for two lines to intersect is not only that their plans and intersections respectively intersect, but also that the point of intersection of the elevations and plans must be in the same projector.

EXERCISES

Note.—The figures relating to these exercises should be drawn larger.

1. What is the difference between the trace of a line, the projection of a line, and the projectors of a line? (*Sc.*, 1881.)

2. When will a line have:—1, No traces. 2, A horizontal trace only. 3, A vertical trace only. 4, Both a horizontal and vertical trace?

3. What is the true length of a line if its plan measures 2·5″ and it is inclined at 25°. (*Sc.*, 1881.) (*Prob.* 286, Chap. XVI.)

4. The plan of a line 3″ long is 1·5″ in length: at what angle is the line inclined? (*Sc.*, 1876.)

5. Represent correctly by their projections on a horizontal and vertical plane:—*a*, A line parallel to both planes of projection. *b*, A line passing through a point in the ground line. *c*, A point equidistant from both planes of projection. *d*, A line equally inclined to both planes of projection.

6. Assume the projections of any line parallel to neither plane of projection. Determine the traces of the line and the real distance apart of those traces. (*Sc.*, 1877.)

7. Show the real distance between the points whose projections are given on Fig. 1. (*Sc.*, 1876.)

8. Obtain the projections of two points P and Q on the given line $a\,b$, $a'\,b'$, Fig. 2; such that P is 1·4″ from the V.P. and Q 2″ from the H.P. Determine and write down the real length of the line P Q. (*Sc.*, 1886.)

9. Determine the vertical trace of the line A B, Fig. 2, and its inclination to the H.P. (*Sc.*, 1882.)

10. From the line A B, Fig. 2, of which the projections are given, cut off a portion, A C, which shall be 2″ long, and show the plan of C. (*Sc.*, 1881.)

11. The plan of a triangle, $a\,b\,c$, is given, the height of a is 1″, of b 2″, of c 3″. Draw the elevation, and determine the true form of the triangle. (*Sc.*, 1878.)

12. b is the plan of a point which is 2¾″ from the given point, $a\,a'$, Fig. 3. Determine its elevation b'. (*Sc.*, 1882.)

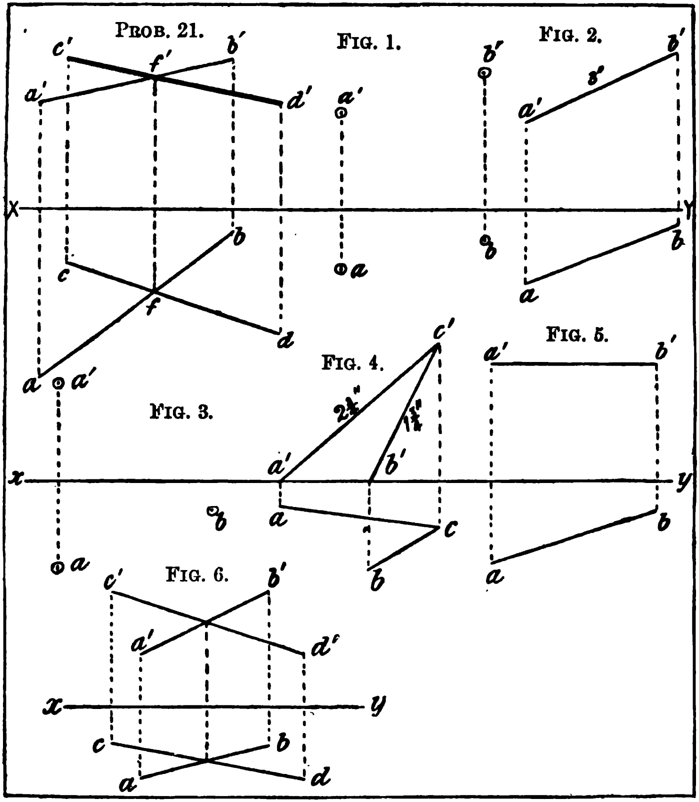
Note.—Join $a\,b$, and obtain elevation of b by Problem 13.

13. Show by their projections:—*a*, A line parallel to the V.P. and inclined at 48° to the H.P. *b*, A line inclined 35° to the V.P., and the plan of which makes 50° with the ground line. (*Sc.*, 1885.) (*Prob.* 15.)

14. A point 1·5″ from both planes of projection is distant 3·25″ from another point 2·25″ from both planes of projection. Obtain the projections of the two points. (*Sc.*, 1888.) (Apply principles of *Prob.* 15, Case 2.)

15. A point M is 1″ above the H.P. and ·75″ in front of the V.P. A point N is at double these distances, and is 2″ from M. Show their plans and elevations. (*Sc.*, 1882.) (*Prob.* 17.)

16. Show by their traces:—*a*, A line inclined at 37° to the V.P. and 53° to the H.P. *b*, A line inclined at 33° to both planes of projection. (*Sc.*, 1884.) (*Prob.* 18.)



17. What is the real angle contained by the lines whose projections are given, Fig. 4?

18. Construct a triangle, *o a b* (*o a*=2¼″, *o b*=3¾″, *a b*=4½″); *a* and *b* are the horizontal traces of the lines *o a*, *o b*, meeting at a point of which *o* is the plan. The height of *o* above the horizontal plane is 1½″. Obtain the real angle contained by the lines. (*Sc.*, 1879.) (*Prob.* 19.)

19. Show by their projections any two equal parallel lines not parallel to either plane of projection. (*Sc.*, 1887.)

20. State exactly what is meant by Figures 5 and 6. (*Sc.*, 1887.)

CHAPTER XVIII

ON PLANES

The two co-ordinate planes which have been dealt with previously are conceived to be always in a fixed position. Other planes in different positions may also be imagined, crossing and intersecting the co-ordinate planes in lines which are called *traces.* (Fig. 4, p. 183.) These secondary planes, of which there may be an infinite number, must intersect one or both of the co-ordinate planes; and, having no limit, they cannot be represented, as objects can, by their projections; but can only be indicated by their *traces.* The traces of planes are always straight lines (Euc. xi. 3), and they intersect on the ground line or are parallel to it.

PROBLEM 22.—Represent by their traces planes in the following positions:—1. Horizontal. 2. Vertical and parallel to the V.P. 3. At right angles to both planes. 4. At right angles to the H.P. and inclined at 45° to the V.P. 5. At right angles to the V.P. and inclined at 45° to the H.P. 6. Inclined at 45° to both planes and parallel to the ground line. 7. Oblique to both planes.

These positions are shown first pictorially to assist the student, who should illustrate them for himself with the help of the cardboard planes. For positions 1-6 a rectangular piece of card should be placed as shown, and the lines *h t* and *v t*, where the card, if produced, would penetrate the co-ordinate planes, should be marked with a pencil. For 7*a* and 7*b* cut out two triangles, one having *acute angles* and the other an *obtuse angle*, and fit them into the angle between the co-ordinate planes, marking where they touch those planes as before. If the vertical cardboard plane be now turned down, the traces will be shown on it as represented under the picture.

Notes.—1. The planes and their traces must not be supposed to terminate as is here shown, but to be without limit.

2. In numbers 1, 2, and 6, the traces are parallel to **X Y**, in all the other cases they intersect on **X Y**.

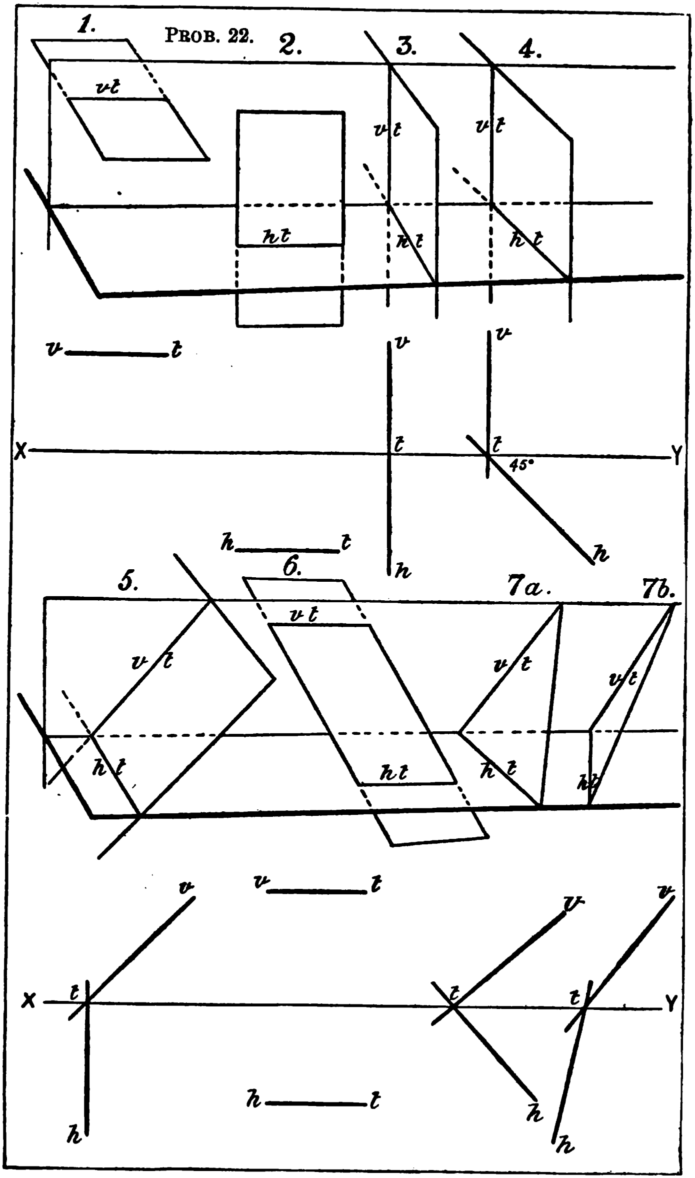
3. Planes 1 and 2, being parallel to one of the co-ordinate planes, have only one trace.

4. In 4, *h t* shows the inclination of the plane to the V.P., and in 5, *v t* shows the inclination of the plane to the H.P.

5. The plane in 6, being equally inclined to both of the co-ordinate planes, has its traces equidistant from **X Y**.

6. In 7*a* and 7*b* the traces do not represent the inclination of the planes to either of the co-ordinate planes.

7. The sum of the inclinations which a plane makes with the co-ordinate planes cannot be less than 90° nor more than 180°.



PROB. 22.

When a plane is in a position similar to the planes represented by Figures 6, 7*a*, and 7*b* of the previous problem, that is, *inclined to both of the co-ordinate planes*, it is termed an **Oblique Plane**, and its traces do not show the angle at which it is inclined to either of the co-ordinate planes.

To determine this angle a special construction is necessary; but before doing this the student must clearly comprehend what is meant by *the inclinations of an oblique plane*. In Fig. 1, let $v\,t\,h$ be an oblique plane intersecting the co-ordinate planes in $v\,t$ and $h\,t$. Take any point, a, in $h\,t$, and draw $a\,c$ in the horizontal plane perpendicular to $h\,t$; from the same point draw $a\,b$ in the oblique plane also perpendicular to $h\,t$; then the angle $c\,a\,b$ is the inclination of the plane $v\,t\,h$ to the horizontal plane. If this angle be supposed to revolve into the vertical plane, as shown by the angle $b\,a'c$, then θ will be the true size of the angle.

If a point be taken in the vertical trace, and two perpendiculars to that trace be drawn, one in each plane, the inclination to the vertical plane will be determined.

PROBLEM 23.—Given the traces of an oblique plane, to determine its inclinations.

Case 1.—Take a point, a, in X Y. Draw $a\,b$ perpendicular to the horizontal trace, and $a\,a'$ perpendicular to X Y. Make $a\,c$ equal to $a\,b$. Join $a'c$. Then the angle $a\,c\,a'$ is the inclination of the plane to the H.P. It will be seen that $a\,a'$ is the axis of a cone and $b\,c$ part of its base. The inclination to the V.P. is found in a similar manner. Take a point, e', in X Y, and draw $e'f$ perpendicular to the vertical trace, and $e'\,e$ perpendicular to X Y. Make $e'\,g$ equal to $e'f$. Join $e\,g$. Then the angle $e\,g\,e'$ is the inclination of the plane to the vertical plane.

Case 2.—This is really an advanced problem, but is given here for the sake of completeness. The traces of the plane form an obtuse angle, and require producing to obtain the points a' and e. The portions dotted fall without the first dihedral angle. The lettering is the same as for Case 1, so that the same explanation applies.

Case 3.—When the traces are both parallel to X Y.

The same construction applies as in Case 1. The angle θ is the inclination to the H.P., and ϕ is the inclination to the V.P. The triangle $a\,c\,a'$ represents a profile view of the planes when cut by a section at right angles to their intersections.

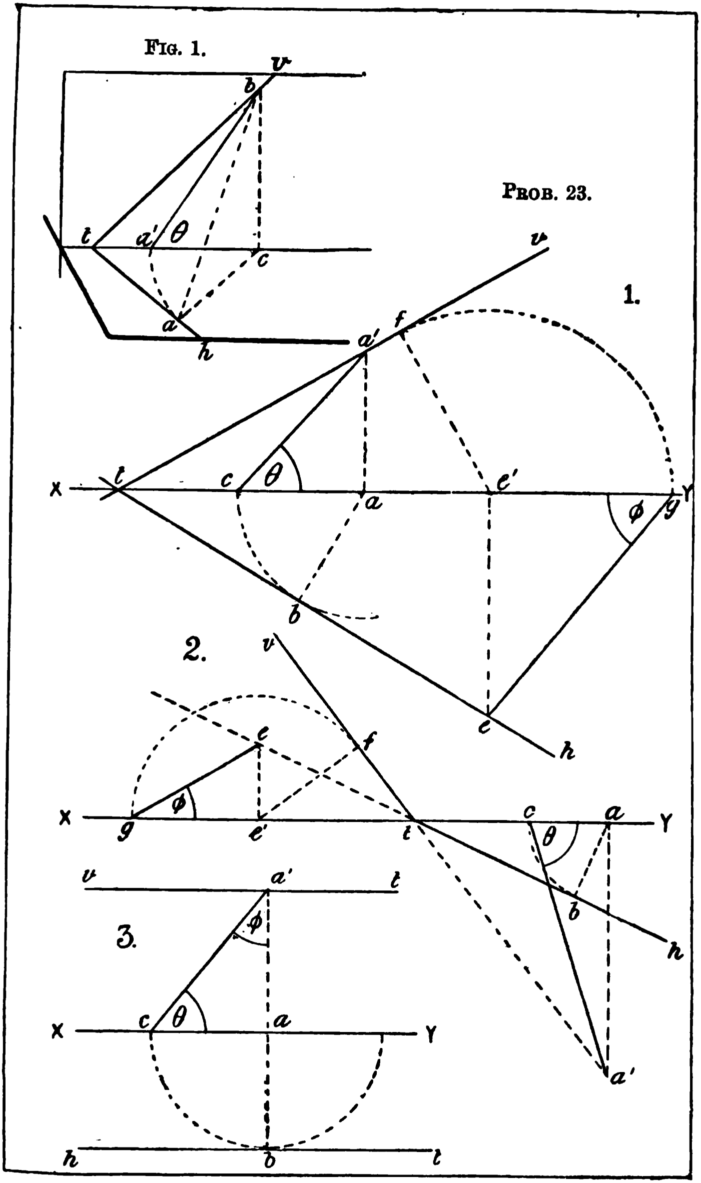


FIG. 1.

PROB. 23.

v
b
t
a'
θ
c
a
h
v
1.
f
a'
X
t
c
θ
a
e'
Y
φ
g
b
e
h
2.
v
e
f
φ
X
g
e'
t
c
a
Y
θ
b
h
a'
v
a'
t
3.
φ
θ
X
c
a
Y
h
b
t

PROBLEM 24.—Given one trace of a plane, and the inclination of the plane to either of the co-ordinate planes, to determine the other trace.

Case 1.—*Given the horizontal trace (h t), and the inclination to the H.P.* 50°. From any point, *a*, in X Y draw *a b* perpendicular to *h t* and *a a′* perpendicular to X Y. With centre *a* and radius *a b* describe the arc *b b′*, part of the base of the cone to which the plane is tangential. At *b′* draw *b′ a′*, making the given inclination (50°) with X Y. Through *a′* draw *v t*, the required trace.

Case 2.—*Given the vertical trace (v t), and the inclination to the H.P.* 50°.

Take any point, *a′*, in *v t*, and draw *a′ a* perpendicular to X Y. At *a′* draw *a′ b′* to meet X Y at an angle of 50°. With centre *a* and radius *a b′* describe the arc *b′ b*. Draw *h t* tangential to this arc.

Case 3.—*Given the vertical trace (v t), and the inclination to the V.P.* 50°.

This is exactly similar in construction to Case 1, and is the same figure inverted, the base of the cone being in the V.P. instead of in the H.P. Take any point, *a′*, in X Y and draw *a′ b′* perpendicular to *v t*. Revolve *a′ b′* into X Y. Draw *b a*, making 50° with X Y. Through *a* draw *h t*, the required trace.

Case 4.—*Given the horizontal trace (h t), and the inclination to the V.P.* 50°.

From any point, *a*, in *h t* draw *a a′* perpendicular to X Y. From *a* draw *a b* to meet X Y at an angle of 50°. Describe the arc, *b b′*, and draw the vertical trace tangential to it.

PROBLEM 25.—Given the traces of a plane, to determine the real angle contained by them.

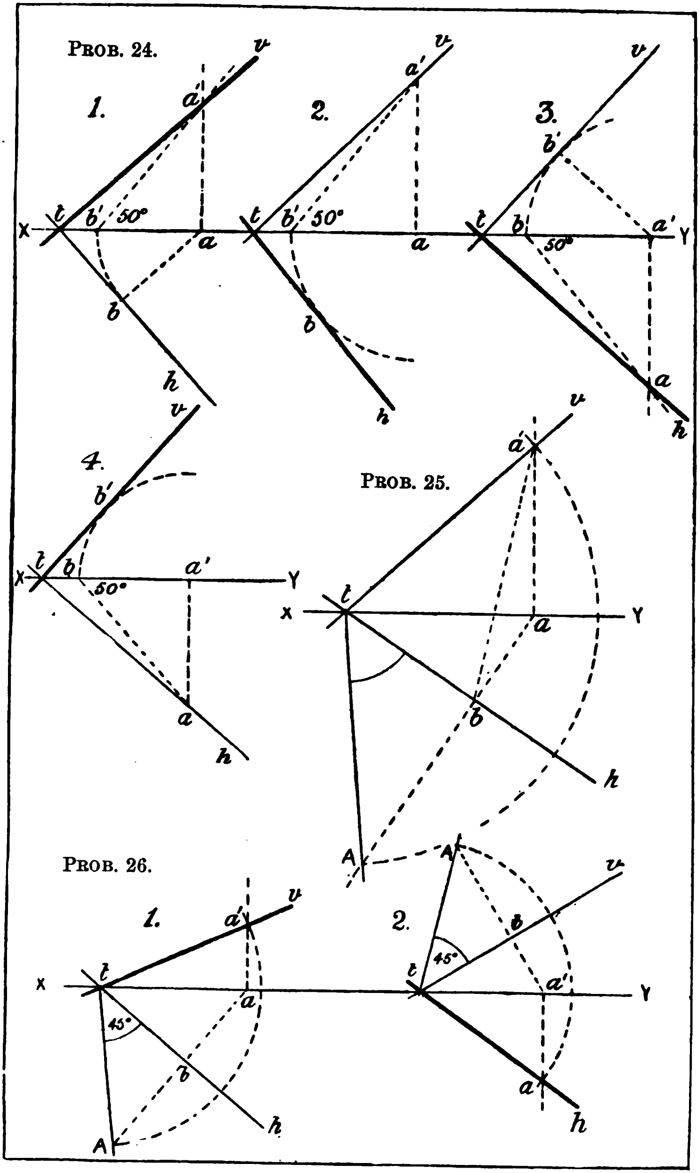
What is required is the true size of the angle *v t h* when the co-ordinate planes are in their proper position; if in Fig. 1. Prob. 23 the triangle *b a t* were to be '*constructed*' into the H.P., that is, supposed to be laid flat upon it, then the angle *b t a* as then represented on the H.P. would be the angle required.

Let *v t* and *h t* be the traces. From any point, *b*, in the horizontal trace draw a perpendicular, *b a*, and from *a* draw *a a′* to meet the vertical trace. Now 'construct' the triangle *b t a′* into the H.P. by describing an arc, with *t* as centre and *t a′* as radius, to cut *a b* produced in A. Join *t* A. Then *b t* A will be the real angle between the traces.

PROBLEM 26.—Given the real angle (say 45°) between the traces of a plane, and one trace, to find the other trace.

Case 1.—*When the horizontal trace (h t) is given.* At *t* draw *t* A, making 45° with *h t*. From any point, A, in *t* A draw A *b* perpendicular to *h t*, and produce it to meet X Y in *a*. Draw *a a′* perpendicular to X Y. With centre *t* and radius *t* A, describe an arc cutting the perpendicular from *a* in *a′*. Then *v t* will be the required trace.

Case 2.—*When the vertical trace is given.* Construct the angle *v t* A of 45°. The same construction applies as in Case 1. The required trace will be *h t*. The reason for the construction will be demonstrated by assuming the traces, and working out the process for finding the real angle as in Problem 25.


PROB. 24.
1.
v
a'
t
b'
50°
X
a
b
h
2.
v
a'
t
b'
50°
a
b
h
3.
v
b'
t
b
a'
Y
50°
a
h
4.
v
b'
t
b
X
a'
Y
50°
a
h
PROB. 25.
v
a'
t
X
Y
a
b
h
A
PROB. 26.
1.
v
a'
t
X
a
45°
b
h
A
2.
A
v
b
45°
t
a'
Y
a
h

PROBLEM 27.—Given the angles of inclination to the co-ordinate planes, to determine the traces of an oblique plane.

This is the converse of Problem 23.

The sum of the angles of inclination must be more than 90°, and less than 180°. Let the inclination to the H.P. be 48°, and to the V.P. 60°. The principle upon which this problem is solved is that the plane must touch two cones whose base angles are 48° and 60° respectively, and which are arranged so that the axis of one is in the H.P. and that of the other in the V.P.; the axes should meet in X Y, and the cones should be placed so that they envelop a common sphere, which has its centre in X Y, where the axes of the cones meet.

In X Y take any point, *a*, and, with any radius, describe a circle. This circle will be both the plan and elevation of the sphere mentioned above. Through *a* draw a perpendicular, *a′ d*, of indefinite length. Draw *b a′* tangential to the circle, and making 48° with X Y. (Draw any line, A B, at 48°, and draw *a′ b* parallel to it.) Then *a′ b* will be the slant side of a cone enveloping the sphere. With centre *a* and radius *a b* describe the base of the cone. Draw *d c* tangential to the circle, and meeting X Y at 60°. With centre *a* and radius *a c*, describe the base of the second cone. Through *d* draw *h t* tangential to the base, *b e*, of the vertical cone. Join *t* and *a′*. Then *h t* and *v t* are the required traces. The vertical trace, *v t*, will touch the base of the cone having the horizontal axis in *f*.

PROBLEM 28.—To determine the distance between two parallel planes given by their traces.

Note.—All parallel planes have parallel traces.

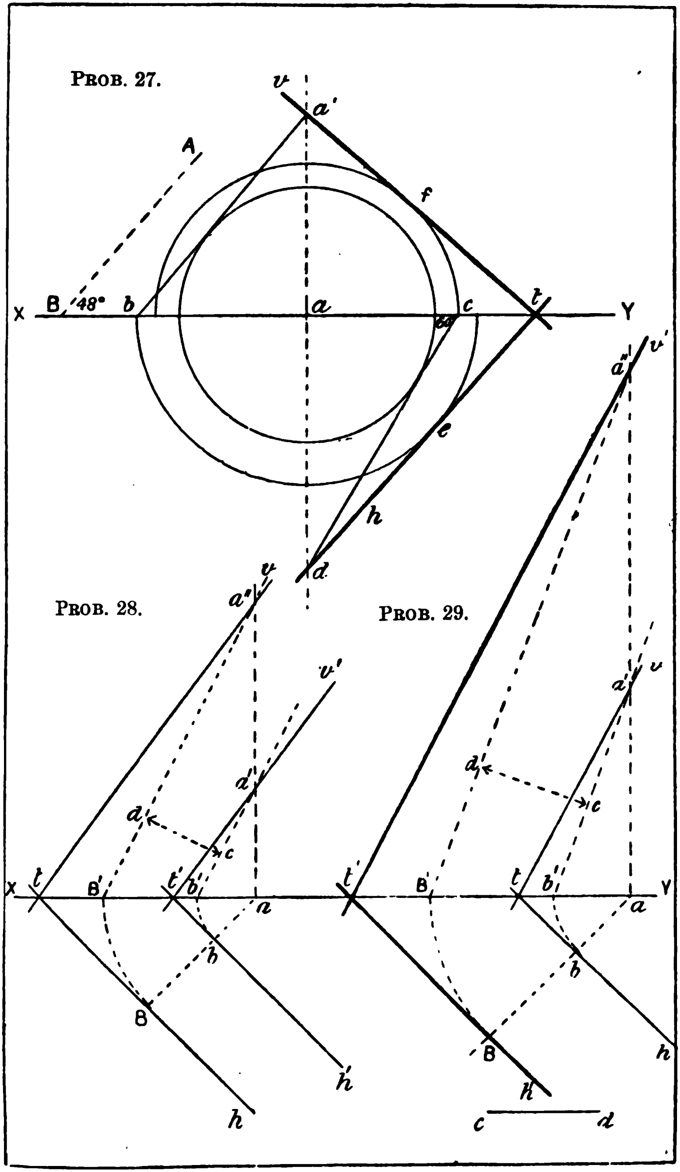
Let *v t h* and *v′ t′ h′* be the two parallel planes.

Take any point, *a*, in X Y, and draw *a* B perpendicular to the horizontal traces, and *a a″* perpendicular to X Y. With centre *a* and radii *a b* and *a* B describe the arcs *b b′* and B B′. Join *b′ a′* and B′ *a″*. The distance *c d* between these two lines is the required distance.

Note.—The vertical plane, of which *a* B is the horizontal trace, has been revolved on its vertical trace, *a a″*, into the V.P.

PROBLEM 29.—To determine the traces of a plane parallel to a given plane, and at a given distance from it.

This is the converse of the previous problem. Let the given plane be *v t h*, and the given distance be equal to *c d*. Take any point, *a*, in X Y, and draw *a b* perpendicular to *h t*, and *a a′* perpendicular to X Y. Revolve *a b* into the V.P., and join *b′ a′*. Draw *a″* B′ parallel to *a′ b′*, and at a distance from it, equal to *c d*. With centre *a* and radius *a* B′ describe an arc, B′ B. Draw *v′ t′* through *a″* parallel to *v t*, and draw *h′ t′* parallel to *h t*.



PROB. 27.

PROB. 28.

PROB. 29.

PROBLEM 30.—To determine the intersection of two given planes.

'When two planes meet, their intersection is a straight line.' (Euc. xi. 3.)

Case 1.—*When the vertical and horizontal traces meet one another respectively.*

Three positions are given. Let the vertical traces, *v t* and *v′ t′*, intersect in the point *a′*, and the horizontal traces, *h t* and *h′ t′*, intersect in the point *b*. The line joining *a′* and *b* would be the line in which the planes intersect, and the problem is simply to obtain the plan and elevation of this line. Project *a′* and *b* to X Y, giving *a* and *b′*. Join *a′ b′* and *a b* for the plan and elevation of the intersection required.

Case 2.—*When either the horizontal or vertical traces are parallel.*

Four positions are shown. In the first position the planes meet like a roof, the plan of their intersection is parallel to the horizontal traces, and the elevation will be the point *a′*.

The second position has the vertical traces parallel, and is similar in its construction to the planes in the first position. In the third case the horizontal traces are parallel, and the elevation of the intersection will be a horizontal line passing through *a′* parallel to X Y, while the plan of the intersection will be a line drawn from *a* parallel to the horizontal traces. The fourth position shows the intersection when the vertical traces are parallel.

Case 3.—*When both traces are parallel to* X Y.

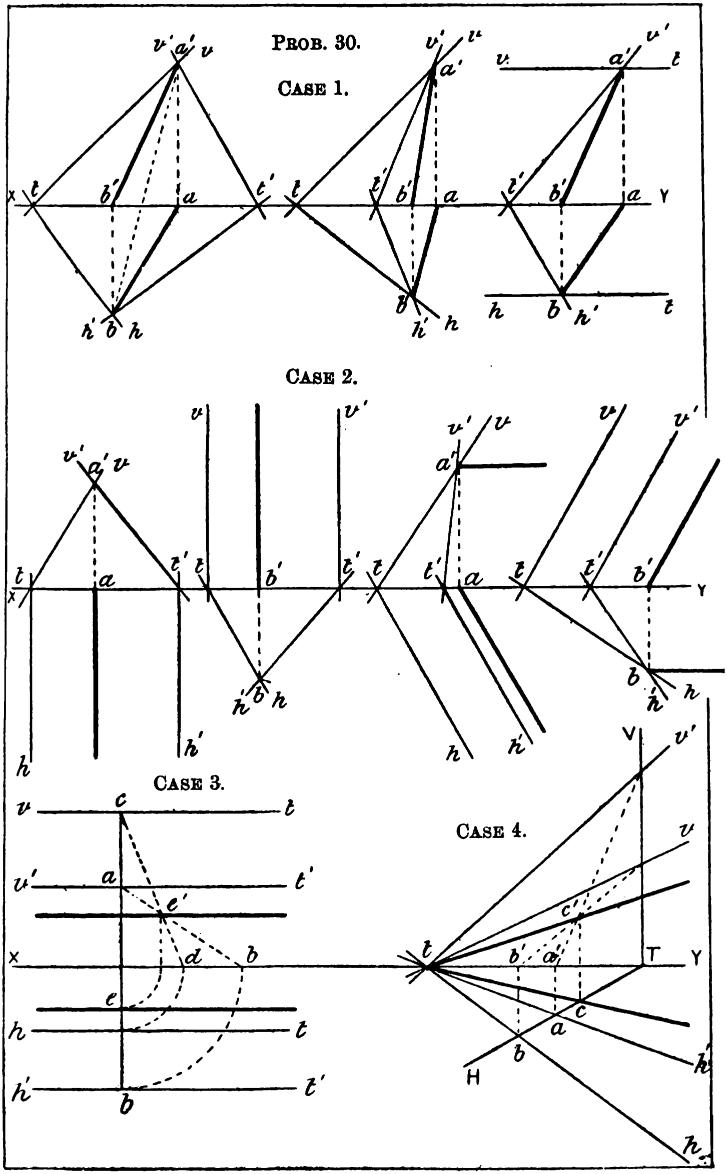
As the traces are parallel to X Y, the intersection will also be parallel to X Y. To determine the intersection in this case, assume a third plane cutting through both planes at right angles to the co-ordinate planes, and obtain a profile view. Let *c b* be the section plane. Obtain the end view, *c d*, of the first plane, *v t*, *h t*; and in the same manner obtain *a b*, the end view of the second plane, *v′ t′*, *h′ t′*. The point *e′*, where *a b* and *c d* intersect, will be a point in the elevation of the section. Through *e′* draw the elevation. For the plan of the intersection, project *e′* to X Y and revolve it back to the section plane at *e*. Through *e* draw the required line.

Case 4.—*When all the traces meet at a point in* X Y.

Let *v t*, *h t*, and *v′ t* and *h′ t* be the traces of the two planes.

The solution is similar to that in Case 3. Assume a third plane, V T, H T, cutting the other two planes, and perpendicular to the H.P. Find the intersection of this plane with each of the given planes as in Case 1. The elevation of the intersection will be drawn as shown through *c′*. Obtain *c*, the plan of *c′*, and draw the plan of the intersection.

When the two planes pass through X Y and contain it, the plan and elevation of the intersection will, of course, be X Y.


PROB. 30.
CASE 1.
v′ a′ v
x t b′ a t′ t t′ b′ a t′ b′ a Y
h′ b h
v′ v a′
b h′ h
v a′ v′ t
h b h′ t
CASE 2.
v′ a′ v
t a t′ t b′ t′ t t′ a t t′ b′
x Y
h h′ h′ b h
v v′ v′ v a′ v v′
h h′ b h h
CASE 3.
v c t
v′ a t′
e′
X d b
e
h t
h′ b t′
CASE 4.
V v′
v
c′
t b′ a′ T Y
c
a
b h′
H h

PROBLEM 30.—(*Continued.*)

Case 5.—*When the traces do not meet within the limits of the paper.*

Let us suppose the horizontal traces not to meet. Project *a′* to X Y for the point *a*. Pass a horizontal plane, A B, through the two planes. Project *b′* and *c′*, for *b* and *c*. Now if the elevation of a line contained by a plane be parallel to the ground line, then the plan of that line will be parallel to the horizontal trace of the plane. The plan of the horizontal line through *b′* will be parallel to *h t*, and the plan of that through *c′* will be parallel to *h′ t′*. The point *d*, where *b d* and *c d* meet, will be a point in the plan of the intersection. Draw *a d*.

Project *d* to *d′*. Then *a′ d′* will be the elevation of the intersection.

When neither traces meet, proceed as shown in the second figure. *Two* horizontal planes, A B and F G, must be passed through the two given planes in order that *two* points may be obtained, through which the plan and elevation of the intersection must be drawn. The solution may be readily followed from the figure.

PROBLEM 31.—To determine the real angle between two planes given by their traces.

The real angle between them will be the angle formed by the lines of intersection which a plane would make by cutting both the given planes at right angles.

Case 1.—Find the plan, *a b*, of the intersection of the two given planes. Draw *f g* at right angles to *a b*. Find the true length of the line A B by revolving its plan, *a b*, into X Y and joining *b′ a′*. Turn *a e* into X Y, and draw *e′ h′* perpendicular to *a′ b′*. This will be the altitude of the triangle made by cutting both the given planes perpendicularly. From *e* set off *e h* equal to *e′ h′*. Join *f h* and *g h*. Then *f h g* will be the real angle between the planes.

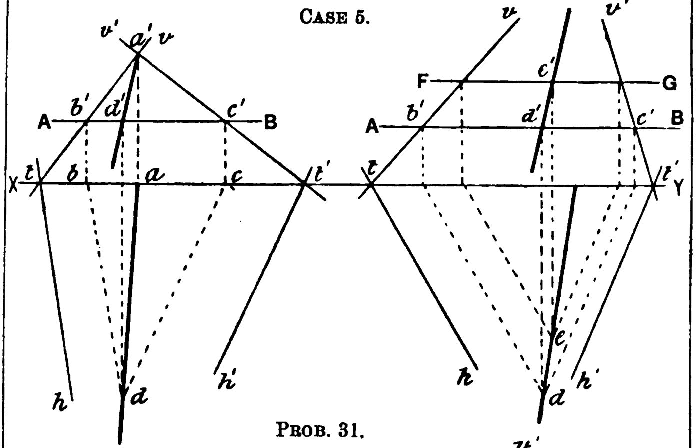
Case 2.—This shows the construction when one of the planes is parallel to X Y. The lettering and description are the same as in Case 1.

PROBLEM 32.—To determine the traces of a plane which makes a given angle with a given plane.

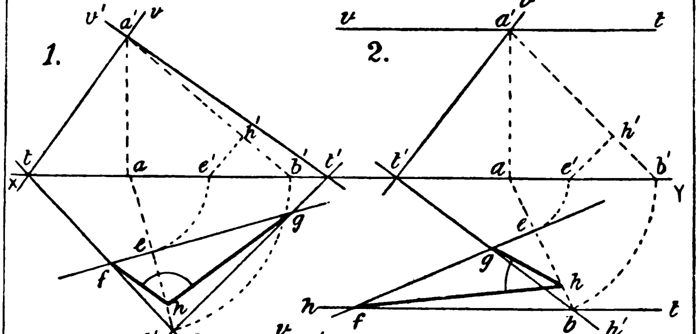
This is the converse of the previous problem. Let A be the given angle and *v t* and *h t* the traces of the given plane.

Assume the intersection of the planes, and let *a b* be the plan of that intersection. Draw *a a′* perpendicular to X Y. Draw *f e g* (the horizontal trace of the plane, which would cut both planes at right angles) perpendicular to *a b*. Find the true length of the intersection by revolving *a b* into the V.P. Make *a e′* equal to *a e*. Draw *e′ h′* perpendicular to *a′ b′*. Set off *e h* equal to *e′ h′*, and join *f* and *h*. At *h* make the angle *f h g* equal to the given angle A. From *b*, through *g*, draw the horizontal trace of the required plane. For the vertical trace, draw from T through *a′*.

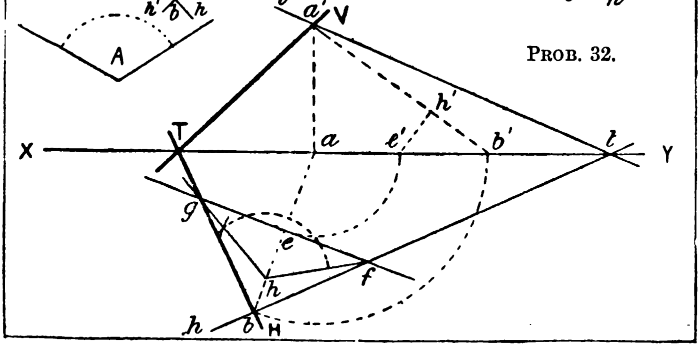
CASE 5.



PROB. 31.



PROB. 32.



CHAPTER XIX

ON PLANES AND LINES

It has already been stated in Problem 30 that if the elevation of a line contained by a plane be parallel to the ground line, then the plan of that line will be parallel to the horizontal trace of the plane. In a similar manner, if the plan of a line contained by a plane be parallel to the V.P., then its elevation will be parallel to the vertical trace of a plane.

In Fig. 1, *a′ b′*, the elevation of a line contained by the plane *v t h*, is parallel to X Y. Find *a*, the plan of *a′*, and draw *a b* parallel to *h t*. Then *a b* is the plan of the line.

In Fig. 2 the plan, *c d*, is parallel to the V.P. Obtain *c′*, the elevation of the point *c*, and draw the elevation, *c′ d′*, of the line parallel to the vertical trace.

Note.—The student may easily see this by taking a book and drawing a line on its cover parallel to the back. If the cover be now opened at any angle while the book lies on the table, the line on the cover will remain horizontal and its plan will be parallel to the back of the book, which represents the horizontal trace of the plane represented by the opened cover.

PROBLEM 33.—Given one projection of a point contained by a given plane, to find the other projection.

Let *v t*, *h t* be the traces of the given plane, and *a* the plan of a point, A, contained by the plane.

Four examples are shown. In the *first case*, the plane being perpendicular to the V.P., the elevation, *a′*, must lie in the vertical trace. In the *second and third cases* draw a line, *a b*, through the point parallel to the horizontal trace. Now this line, being parallel to the horizontal trace, will have its elevation parallel to X Y. Obtain *b′*, the elevation of *b*. A line parallel to X Y, through *b′*, must contain the point A. Draw *b′ a′* and project *a* to it.

In the *fourth case*, the plane being parallel to X Y, a construction similar to that employed in Problems 10, 23, &c., must be used. Pass a vertical plane through the point A, and proceed as shown in the figure. The triangle *b′ c′ b* represents a profile view of the planes.

If the elevation of the point A were given, the plan would be found in a similar manner by considering the given figures inverted.

Another method of solving the fourth case is to imagine an inclined line to pass through the point A and lie in the given plane. Draw any oblique line, *b c*, through *a*. Obtain *b′ c′*, the elevation of this line. Now as *b c* contains *a*, the elevation, *b′ c′*, must contain *a′*. Project as shown.

FIG. 1.

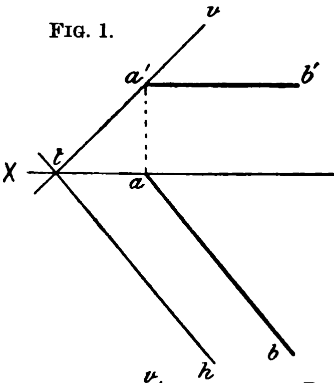
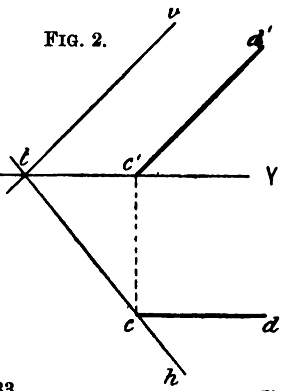
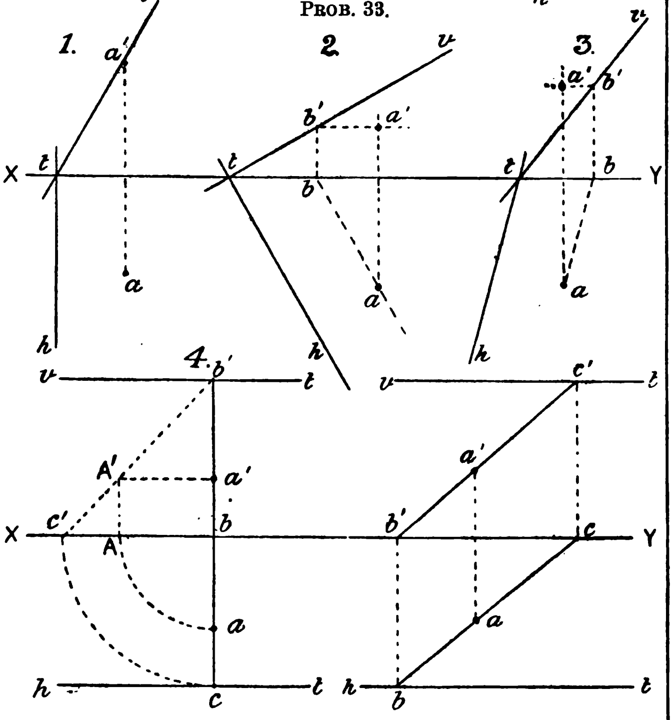


FIG. 2.



PROB. 33.



PROBLEM 34.—Given one projection of a line contained by a plane, to determine its other projection.

Let the plan, *a b*, of the line be given. The extremities of the given line are points. Project each point, as in the previous problem, and join the projections. One case is shown; the others should be worked out as exercises by the student.

PROBLEM 35.—To determine a plane which shall be parallel to a given plane and shall contain a given point.

Let *v t* and *h t* be the traces of the given plane, and *a*, *a′* the projections of the given point A. Through *a* draw *a b* parallel to *h t*. As *a b* is parallel to the horizontal trace its elevation will be parallel to X Y; then draw *a′ b′* parallel to X Y and project from *b* to meet it. This point *b′* must be in the vertical trace of the plane. Through *b′* draw *v′ t′* parallel to *v t*, and through *t′* draw *h′ t′* parallel to *h t*. Then *v′ t′ h′* will be the required plane.

PROBLEM 36.—To draw the traces of a plane which shall contain three given points.

Let *a*, *b*, *c*, and *a′*, *b′*, *c′* be the projections of the points. Join the points, thus obtaining the projections *a′ b′*, *b′ c′*, and *a b*, *b c* of two intersecting lines. If a line be contained by a plane, its traces must be in the traces of that plane. Produce *a b* and *a′ b′* both ways, and find the horizontal and vertical traces, *h t* and *v t*, of the line A B by *Problem* 5. These will be points in the horizontal and vertical traces of the plane. Find the horizontal and vertical traces, H T and V T, of the line B C in the same manner. These will also be points in the traces of the required plane. Through V T, *v t* draw the vertical trace, and through H T, *h t* draw the horizontal trace of the plane.

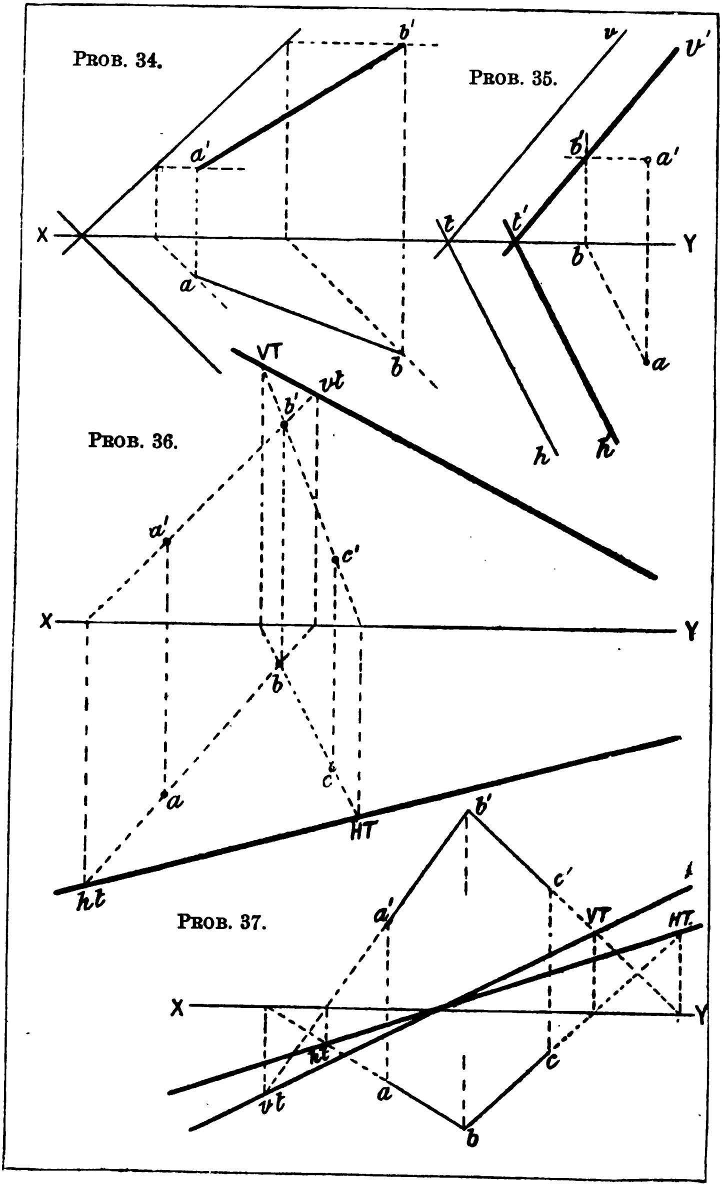
PROBLEM 37.—To draw the traces of a plane which shall contain two intersecting lines.

In the previous figure *a b*, *b c* and *a′ b′*, *b′ c′* will represent the projections of the two intersecting lines. The construction will be the same.

The *second figure* shows the solution when the traces of the lines do not all fall above the H.P. and in front of the V.P.

The same description will apply.

Note.—This problem offers another solution to Problem 34, when the projection, *ab*, being produced, meets X Y and the horizontal trace.


PROB. 34.

PROB. 35.

PROB. 36.

PROB. 37.

PROBLEM 38.—To determine the traces of a plane which shall contain two parallel lines.

Let *a b*, *a′ b′* and *c d*, *c′ d′* be the projections of the two lines.

Case 1. Let *a b* and *c d* be parallel to X Y. Apply *Problem* 5 and find the horizontal traces H T, *h t* of each line. Through these two points draw the horizontal trace of the plane. The vertical trace will be parallel to the elevations of the lines. (*See page* 212.)

Case 2. When both the plans and the elevations of the planes are parallel to X Y we must apply the converse of the solution to *Problem* 38, *case* 4. Intersect the projections by a vertical plane. Suppose it to revolve and turn *e* and *f* into X Y. Obtain the actual positions, E and F, of these points with regard to the vertical plane of projection. Join E and F, and produce the line to meet the vertical plane in V T, and X Y in H. Revolve the plane back, when H will come to H T. Through V T and H T draw the traces of the plane.

PROBLEM 39.—To determine the intersection of a given line and a plane.

Let *v t h* be the given plane and *a b*, *a′ b′* the projections of the given line.

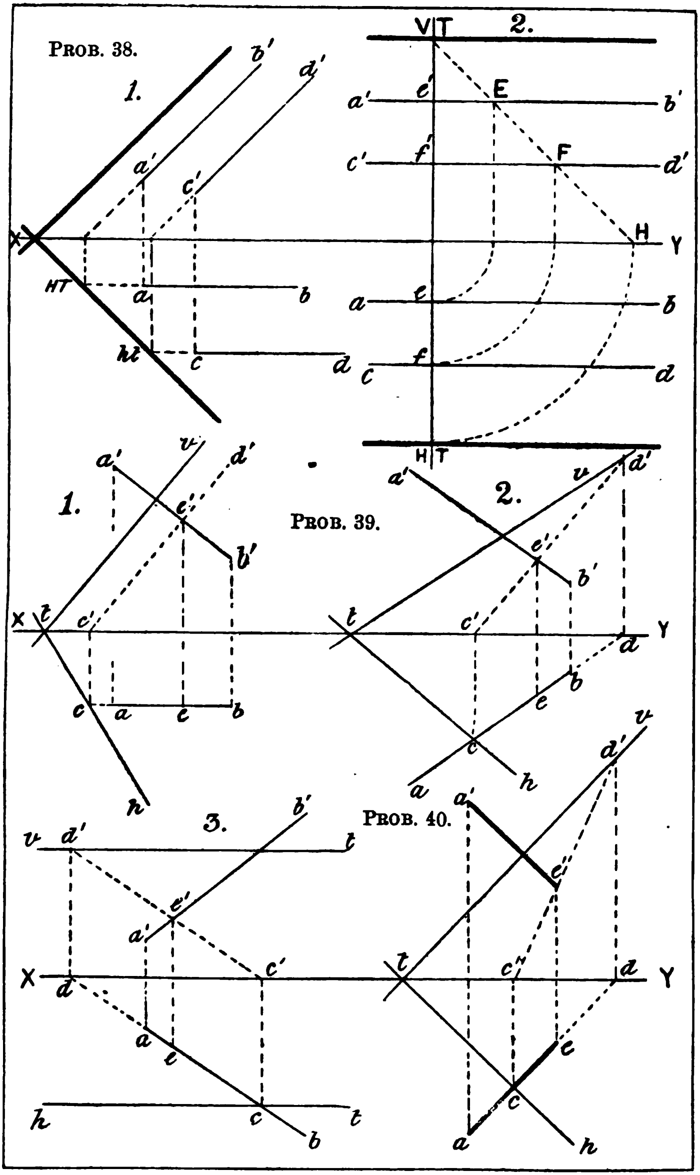
Case 1. *When the given line is parallel to one of the co-ordinate planes.* Assume a vertical plane which shall contain the given line and find its intersection with the given plane. Produce *a b* to cut *h t* in *c*. Obtain *c′*, the elevation of *c*. Draw *c′ d′*, the elevation of the intersection of the two planes. The point *e′*, where *c′ d′* cuts *a′ b′*, is the elevation, and *e* is the plan of the point where the given line intersects the given plane.

Case 2. *When the given line is inclined to both of the co-ordinate planes.* Produce *a b* to cut X Y and *h t*. Obtain *c′* and *d′*; *c′ d′*, as before, will be the elevation of the intersection of the vertical auxiliary plane with the given plane, and *e′* and *e* will be the elevation and plan of the intersection.

Case 3. *When the traces of the given plane are parallel to* X Y. This may be easily followed from the figure. The lettering is the same as in *Case* 2.

PROBLEM 40.—From a given point to draw a line perpendicular to a given plane.

The projections of a line which is perpendicular to a plane are perpendicular to the traces of that plane. Let *v t h* be the given plane, and *a*, *a′* the projections of the given point. From *a* and *a′* draw perpendiculars to *v t* and *h t*. Assume a vertical plane to contain these perpendiculars, as in *Problem* 39, and find the intersection of this plane with the given plane. E will be the point where the perpendicular penetrates the plane, and *a′ e′* and *a e* will be the projections of the perpendicular to the given plane.



PROB. 38.

PROB. 39.

PROB. 40.

PROBLEM 41.—From a given point in a given plane to erect a perpendicular of a given length.

Let *a* and *a′* be the projections of the given point, and *v t h* the given plane. From *a* and *a′* draw lines of indefinite length at right angles to *h t* and *v t*. Take the projections *b*, *b′* of any point in the lines. Find the true length of the line A B by 'constructing' it into the H.P. (*Problem* 8.) From A set off A C equal to the required length. Obtain *c* and *c′*. Then *a c* and *a′ c′* will be the projections of the required perpendicular.

Note.—If only one projection of the point were given, as *a′*, then the plan *a* must first be obtained by *Problem* 33.

PROBLEM 42.—To draw the traces of a plane which shall contain a given point and be perpendicular to a given line.

Let A B be the given line and C the given point in it. The traces of the plane must be perpendicular to the projections of the line. Through *c* draw *c d* perpendicular to *a b*. From *d* draw *d d′* perpendicular to X Y, to meet a line parallel to X Y from *c′*. Through *d′* draw *v t* at right angles to *a′ b′*, and from *t* draw *h t* parallel to *c d*. Then *v t* and *h t* will be the traces of the required plane.

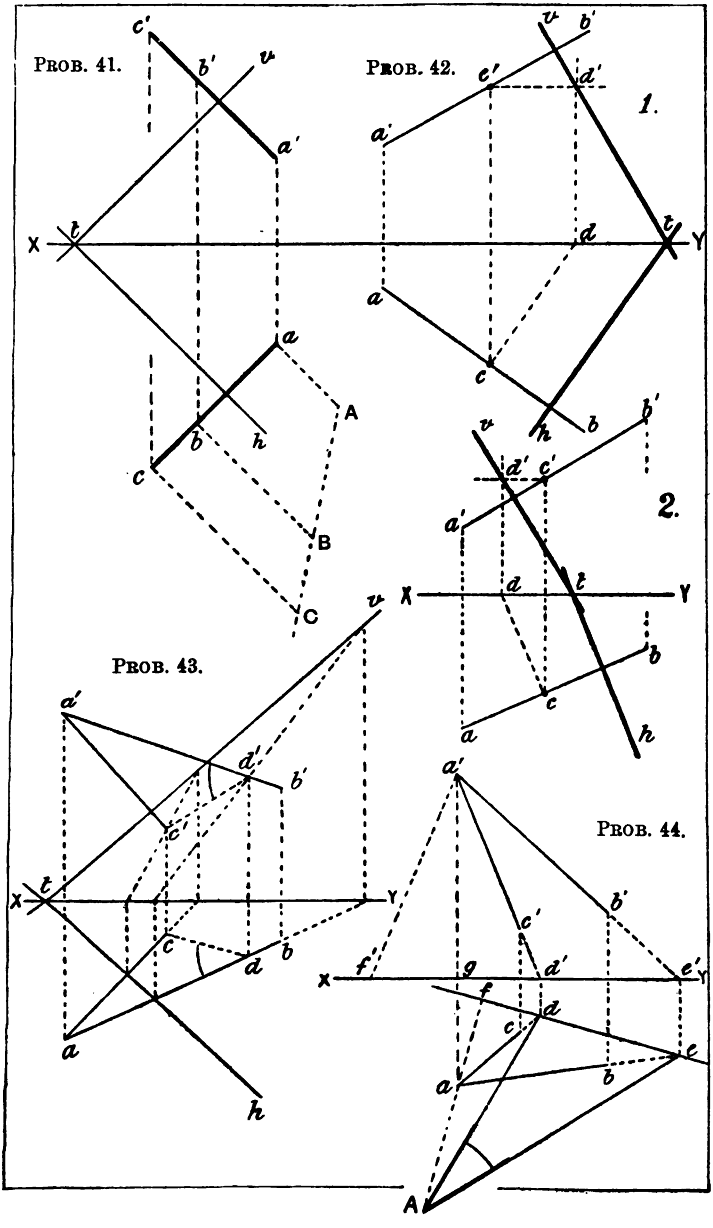
Note.—Two positions of the line are given. The description applies to both.

PROBLEM 43.—To determine the angle which a given line makes with a given plane.

Let *v t h* be the given plane, and *a b*, *a′ b′* the projections of the given line. From any point (*a*, *a′*) in the line draw a perpendicular to the given plane. (*Problem* 40.) Its projections will be *a c*, *a′ c′*. Find the intersection *d*, *d′* of the given line A B with the given plane *v t h*. (*Problem* 39.) If C and D, the points of intersection respectively of the perpendicular A C and the given line A B with the given plane, be now joined, a triangle will be formed and the true size of the angle A D C represented by its projections *a d c*, *a′ d′ c′* will be the required angle. To find the size of the angle determine the true lengths of the sides, and construct the triangle by *Problem* 14, or proceed as shown in the following problem.

PROBLEM 44.—To determine the angle between two straight lines. (Another method for solving Problem 19.)

Let *a b*, *a c* and *a′ b′*, *a′ c′* be the projections of the given line. Obtain *d* and *e*, the horizontal traces of the lines. Through *d* and *e* draw a line, and with this line as an axis of rotation 'construct' the triangle thus formed into the H.P. The point, *a*, will travel in a plane perpendicular to *d e*. Draw *a f* perpendicular to *d e* and produce it beyond *a*. To obtain the actual distance of A from *f*, construct a right-angled triangle having its base, *g f′*, equal to *a f*, and its perpendicular *a′ g*. Then its hypotenuse, *a′ f′*, will be the actual distance required. Make *f* A equal to *a′ f′*. Join *d* A and *e* A. The angle *d* A *e* is the angle required.


PROB. 41.
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h
c
B
C
PROB. 42.
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b′
e′
d′
1.
a′
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a
c
h
b
2.
v
b′
d′
c′
a′
X
d
t
Y
b
c
a
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PROB. 43.
v
a′
d′
b′
c′
X
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Y
c
b
d
a
h
PROB. 44.
a′
b′
c′
f′
g
d′
e′
X
Y
f
d
c
e
a
b
A

PROBLEM 45.—To draw the traces of a plane which shall have a given inclination to either of the co-ordinate planes, and shall contain a given line.

Let *a b*, *a′ b′* be the projections of the given line, and let the required plane be inclined to the H.P. at 65°. Find the traces, *v* and *h*, of the given line; these must be points in the traces of the required plane. From *v* draw a line to meet X Y at 65° to represent the slant side of the cone on which the plane may be assumed to rest. Draw the plan of this cone. From *h* draw a tangent to the plan of the cone, meeting X Y in *t*. This will be the horizontal trace of the plane. Join *v t* for the vertical trace.

The *second figure* shows the solution when the inclination to the V.P. is given. The same method of working is followed, the base of the cone being now in the vertical plane.

Notes.—1. The inclination of the line cannot be greater than the inclination of the plane, as, if it were so, the horizontal trace would fall within the base of the cone and no tangent could be drawn.

2. Two planes may be drawn satisfying the conditions, one on each side of the cone.

PROBLEM 46.—In a given plane to place a line inclined at a given angle.

Case 1.—*Inclined to the horizontal plane.*

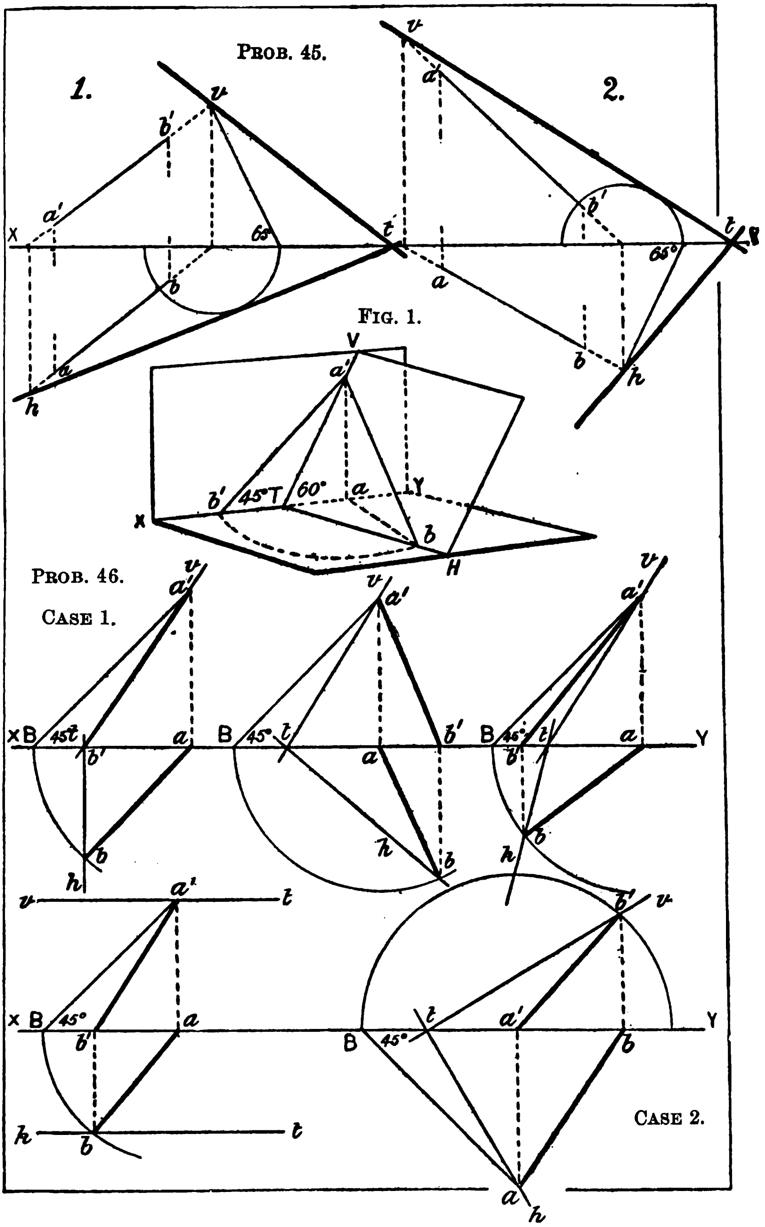
By referring to the diagram, Fig. 1, the reason for the solution will be easily seen. Let V T H be a plane inclined at 60° to the H.P., and let it be required to place in it a line inclined at 45° to the H.P. In V T take any point *a′* and draw *a′ a* perpendicular to X Y. From *a′* draw *a′ b′* to meet X Y at an angle of 45°. Now imagine the triangle *a a′ b′* to be revolved on *a a′* as an axis until the point *b′* reaches the horizontal trace of the given plane in *b*. The triangle will then occupy the position shown by the triangle *a′ b a*, and *a′ b* will be the required line.

Four different planes are shown; the construction is the same in all cases. Let *v t*, *h t* be the traces of the plane and let the line be inclined at 45°. In *v t* take any point, *a′*, and draw *a′ a* perpendicular to X Y. From *a′* draw *a′* B to meet X Y at an angle of 45°. With centre *a* and radius *a* B describe an arc cutting *h t* in *b*. Join *a b*. This will be the plan of the line. Project *b* to *b′*, and join *a′ b′* for the elevation.

Case 2.—*Inclined to the vertical plane.*

The solution is exactly similar to the previous case; it is only necessary to imagine the preceding figures reversed. One figure only is shown, the rest may be easily worked out by the student.

Take any point, *a*, in *h t* and draw *a a′* at right angles to X Y. From *a* draw *a* B meeting X Y at 45°. With centre *a′* and radius *a* B describe an arc cutting *v t* in *b′*. Join *a′ b′* for the elevation. Project *b′* to X Y and join *a b* for the plan.



PROB. 45.

1.

2.

FIG. 1.

PROB. 46.

CASE 1.

CASE 2.

PROBLEM 47.—From a given point to draw a line parallel to a given plane and inclined at a given angle.

Let *a*, *a′* be the projections of the given point, *v t h* the given plane, and let the line be inclined at 50° to the H.P. First obtain the traces, *v′ t′ h′*, of a plane parallel to the given plane, and containing the given point. (*Prob.* 35.) From *a′* draw *a′ c′* to meet X Y at 50° and obtain its plan *a c*. With centre *a* and radius *a c* describe an arc cutting *h′ t′* in *d* and *e*. Join *a d* for the plan, and project the elevation *a′ d′*. Two lines may be drawn from the point, *a e* and *a′ e′* being the projections of the other line.

PROBLEM 48.—In a given plane to place a line of a given length and inclined to the H.P. at a given angle.

Let *v t h* be the given plane, and let the given line be inclined to the H.P. at 45°. In *v t* take any point, *a′*, and draw *a′ a* perpendicular to X Y. Draw *a′* B to meet X Y at 45°. Obtain *a b* and *a′ b′* as previously shown. On *a′* B set off B C, the required length. Project C to *c′*, and *c′* to *c*. Then *b c*, *b′ c′* will be the projections of the required line.

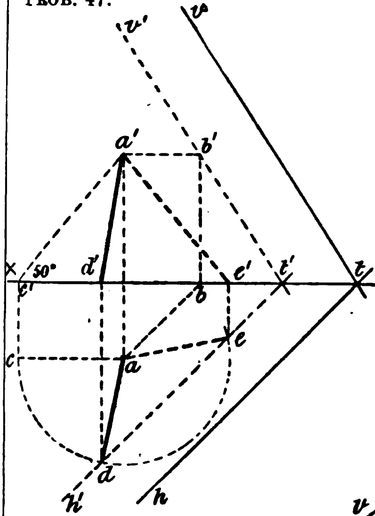
PROBLEM 49.—Given the vertical trace of a plane which is inclined to the H.P. at 50°, and the plan of a line lying in the plane, to determine the elevation, true length, and inclination of the line.

Let *v t* be the vertical trace and *a b* the plan of the line. Obtain the horizontal trace. (*Prob.* 24.) Find the elevation of each of the extremities of the line, and join *a′* and *b′*. (*Prob.* 34.) For the true length 'construct' the line into the H.P. as A B. (*Prob.* 8.) For the inclination produce *a b* to meet A B, giving θ the inclination to the H.P. The inclination to the V.P. may be found in a similar manner.

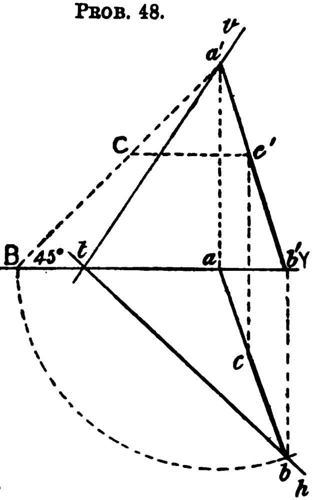
PROBLEM 50.—Draw the traces of two planes not at right angles to either plane of projection, and determine the angle which the intersection of these two planes makes with the vertical plane of projection. (*Sc.*, 1889.)

Draw the traces of two oblique planes *v t h* and *v′ t′ h′*. Determine their intersection *a b*, *a′ b′*. Draw *b′* B at right angles to *a′ b′*. Make *b′* B equal to *b′ b*. Join B *a′*. Then ϕ will be the inclination of the intersection to the V.P.

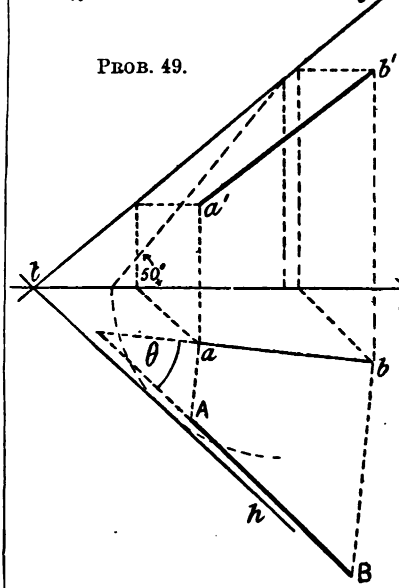
PROB. 47.



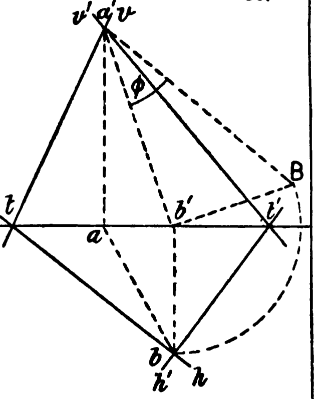
PROB. 48.



PROB. 49.



PROB. 50.



EXERCISES

CHAPTER XVIII

Note.—Figures to which these questions relate will be found on the opposite page. They must be drawn to a much larger scale.

1. State precisely what is represented by each of the four given figures, *a*, *b*, *c*, *d*. (*Sc.*, 1883.)

2. Show by their traces two vertical planes making 60° with each other, and with the vertical plane of projection. (*Sc.*, 1887.)

3. Define the 'traces of a plane.' Represent by their traces two parallel vertical planes 1″ apart, and inclined 35° to the vertical plane of projection. (*Sc.*, 1878.)

4. Taking *a b* as the horizontal trace of a plane inclined at 60°, draw the vertical trace. (*Se.*, 1882.)

5. Draw a plane parallel to the given plane and 1″ from it. (*Sc.*, 1882.)

6. Through the given point, *p p′*, draw a plane parallel to the given plane. (*Sc.*, 1877.)

7. Determine the projections of the intersection of the two given planes and also the inclination of this intersection. (*Sc.*, 1877.) (*Probs.* 30 and 8.)

8. The traces of a plane are given. Determine its vertical trace on a new vertical plane, *a b*. (*Sc.*, 1878.)

9. Find the intersection of two planes whose horizontal traces are parallel and 1″ apart, and which are inclined in the same direction at 25° and 50° respectively. (*Sc.*, 1881.)

10. O *a* and O *b* are the horizontal traces of two planes. O *i* is the plan of their intersection. The plane of which O *a* is the horizontal trace is inclined to the H.P. at 65°. Determine and write down the inclination of the second plane to the H.P. (*Sc.*, 1888.)

Note.—Imagine the two planes to meet like two of the sides of a pyramid. Make a vertical section of the plane having the given inclination. Draw *c d* perpendicular to O *a*, *c e* perpendicular to *c d*, and *d e* at 65° to *c d*. Imagine this triangle to be vertical, then *e* would be a common point in both planes. If *c f* be drawn perpendicular to O *b*, and on it a triangle be drawn having its altitude equal to *c e*, then the angle *c f e′* will be the required inclination. The dotted lines show the working.

CHAPTER XIX

1. Draw the projections of a horizontal line lying in the given plane and 1·25″ above the horizontal plane of projection. (*Sc.*, 1888.) (*Fig.* 1, *page* 213.)

2. The traces of a plane parallel to the ground line, *v v*, *h h*, are given; *a* is a point on the ground line. Determine the length of a perpendicular drawn from *a* to the given plane. (*Sc.*, 1888.)

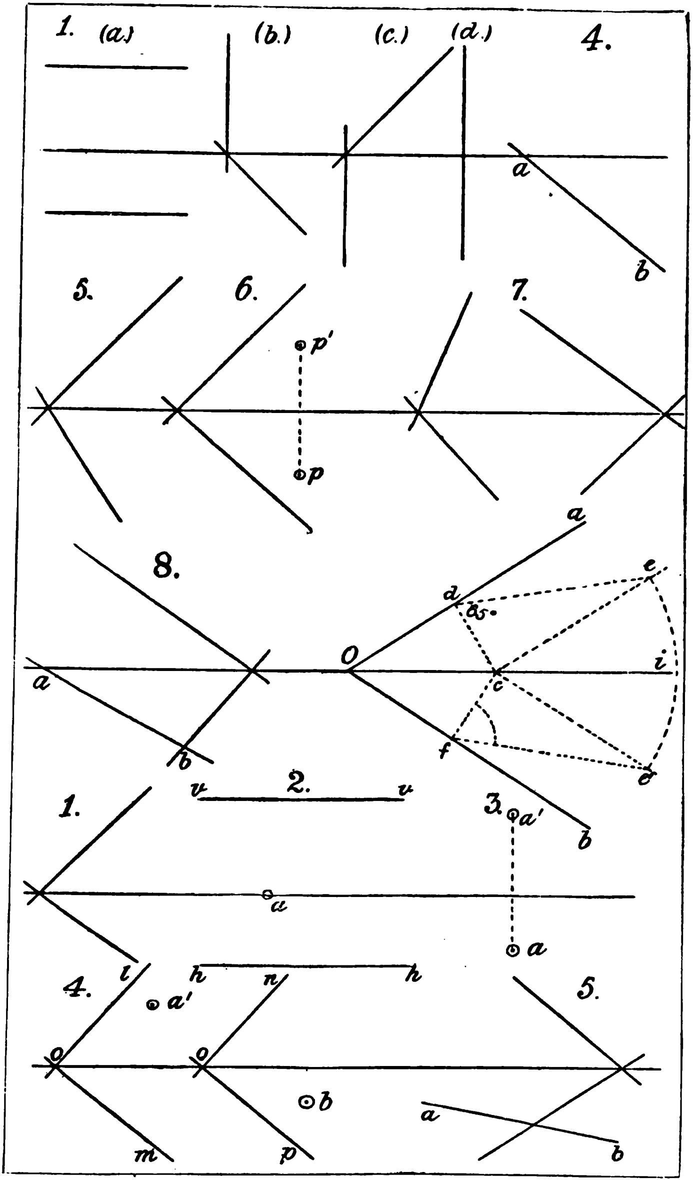
Note.—The perpendicular drawn from *b* to *b′* C in *Problem* 33, *case* 4, is the distance required.

3. Represent by its traces a plane inclined at 65° to the H.P., and containing the given point *a a′*. (*Sc.*, 1884.) (*Prob.* 33, *case* 4.)

4. *a′* is the elevation of a point in the plane, *l o m*; *b* is the plan of a point in the plane, *n o p*. Determine the real length of the line joining these points. (*Sc.*, 1887.)

Note.—Find the other projections of each point and determine the true length of the joining line by *Problem* 8.

5. *a b* is the plan of a line lying in the given plane. Determine the inclination of this line to the H.P. (*Sc.*, 1884.) (*Probs.* 34 and 8.)


1. (a.) (b.) (c.) (d.)
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d
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l
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6. *g h* is the plan of a line lying in the plane. Determine its elevation and real length. (*Sc.*, 1883.)

7. The projections of a line and point (*p p'*) are given. Determine the traces of a plane containing the line and point. (*Sc.*, 1878.) (*Prob.* 36.)

8. *a b*, *a' b'* are the projections of a given line; *c d* is the plan, and *p'* a point in the elevation of a second line intersecting the first. Determine the traces of the plane containing the two lines. (*Sc.*, 1886.) (*Prob.* 37.)

9. The plans of two parallel horizontal lines are 1·75″ apart. The heights of the line are 0·5″ and 1·75″ respectively above the H.P. Determine the inclinations of the plane containing the two lines. (*Sc.*, 1884.) (*Probs.* 38 and 23.)

10. Find the intersection of the given line **C D** with the given plane. (*Sc.*, 1882.)

11. Draw a plane parallel to the ground line, 1″ distant from it, and making equal angles with the plane of projection; find the projections of the point in which the given line **A B** meets this plane. (*Sc.*, 1885.)

12. From the given point, *p p'*, draw a perpendicular to the given plane and determine the real length between *p p'* and the point in which it intersects the plane. (*Sc.*, 1885.)

13. The plan, *p*, of a point lying in the given plane is shown. Determine the elevation of the point and draw through it a perpendicular to the given plane. (*Sc.*, 1878.)

14. A vertical plane of which *h t* is the horizontal trace cuts the two given planes. Draw the elevation of the intersections and determine the real angle they contain. (*Sc.*, 1887.)

Note.—Obtain the vertical trace, *h h'*, of the vertical plane, and determine the elevation of its intersections with each of the given oblique planes. (*Prob.* 30.) From *a'*, where these intersections cut, draw *a' a*, and determine the real angle between *b a'* and *c a'* by *Prob.* 44. The working is shown in dotted lines.

15. Draw a plane to contain the line *a b*, *a' b'*, Fig. 11, and the horizontal trace of which makes 45° with the ground line. (*Sc.*, 1885.) (*Prob.* 45.)

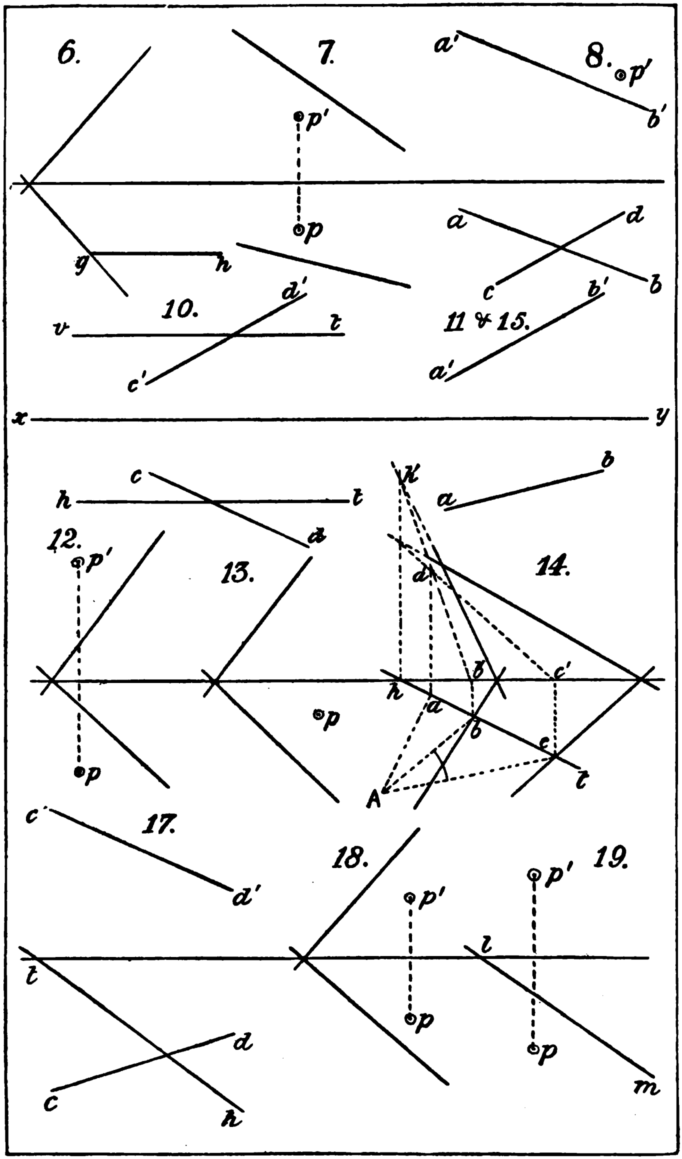
16. Determine the horizontal and vertical traces of a plane containing the given line **A B** (Fig. 11) and inclined at 55°. (*Sc.*, 1881.)

17. Find the intersection of the line **C D** with the plane inclined at 45° whose horizontal trace is given.

18. From the given point, *p p'*, in the given plane draw two lines in the plane inclined at 29°, and determine the real angle they contain. (*Sc.*, 1883.) (*Probs.* 47 and 44.)

19. *l m* is the horizontal trace of a plane inclined at 55° to the H.P. Draw its vertical trace, and from the point whose projections are *p* and *p'* draw a horizontal line meeting the plane in a point 2″ from *p p'*. (*Sc.*, 1885.)

20. The horizontal trace of a vertical plane makes 42° with the ground line. Determine the elevation of a line lying in this plane, inclined at 30°, and passing through the point where the given plane cuts the ground line. (*Sc.*, 1889.)


6.
7.
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p'
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10.
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CHAPTER XX

PROJECTIONS OF OBLIQUE SURFACES

In representing the projections of oblique surfaces it is more convenient to *first* assume that the plane upon which the surface is supposed to rest is perpendicular to the V.P., and inclined *only* to the H.P. The elevations of any points in that surface will then lie in the vertical trace of the inclined plane. If other elevations are required inclined to the V.P., they may then be easily obtained by altering the ground line. For instance, if a book be placed on the H.P. with its back perpendicular to the V.P. and the cover be opened at any angle, the elevation of the cover will be a line, and the elevation of any surface lying on the cover will lie in that line. If now an elevation were required when the cover is also inclined to the V.P. it would only be necessary to draw a fresh ground line inclined to the V.P. at the required angle, and project as shown previously in Chapter XVI., *Problem* 297, &c. The consideration of the following examples will demonstrate the advantage of this method of procedure.

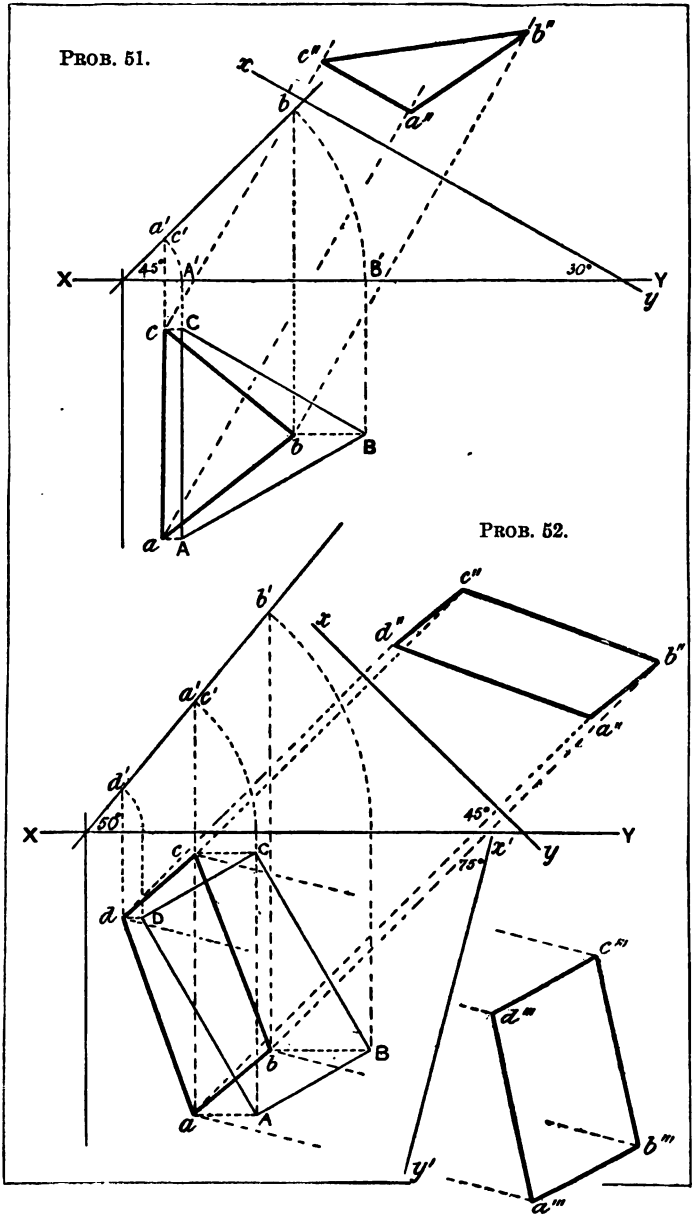
PROBLEM 51.—An equilateral triangle is inclined to the H.P. at 45°, and one of its edges is horizontal. Draw its plan and also an elevation on a ground line inclined at 30° to the V.P.

As the surface is inclined to the H.P. at 45°, commence by drawing the traces of a plane inclined at that angle. Draw the plan of the triangle (A B C) when lying in the horizontal plane, with its side A C parallel to the horizontal trace of the plane. Project to X Y, and rotate this elevation (A′ B′) into the vertical trace, giving *a′ b′*, the elevation of the triangle. Obtain the plan by dropping perpendiculars from *a′*, *b′* to meet lines drawn from A, B, and C parallel to X Y. Join *a*, *b*, and *c*, giving the required plan.

For the *fresh elevation* draw *x y* making 30° with the V.P. Project from *a*, *b*, and *c*, and make *a″*, *b″*, and *c″* the same height above *x y* that *a′*, *b′*, *c′* are above X Y.

PROBLEM 52.—Draw the plan of a rectangle when its surface is inclined at 50°, and one of its diagonals is horizontal. Also obtain elevations on ground lines inclined to the V.P. at 45° and 75° respectively.

This problem is similar to the last. Draw the rectangle A B C D with one of its diagonals A C parallel to the horizontal trace of the plane, obtain the elevation *d′ b′*, and the plan *a b c d* as before. The rest of the working may be easily followed from the figure.



PROB. 51.

PROB. 52.

PROBLEM 53.—An octagon lies in a plane inclined at 40°, one of its diagonals is horizontal. Draw its plan and elevation.

First draw the octagon *a* B C D *e* F G H with its diagonal *a e* perpendicular to X Y. Produce *a e* to *a′* and consider this line to be the horizontal trace of the inclined plane. Through *a′* draw the vertical trace. Project the angular points of the octagon to X Y. With centre *a′* rotate the points G′, H′, B′, C′, into the vertical trace. From the points *g′*, *h′*, *b′*, *c′*, project to meet lines parallel to X Y from the corresponding points G, H, B, &c. of the plan. Join the points thus obtained.

Note.—This is only another way of arranging the figure, a similar result would be obtained by drawing the traces in the same position as in the previous problems. The circle and any of the other polygons may be treated in a similar manner.

PROBLEM 54.—*a b c d* is a four-sided figure; draw its plan when *b d* is horizontal, and *a c* inclined at 62° to the H.P. (*Sc.*, 1888.)

Draw the traces of a plane inclined at 62°. Place the figure so that *b d* is on the horizontal trace. Proceed as in the previous problem.

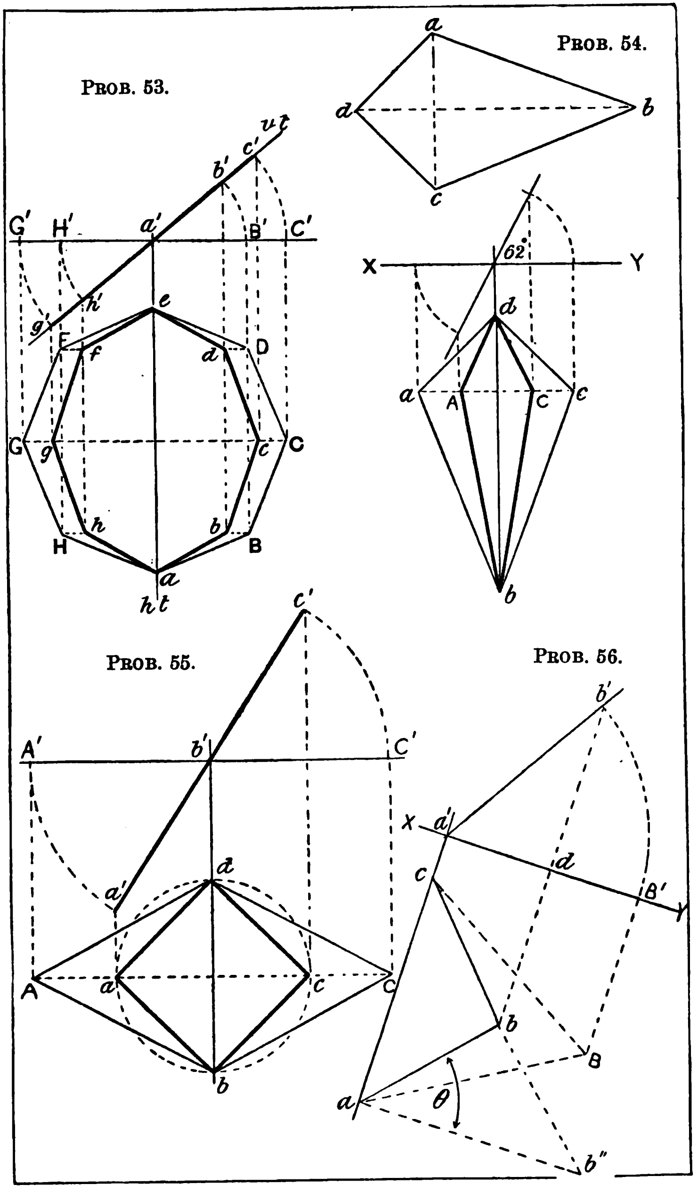
PROBLEM 55.—A rhombus revolves about its shorter diagonal until the plans of its smaller angles are right angles. Draw its projections, and show the inclination of its surface to the H.P.

Place the rhombus A *b* C *d* with its shorter diagonal *b d* perpendicular to X Y. On *b d* describe a circle, and join the points *d*, *c*, *b*, *a*. The angle in a semicircle is a right angle (Euc. iii. 31), therefore *a b c d* will be the plan. Project from A, *b*, and C to X Y. With centre *b′* describe arcs from A′ and C′. Project from *a* and *c* to intersect these arcs in *a′* and *c′*. Through *b′* draw *a′ c′*, the required elevation. The angle *c′ b′* C′ will represent the inclination of the rhombus to the H.P.

PROBLEM 56.—The given isosceles triangle *a b c* is the plan of an equilateral triangle, whose base *a c* is in the H.P. Determine the height of B above the H.P., and the angle at which the sides *a b*, *b c* are inclined.

On *a c* construct the equilateral triangle *a* B *c*. Produce *a c* and draw X Y at right angles to it. Project from B to B′. With centre *a′* describe the arc B′ *b′* to meet a projector from *b*. Join *a′ b′*. The angle *b′ a′* B′ is the inclination of the plane of the triangle, and *d b′* is the height of its vertex B above the H.P. To find the *inclination* of the sides, draw *b b″* perpendicular to *a b* and equal to *d b′*. Join *a b″*. The angle *b a b″* will be the inclination required.

Note.—If the plan and true shape of a figure are given, its *inclination* may always be determined; if the inclination and true shape are given, then the *plan* may be found; and if the inclination and plan are given, then the *true shape* of the figure may be ascertained.



PROB. 53.

PROB. 54.

PROB. 55.

PROB. 56.

PROBLEM 57.—Draw the projections of a square prism when one of its ends is inclined at 60°, and none of its edges are horizontal. Also obtain another elevation on a line inclined at 75° to X Y.

Draw the traces of a plane inclined at 60°. As none of the edges are horizontal, draw the square A B C D with none of its edges parallel to the horizontal trace. Project to X Y and obtain a', b', c', d' as in *Prob.* 52. Set off the long edges at right angles to the vertical trace, make them the required length, and join e', g' for the elevation. Project vertically from each end to meet lines parallel to X Y from A, B, C, D for the plan.

For the *second elevation* draw $x\,y$ making 75° with X Y, and project from each angle of the plan as shown. Make each of the points a'', b'', &c., the same distance above $x\,y$ that a', b', &c., are above X Y.

THE INCLINATION OF THE PLANE of any figure, AND OF ANY LINE connected with it, being given, to determine its projections.

In *Probs.* 51–57, only the inclination of the plane of the figure was given; if, in addition to the inclination of the plane, the inclination of a line belonging to the figure is also given, then *the exact position of the figure on the oblique plane is fixed.* Thus if a triangle has its surface inclined at 60°, and one of its sides 30°, then its position with regard to the planes of projection is definitely fixed.

PROBLEM 58.—To determine the projections of an equilateral triangle, when its plane is inclined at 60°, and one of its sides 45°.

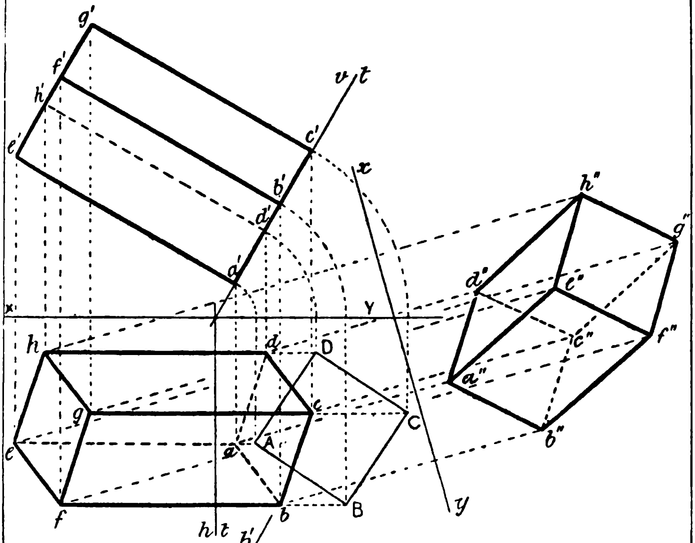
Commence by drawing the traces of a plane inclined 60°, and then by *Prob.* 46 place a line inclined 45° on this plane thus:—In the vertical trace take any point, b', and draw $b'\,b$ perpendicular to X Y, and b' A to meet X Y at 45°. With centre b, and radius b A, describe an arc cutting the horizontal trace in a. Join $a\,b$. Then $a\,b$, $a'\,b'$ will be the plan and elevation of a line inclined 45°, lying on a plane inclined 60°. 'Construct' this line into the H.P., by turning $a'\,b'$ into X Y as a' B, and joining a B. (*Then* a B *will be the true length of the line.*) On aB set off C D the side of the required figure, and on it construct the triangle C D E. The remainder of the construction is exactly similar to the preceding problems. Project from C, D, E to X Y, and turn each of the points C′, D′, E′ into the vertical trace, giving $c'\,d'\,e'$ the elevation of the triangle. Project from c', d', e' to meet lines parallel to X Y from C, D, E. Join the points c, d, e for the plan.

Notes.—1. The plans of the points C and D must fall on $a\,b$.

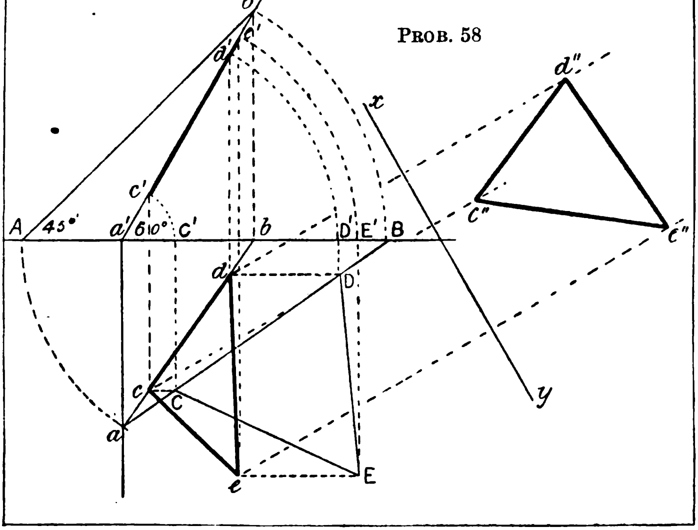
2. The inclination of the given line can never exceed the inclination of the plane, as will be seen by reference to Fig. 1, Prob. 46.

8 An additional elevation is shown on the ground line $x\,y$.

PROB. 57.



PROB. 58



PROBLEM 59.—To determine the projections of a regular hexagon when its plane is inclined 50°, and one of its diagonals inclined 30°.

First draw the traces of a plane inclined 50°, and obtain *a b* the plan of a line in the plane inclined 30°. 'Construct' this line into the H.P. as before. On *a* B set off C F the length of the diagonal, and construct the regular hexagon by describing a circle with C F as its diameter. Project each angle of the hexagon to X Y, and obtain the elevation and plan as before.

PROBLEM 60.—A cube has one of its faces inclined 45°, and an edge of that face inclined 25°. Determine its projections.

Draw the plan, *a b*, of the line having the given inclination as before, and 'construct' it into the H.P. as *a* B. On *a* B construct a square, C D E F, of the required size representing a face of the cube. Obtain the elevation, *c′ d′ e′ f′*, as before. At each of the points *c′*, *d′*, *e′*, *f′* draw lines perpendicular to the vertical trace. Make them equal to C F and complete the elevation.

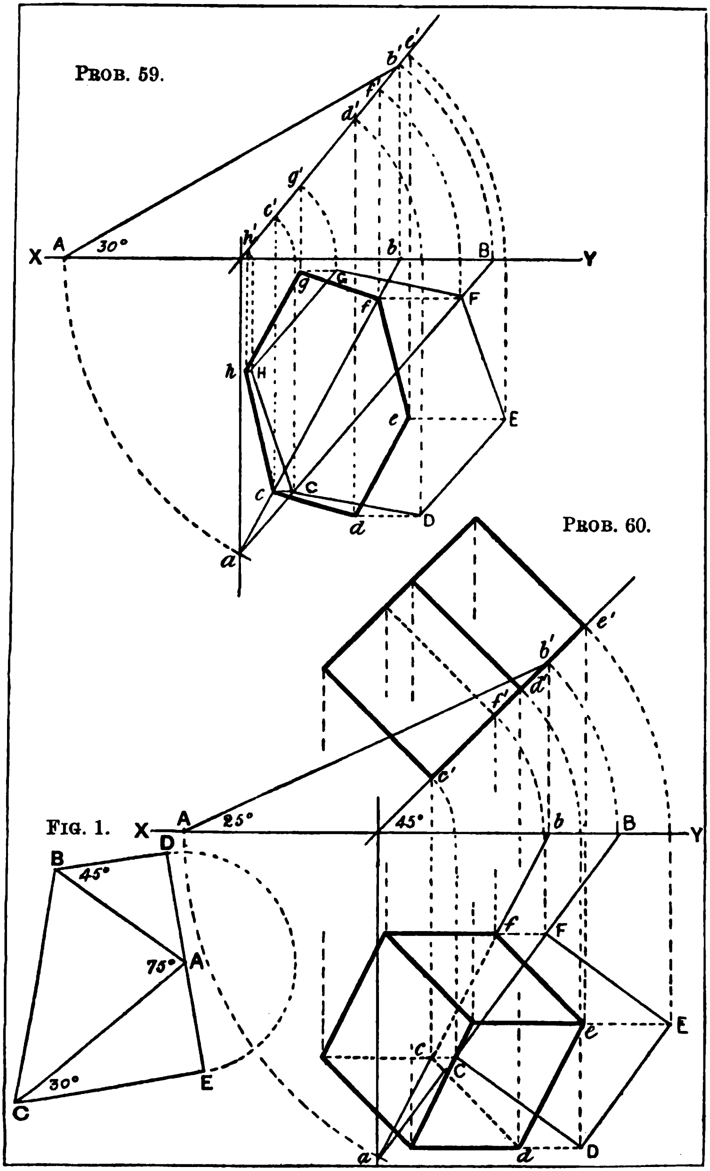
For the plan, project from each of the corners of the figure as shown to meet lines parallel to X Y from the points C, D, E, F.

THE INCLINATION OF ANY TWO INTERSECTING LINES connected with a figure being given, to determine its projections.

It has already been shown that when the inclination of the plane of a figure and the inclination of any line connected with it are given, its exact position with regard to the planes of projection is fixed. This is also the case if the inclinations of any two intersecting lines of the figure including a known angle are given. For instance, if the inclinations of two sides of a square are 30° and 60° respectively, then its plan and elevation will always be the same wherever the square may be placed. The point of difference between this and the previous problems is, that we have previously assumed the plane, while in this case it must first be determined.

To understand clearly the reason for the construction used, the student should actually cut out the figure in cardboard or paper. For example, let two lines A B, A C meet at an angle of 75°, and be inclined to the H.P. at 45° and 30° respectively. Draw A B, A C (Fig. 1) meeting at an angle of 75°. In A B take any point B, and from it draw B D making 45° with A B. From A draw A D perpendicular to B D. Draw A E equal to A D and making 60° (the complement of the given inclination 30°) with A C. From E draw E C perpendicular to A E, and meeting A C at an angle of 30°.

If the figure B C E A D be now cut out, and folded on the lines A B and A C until the point E coincides with the point D, then the inclinations of the plane of the figure, and the two lines will be clearly seen. The line C B will be the horizontal trace of the plane which contains the lines A B and A C.



PROB. 59.

PROB. 60.

FIG. 1.

PROBLEM 61.—To determine the traces of a plane which shall contain two lines intersecting at 60°, and inclined to the H.P. at 50° and 40° respectively.

Note.—The sum of the inclinations of the lines together with the angle contained by them must not exceed 180°. If it equals 180° the lines will lie in a vertical plane.

Draw any two lines A *b*, A *c* intersecting at 60°. At any point, *c*, in A *c* draw *c* D, making an angle of 50°, and from A draw A D perpendicular to it. With centre A and radius A D describe an arc. Draw the line *b* E tangent to this arc, and making 40° with A *b*. (The best way to do this is to draw any line at an angle of 40° with A *b*, and draw the tangent parallel to this line.) Join *b* and *c*. The line *b c* will be the *horizontal trace* of the plane. Draw X Y at right angles to *b c*. As A D represents the actual height of the point A above the H.P., draw a line parallel to X Y and at a distance from it equal to A D. Project A to X Y. With centre *b'* and radius *b'*A' describe an arc intersecting the parallel line in *a'*. The line *b' a'* will be the *vertical trace* of the plane, and will contain the elevation of the lines A *b*, A *c*. The plan of the lines may be obtained by projecting from *a'* to meet the line parallel to X Y from A, and joining *a* with *b* and *c*.

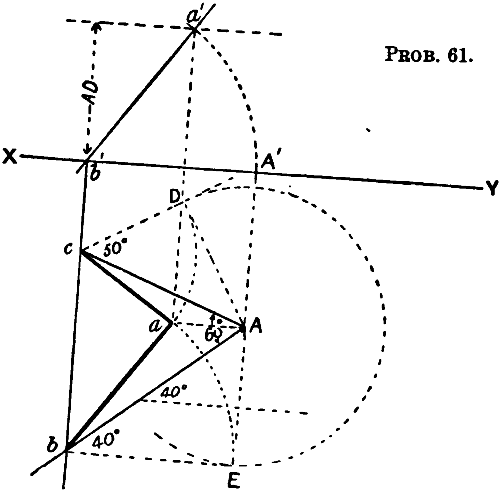
Note.—The lines *c* D and *b* E also represent the plans of *c* A and *b* A, as may be seen from the paper model, and are therefore equal to *c a* and *b a*. If with centre *c* and radius *c* D, an arc be described to intersect another arc described from centre *b* with radius *b* E, another solution for finding the point *a* will be obtained.

PROBLEM 62.—To draw the projections of a rectangle when its diagonal and one of its sides are inclined 40° and 35° respectively.

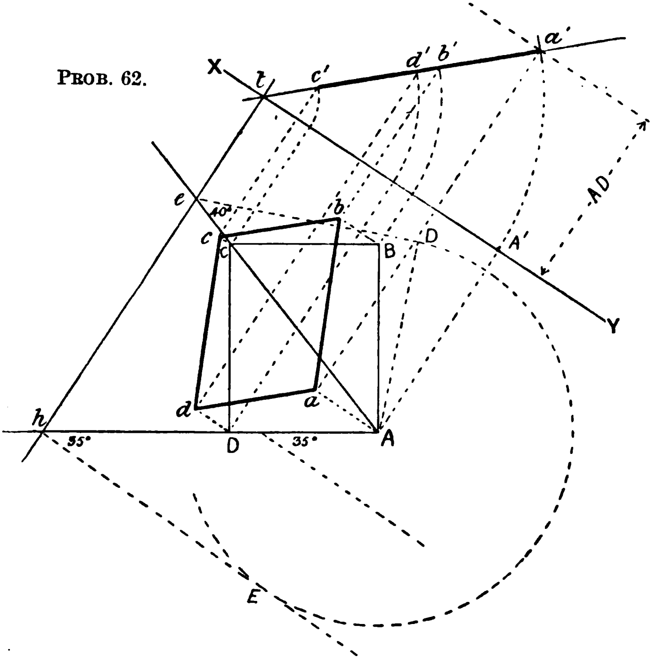
Draw A B C D, the true shape of the rectangle. The plane must now be determined which contains the two lines A C and A D. Produce these two lines indefinitely from A. In A C produced take any point, *e*, and draw *e* D at an angle of 40° with A *e*. Draw A D perpendicular to *e* D, and with centre A and radius A D describe an arc. Draw *h* E tangent to this arc, and making 35° with *h* A. Draw *h t* the *horizontal trace* of the plane. Draw X Y at right angles to *h t*. Draw a line parallel to X Y at a distance equal to A D. Project from A, B, C, D, to X Y, and with centre *t* and radius *t* A' intersect the parallel already drawn in *a'*. The *vertical trace* of the plane will be *t a'*. Obtain the elevation *c' d' b' a'* as shown. For the plan, project from each point of the elevation to meet lines parallel to X Y from A, B, C, D. Then *a b c d* will be the plan.

Notes.—1. Other plane figures may be treated in a similar fashion, and fresh elevations may be determined in exactly the same manner as in previous problems.

2. The student must remember that when the position of the figure, and the traces of the plane containing it are determined, its projections are drawn exactly the same process as shown in Prob. 51.



Prob. 61.



Prob. 62.

PROBLEM 63.—To determine the projections of a plane figure having GIVEN THE HEIGHTS OF THREE POINTS in it.

Note.—The difference between the heights of any two points must not exceed the distance between them.

If the three given points be joined a triangle would be formed, and the traces of the plane must be determined which will contain this triangle. The theory of the construction is similar to that of the two previous problems.

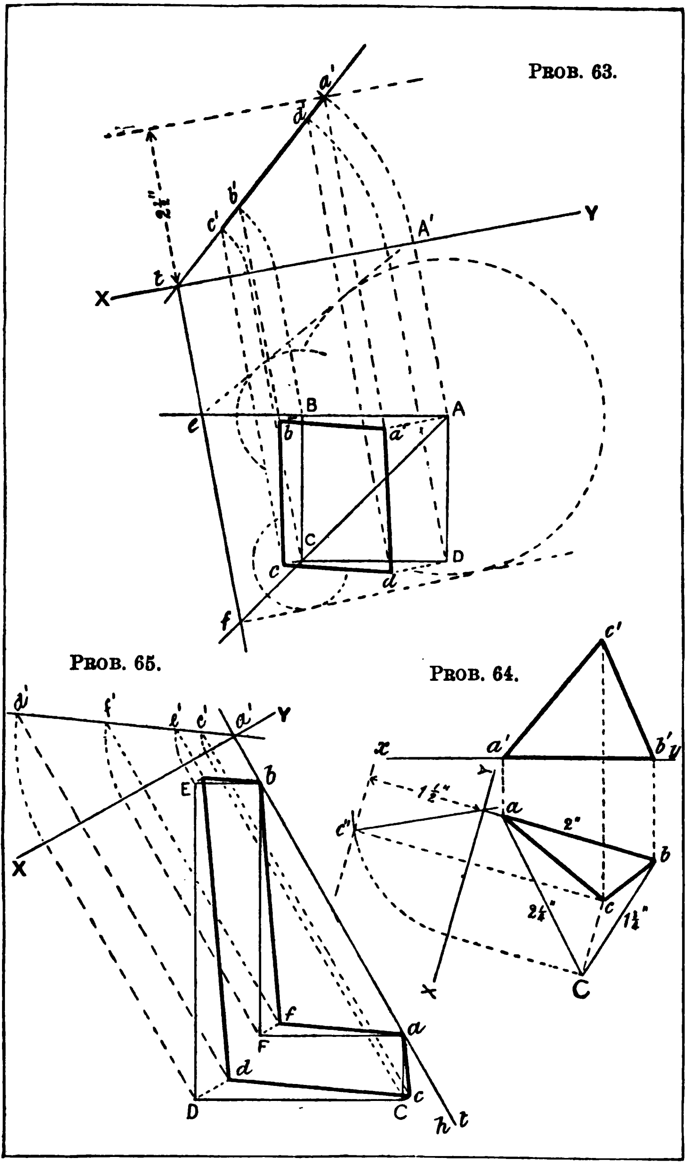
Let the figure be a square, and let the heights of three of its corners A, B, C be 2½″, 1″, and ¾″ respectively. Draw a square A B C D of the given dimensions in any position on the H.P. Produce the side A B, and the diagonal A C indefinitely from A. With centre A and radius 2½″, describe an arc; with centre B and radius 1″ describe a second arc, and with centre C and radius ¾″ describe a third arc. Draw a line tangential to the arcs described from A and B to meet A B produced in *e*, and another line tangential to the arcs described from A and C to meet A C produced in *f*. Join *f* and *e* and produce. This line will be the *horizontal trace* of the plane. Draw X Y at right angles to *f e*. At a distance equal to the height of A (2½″), draw a line parallel to X Y. Project from the point A to X Y, and with centre *t* and radius *t* A′ describe an arc intersecting the parallel in *a′*. Then *t a′* will be the *vertical trace* of the plane. Project the other points B, C, D to X Y, and obtain the elevation and plan as in the previous problems.

PROBLEM 64.—A line, *a b*, 2″ long, lying in the H.P. is the base of a triangle, the real lengths of the other sides are 2¼″ and 1¾″. The vertex of the triangle is 1½″ above the H.P. Complete the plan, and draw the elevation on *x y*. (*Sc.*, 1887.)

On *a b* construct the triangle *a b* C having the given dimensions. Assume *a b* to be the horizontal trace of a plane containing the triangle, and draw X Y perpendicular to it. Draw a line parallel to X Y at a distance of 1½″. Project from C to X Y, and describe an arc intersecting the line in *c″*. Project from *c″* to meet a line parallel to X Y from C. Join *c a*, *c b* to complete the plan. For the elevation let *c′* be 1½″ above *x y*.

PROBLEM 65.—The letter L as given rests on a plane inclined at 35°, with the points *a* and *b* on the horizontal trace of the plane. Draw its plan. (*Sc.*, 1890.)

Through the points, *a* and *b*, draw the horizontal trace of the plane, and at any point, *a′*, draw X Y perpendicular to *a b*. Draw the vertical trace of the plane inclined at 35°. From each angle project to X Y and rotate into the vertical trace. From each point of the elevation project to meet lines parallel to X Y from each angle.


PROB. 63.

PROB. 65.

PROB. 64.

CHAPTER XXI

PROJECTIONS OF SOLIDS AND OBJECTS OF SIMPLE FORM

In Chapters XVI and XX, the representation of solids in simple positions, the projection of sections, and the construction of fresh elevations on other given ground lines have already been dealt with; it will only be necessary here to deal with a few of the more difficult projections. The solids and objects given are selected mostly from recent examination papers, and *must be drawn to a large scale.* The principles used should be applied to other solids in a similar manner.

PROBLEM 66. Draw the plan of a cube:—1. When its diagonal is vertical. 2. When its diagonal is horizontal.

1. The cube must first be placed so that the diagonal will be shown its full length. Draw the plan A B C D with its diagonal, A C, parallel to the V.P. Obtain the elevation. Then $A'G'$ will represent the diagonal of the cube. If a ground line, X_1 Y_1, be now drawn at right angles to $A'G'$, then $A'C'G'E'$ will represent the elevation of the cube when the diagonal is vertical. Project from each angle, making $b\ d$ equal to B D.

2. Draw the ground line, $x\ y$, parallel to the diagonal $A'G'$, and project as in the other figure.

Notes.—1. For other positions of the cube see Chapters XVI and XX.

2. The plan when the diagonal is vertical is a regular hexagon.

3. The solid must always be arranged so that the line whose inclination is given is shown its full length in either plan or elevation.

4. The student should remember that when fresh plans or elevations are made, their distances from their corresponding ground lines are the same. In Problem 66, D, d, and d_1 are the same distances from their respective ground lines.

PROBLEM 67. A square prism 3″ long, and 1·5″ square, lies with one of its side edges horizontal, and one of its sides inclined at 30°. Make an elevation of it on a V.P., which is at 50° to its axis. Also draw the section made by a V.P., which is at right angles to the above plane of elevation, and which cuts the lowest edge of the prism at 1″ from one end of it.

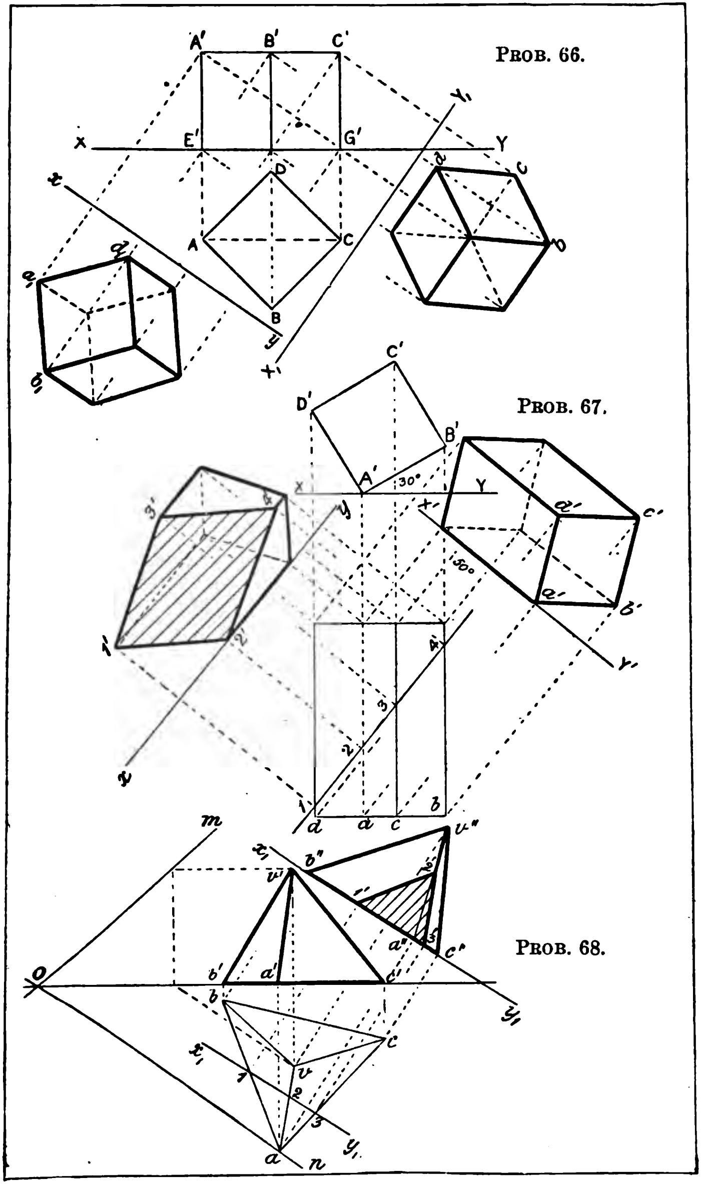
First draw the elevation $A'B'C'D'$ when one end of the prism is parallel to the V.P., and from this obtain the plan. The edges will now be shown their full length. Draw $X'Y'$ making 50° with b B'. Project from each angle of the prism, making $a'\ b'\ c'\ d'$ the same height above $X'Y'$ that A', B', C', D' are above X Y.

For the *section*, find a point, 2, on the lowest edge of the prism, 1″ from a, and draw the section plane at right angles to $X'Y'$. Draw $x\ y$ parallel to this plane, and project the end which is not cut away. From each point of the section project to meet the lines parallel to $x\ y$ from each corner of the end.

PROBLEM 68.—The plan $a\ b\ c\ v$ of a triangular pyramid standing on the H.P. is given. The vertex of the pyramid is in the given plane $m\ o\ n$. Draw the elevation, and also a section on $x_/\ y_/$.

Obtain v' byProblem 33, and complete the elevation.

For the *section* draw $x_/\ y_/$ parallel to the section line, and on it btain another elevation. Project each of the points, 1, 2, 3, and n as shown.



PROB. 66.

PROB. 67.

PROB. 68.

PROBLEM 69.—Draw the projections of a hexagonal pyramid:—1. When one face is in the H.P. 2. When one face is vertical. 3. Show a section made by a plane passing through the centre of the base of the pyramid parallel to the vertical face.

1. Draw the plan A B C D E F, and place it so that two sides are perpendicular to X Y. Obtain the elevation. If a ground line, X′ Y′, be drawn containing $b'v'$, then $f'b'v'$ will represent the elevation of the pyramid when one face is horizontal (turn the paper so that X′ Y′ is in a horizontal position). From this elevation project the fresh plan, making each point as far from X′ Y′ as the corresponding point is from X Y ($b'c$ and $b'b = b'$C and b'B respectively, and $v'v = a'$V).

2. Draw xy perpendicular to the vertical face, $v'f'$. Then $v'f'b'$ will represent the elevation when $v'f$ is vertical. Project a new plan, making, as before, each point as far from xy as the corresponding point is from X Y ($f'e_{\prime} = f'$E, &c.). The vertex will fall in the middle of one side.

3. Draw the section line L M passing through the centre of the base and parallel to the vertical face. Project as shown from the elevation, and obtain the widths from the plan.

Note.—This is a very instructive problem, and should be drawn to a large scale and carefully worked out.

PROBLEM 70.—Draw the plan and elevation of a tetrahedron when one face is inclined 50°, and one side of that face 30°.

Draw the traces of a plane inclined 50°, and obtain a line, $a\,b$, lying in that plane and inclined 30°. ‘Construct’ this line into the H.P. as aB, and on it draw the plan of the tetrahedron. Obtain the elevation and plan as shown. The height will be found as in *Problem* 310.

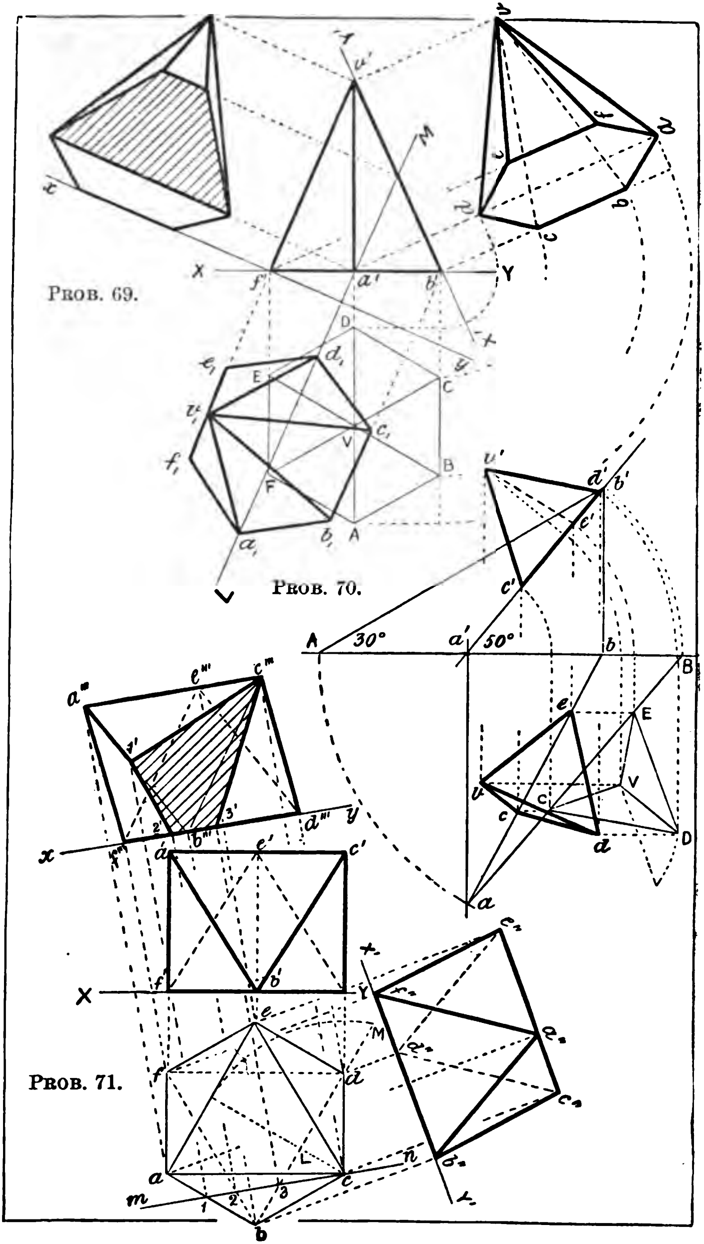
PROBLEM 71.—The plan of an octahedron with one face in the H.P. is given. Draw:—1. an elevation on X Y. 2. Another elevation when one edge of the horizontal face makes 70° with the V.P. 3. A sectional elevation made by a vertical plane $m\,n$.

1. Project from each angle of the plan. To obtain the height, produce $d\,b$, and with centre c and radius equal to the altitude of the equilateral triangle, $a\,e\,c$, describe an arc cutting $d\,b$ in M, then L M will be the distance between two parallel faces of the solid. Make $f'a'$ equal to L M, and complete the elevation.

2. Draw X′ Y′ making 70° with $d\,f$. Project from f, b, and d to X′ Y′. Set off the height equal to L M as before, and project from a, c, and e. Join the points obtained.

3. Draw a ground line $x\,y$ parallel to $m\,n$, and project an elevation as in the last. From each point where the section plane cuts an edge of the figure, project to the corresponding line in the elevation. Complete the section as shown. Point 1 is on line $a\,b$, therefore 1′ will be on $a'''\,b'''$, 2 and 3 are on the H.P., therefore their elevations will lie in $x\,y$.

Note.—The student is advised to project the face of the octahedron which lies on the H.P. first, and carefully letter each point as it is obtained.



PROB. 69.

PROB. 70.

PROB. 71.

PROBLEM 72.—Given the plan and elevation of a solid, made up of two square prisms. Draw the plan when the edges through the points A and B are both in the H.P. Also draw the section made by the vertical plane 1 4.

As the edges containing the points A and B are to be in the H.P., draw $x_, y_,$ through $a' b'$ and project as shown, making each point in the new plan the same distance from $x_, y_,$ as the corresponding point is from $x y$ $(a' a = a' a_,)$.

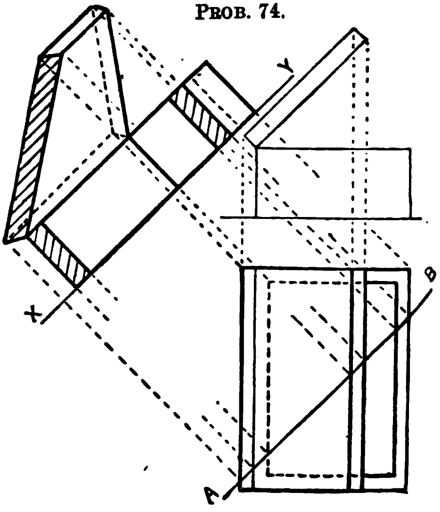
For the *section* draw X Y parallel to 14, and project from the plan, obtaining the heights from the given elevation.

PROBLEM 73.—Given the elevation of the letter A cut out of material ¾″ thick, and standing on the H.P. Draw the plan, and make an elevation on a plane parallel to a diagonal of the rectangle at the top. Also make a sectional elevation on a line passing through the plan of the point b', and making 35° with the plan of the line $b' c'$.

Draw the plan, making it ¾″ broad. Draw the diagonal, $l m$, of the rectangle at the top, and draw X Y parallel to it. First project the elevation of the front of the letter, $a'' d'' m''$, and then the back lines. The principal projectors only are shown on the figure.

For the *section* draw a line through b making 35° with $b c$, and draw $x' y'$ parallel to it. Project an elevation on this ground line as before. From each of the points of section, b, 1, 2, 3, 4, 5, project to the corresponding line in the elevation. The projector from b will of course pass through b'''. Point 2 is where the inner lines of the top of the letter meet.

PROB. 74.

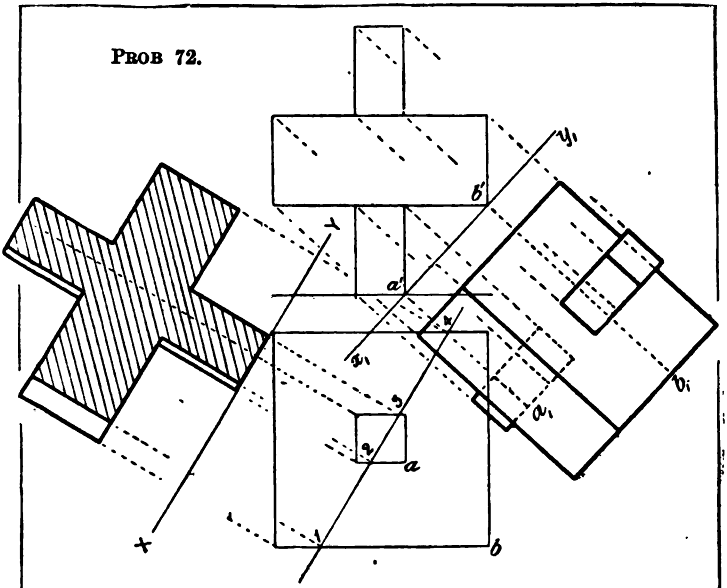


PROBLEM 74.—Given the end elevation and part of the plan of a box $\frac{3}{16}$″ thick. Complete the plan and obtain a sectional elevation on A B.

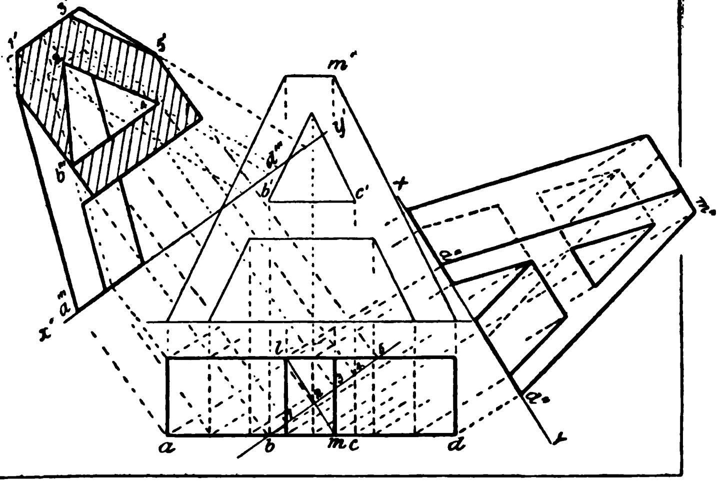
Complete the plan by setting off the thickness of the wood, and projecting the lid.

Draw X Y parallel to A B, and project the section, making the heights the same as in the given elevation.

PROB 72.



PROB. 73



EXERCISES

CHAPTERS XX. and XXI

1. The given figure represents the plan of a square, one of whose diagonals is horizontal. Find the side of the square and the inclination of its plane. (*Sc.*, 1876.)

2. *a b c d* represents a blackboard upon which an equilateral triangle is drawn as shown. Draw the plan of the board and triangle when the plane of the former is inclined 45° and the long edges (*a b*, *c d*) are horizontal. (*Sc.*, 1879.)

3. Draw the plan of an equilateral triangle of 2″ sides lying in a plane inclined at 45°, and with one of its sides inclined 60°. (*Sc.*, 1882.)

4. Draw the projections of a square of 2¾″ sides; one side to be in the ground line. The opposite side to be equidistant from both planes of projection. (*Sc.*, 1884.)

5. The T square, supposed to have no thickness, rests with the angles *a* and *b* on the H.P. and its own plane is vertical. Draw an elevation on a plane making 35° with the plane of the T square. (*Sc.*, 1886.)

6. Draw the plan of a regular hexagon of 1⅛″ side in any position such that its plane is neither horizontal nor vertical. (*Sc.*, 1886.)

7. Draw the plans of two spheres, 1″ and 2½″ in diameter, touching each other, and both planes of projection. (*Sc.*, 1887.)

8. An hexagonal right pyramid, side of base 1½″, height 3½″, stands on the H.P. It is truncated by a plane parallel to and 1⅝″ from its base. Suppose this truncated pyramid to be tipped about an edge of the base till the plane of one of the six side faces is horizontal. Draw the plan. (*Sc.*, 1889.) (*Apply Prob.* 69.)

9. *a b c* represents the plan of a wedge-shaped slice cut out of a cylindrical cheese, whose height is equal to half the radius, *a b*. Draw the plan of the slice resting with one of its rectangular faces on the H.P., and construct an elevation on a line making 25° with the short sides of that face. (*Sc.*, 1888.)

10. Four equal bricks are arranged as shown, their side faces being all in one plane. The dimensions of each brick are 9″ × 4½″ × 3″. Draw the plan of the pile, and an elevation on a plane inclined at 35° to the long edges of the three lower bricks. *Scale ⅛ full size.* N.B. The figure is not to be merely reproduced. (*Sc.*, 1887.)

11. Make a sectional elevation of the pile of bricks by a vertical plane containing the diagonals of the horizontal faces of the brick **A**.

12. The plan and elevation of a simple solid are given. Draw an elevation on a line parallel to **X Y**. (*Sc.*, 1886.)

13. Make a sectional elevation of the solid on the line **A B**.

14. The elevation of the letter **X** is given. Supposing it cut out of wood of thickness equal to the breadth of the bars, draw its plan when standing 1½″ in front of the V.P. Draw an elevation on the ground line **X Y**. (*Sc.*, 1885.)

15. The front and side elevation of a hencoop are given. Draw its plan and an elevation on the ground line **X Y**. (*Sc.*, 1885.) (*Draw the figure twice the given size.*)

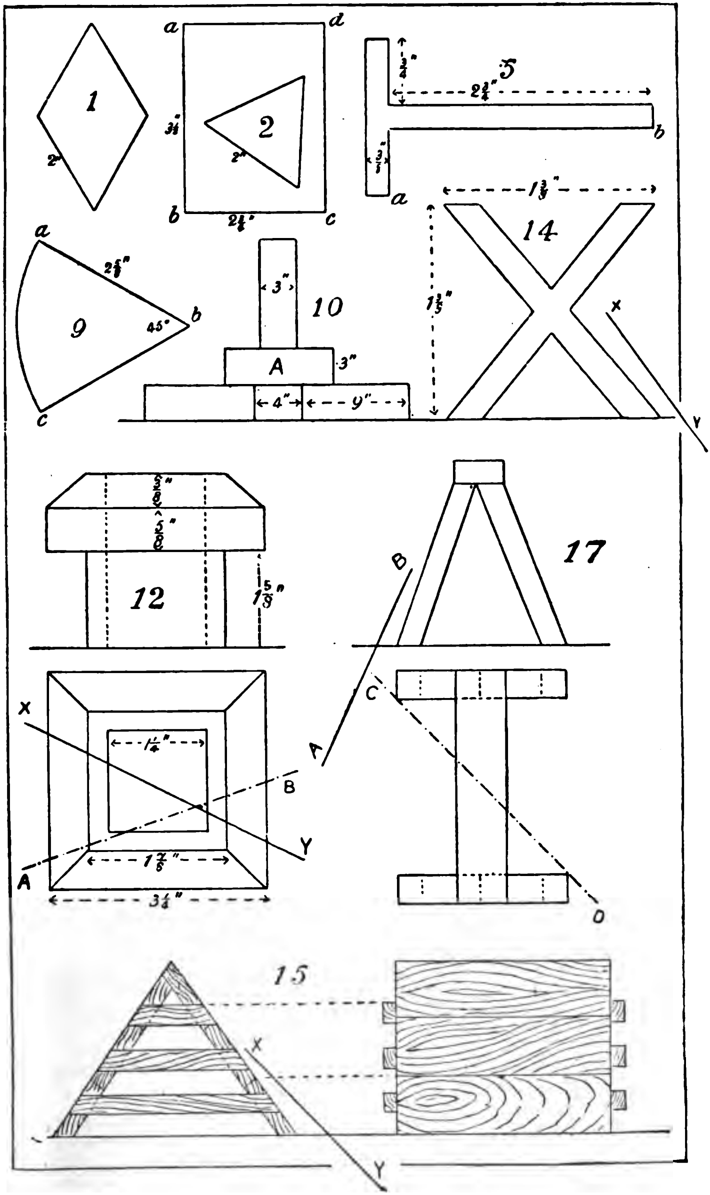
16. Draw a section of the hencoop made by a V.P. passing through the centre of the plan and making an angle of 55° with the sides.

17. Given the plan and elevation of a trestle. Draw an elevation on **A B**. (*Sc.*, 1884.) (*Draw the figure twice the given size.*)

18. Draw a sectional elevation of the trestle on **C D**.

19. An octahedron of 2″ edge lies with one face in the H.P. Draw an elevation of it on a vertical plane which makes an angle of 80° with one edge of the base. (*Sc.*, 1881.) (*Prob.* 71.)

20. Assuming the octahedron in Question 19 to be cut by a vertical plane passing through its centre and perpendicular to the vertical plane above mentioned, draw a section of it and show also what is seen in the elevation.


1
2"
a
d
b
c
3¼"
2½"
2
2"
5
¾"
2¾"
⅜"
a
b
a
b
c
2⅝"
9
45°
10
3"
A
3"
4"
9"
14
1⅓"
1⅓"
X
Y
12
⅝"
⅝"
1⅝"
17
B
A
C
D
X
Y
A
B
1¼"
1⅞"
3¼"
15
X
Y

CHAPTER XXII

GRAPHIC ARITHMETIC

In ordinary arithmetic we use arbitrary signs to represent numbers, and to perform calculations with. We may also represent numbers by lines, and obtain ordinary arithmetical results by means of geometrical constructions. This use of lines is termed **Graphic Arithmetic.** The processes involved are applications of the principles of proportion, as shown in Chapter IV.; there the student has already been shown that the arithmetical and geometrical solutions produce the same result. The greatest possible care and accuracy in the constructions are absolutely necessary, as the student will find by checking his geometrical result by arithmetic.

If a line 3″ long be taken to represent 9, then a line 6″ long would stand for 18, a line 1″ long for 3, and a line $\frac{1}{3}$″ in length would represent 1. This distance of $\frac{1}{3}$″, which stands for the number 1, is called the **unit of measurement**; it may be of any length, as yards, feet, inches, &c., but for the convenience of geometrical construction the unit must be selected according to the size of the quantity to be measured. If the quantities to be measured are small, units of 1″, $\frac{1}{2}$″, &c. might be suitable; while to represent larger quantities, units of $\frac{1}{100}$″ or $\frac{1}{1000}$″ might be more convenient; but whatever distance is taken to represent the unit, *the result obtained must be on the same scale as that on which the unit represents a number.*

PROBLEM 75.—The line *a b* represents 17. To find the unit.

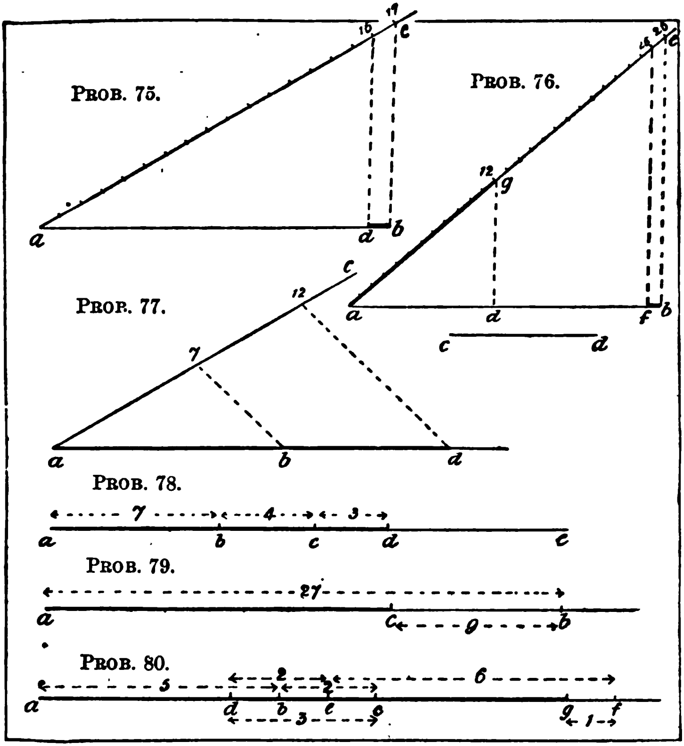
Draw *a c* at any angle to *a b*, and set off from the scale 17 equal distances to any convenient scale (say $\frac{1}{8}$″). Join *b c*, and from 16 draw a line parallel to *b c* and cutting off *d b*, $\frac{1}{17}$ of the given line, and hence the required unit.

PROBLEM 76.—If *a b* represents 26, what does *c d* represent, and what is the unit?

Draw *a e* and set off 26 convenient units from the scale. Join *b* and *e*, and from 25 draw a parallel cutting off the unit *b f*. To find what *c d* represents, set off *a d* equal to *c d*, draw *d g* parallel to *b e*, then *a g* (12) will be the number of units represented by *c d*.

PROBLEM 77.—The line *a b* represents 7 lbs. Draw the line which shall represent 12 lbs.

Draw *a c*, and set off 7 + 5 equal parts. Join 7 *b*, and from 12 draw 12 *d* to meet *a b* produced in *d*. Then *a d* will represent 12 lbs.



PROBLEM 78.—To find the SUM of 7 + 4 + 3. Unit $\frac{1}{8}''$.

Draw any line $a\,e$, set off $a\,b = 7$ units, $b\,c = 4$ units, and $c\,d = 3$ units. If the scale be now applied to the line, $a\,d$ will measure 14 units.

PROBLEM 79.—To find the DIFFERENCE between 27 and 9. Unit $\frac{1}{10}''$.

On any line set off $a\,b = 27$ units. From b set off $b\,c = 9$ units. Then $a\,c$ will be found to measure 18 units on the scale.

PROBLEM 80.—To draw a line to represent the value of $5 + 2 - 3 + 2 + 6 - 1$. Unit ·25″.

Draw a line of indefinite length, and set off $a\,b = 5$ units and add $b\,c$, 2 units more. From c set off $c\,d = 3$ units to the left, and from d add to the right $d\,e$ and $e\,f = 2$ and 6 units respectively. From f subtract $f\,g = 1$ unit. Then $a\,g = 11$ units will be the result.

Note.—Quantities to be added are called **positive**, and are measured to the right, those to be subtracted are called **negative**, and are measured to the left.

MULTIPLICATION

It has already been shown in Chapter IV., that when four numbers are in proportion, *the product of the extremes equals the product of the means.* If one of the extremes be *unity*, then the product of the means equals the other extreme. If we take a numerical example to illustrate this, as the proportion $1 : 4 :: 3 : 12$ we find that $4 \times 3 = 12 \times 1$ and that $12 = \frac{4 \times 3}{1}$. But dividing by unity does not alter the result, therefore if we take lines to represent the numbers, the product of the numbers may be found by finding a fourth proportional (as in Chapter IV. Problem 27) between the three given lines, viz.: the unit and the two lines to be multiplied.

PROBLEM 81.—To multiply the lines A and B together. Unit = C.

The result is first shown when obtained as in Chapter IV. Draw two lines at any angle to each other, set off O C equal to the unit C, O A equal to A, and O B equal to B. Join A C, and from B draw B D parallel to A C; then O D will be the product.

The more convenient method is to arrange the lines as in the second figure. Draw O X of indefinite length. Set off O C equal to the unit, C. Draw C A perpendicular to O C and equal to A. Join O A and produce it. Set off O B equal to B, and draw B D parallel to C A. Then B D will be the product, and must read to the same scale as the unit. The two results will be found to be equal.

Note.—The triangles O A C and O D B are similar and the corresponding sides will be in the same proportion. Therefore O C : C A :: O B : B D, and C A × O B = B D × unity.

PROBLEM 82.—Find the continued product of three or more lines.

Let A, B, C be the given lines, and D the given unit. Find the product of two of the lines and multiply this product by the other line. Draw O X of indefinite length and set off the unit, O D. Draw D A = A. Join O A and produce. Set off O B = B, and draw B E the product of A and B. Make D F = B E. Join O F and produce. Set off O C = C, and draw C G parallel to D F and meeting O F produced. Then C G will be the product of A, B, and C.

DIVISION

If it is required to divide 12 by 3 the quotient might be expressed thus: $\frac{12}{3} = 4$. Dividing the quotient by unity will not alter its value, so that $\frac{12}{3} = \frac{4}{1}$ or $3 : 12 :: 1 : 4$. Division, then, graphically treated will be the finding of a fourth proportional to the statement :—*Divisor* : *dividend* :: *unity* : *quotient*.

PROBLEM 83.—To divide the line A by the line B. Unit = C.

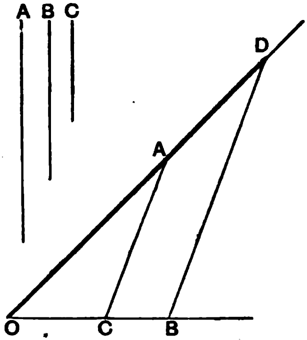
Draw two lines at any angle. Set off O B equal to the divisor B, and O A equal to the dividend A. On O B set off the unit, O C. Join A and B, and from C draw C D parallel to B A. Then O D will be the required quotient.

Note.—O B : O A :: O C : O D. Then $\frac{OA}{OB} = \frac{OD}{OC} = \frac{OD}{1} = OD$.

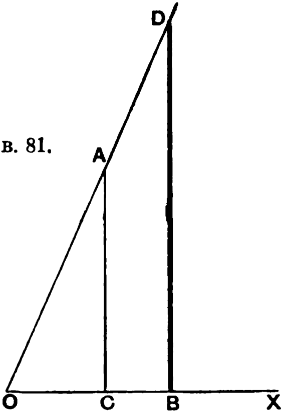
PROBLEM 84.—Find the value of the fraction $\frac{4}{5}$. Unit $\frac{1}{2}''$.

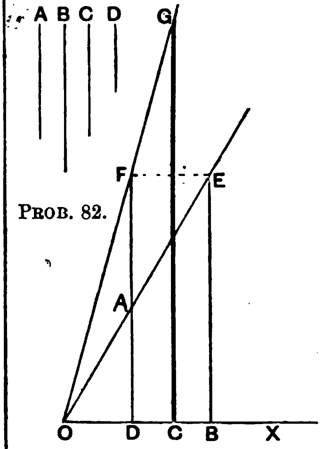
Proceed as in the previous problem. As $5 : 4 :: \frac{1}{2}''$: *quotient*.

Draw two lines at any angle and set off O B = 5, and O A = 4 units. Make O C = $\frac{1}{2}''$. Draw C D parallel to B A, cutting off O D, the required quotient.



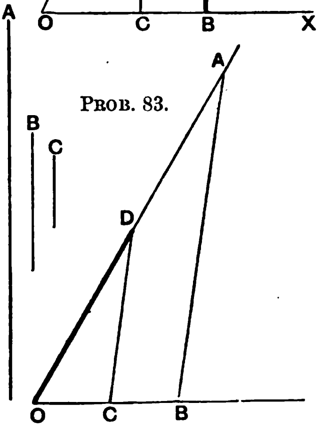
PROB. 81.



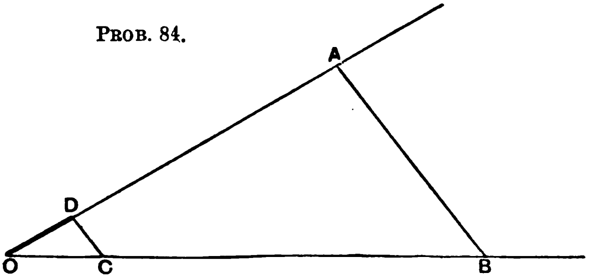


PROB. 82.

PROB. 83.



PROB. 84.



PROBLEM 85.—The given line x represents the sum of the two lines a and b divided by c $\left(\text{in other words } x = \frac{a+b}{c}\right)$. Determine and write down the length of the unit. (*Sc.*, 1888.)

This is an exercise on the preceding problems; the quotient being given instead of the unit. *Divisor* : *dividend* :: *unity* : *quotient*. If we re-arrange the proportion by placing the term to be found in the fourth place, it will read thus:—*Dividend* : *divisor* :: *quotient* : *unity*, and we have simply to find the fourth proportional as before. Draw $o\,c = c$, and $o\,b = a + b$. Set off $o\,d$ equal to the quotient, x, on the dividend, $o\,b$. Draw $d\,e$ parallel to $b\,c$. Then $o\,e$ will be the unit.

FRACTIONS

PROBLEM 86.—To reduce the fractions $\frac{3}{4}$, $\frac{5}{8}$ to equivalent fractions having a common denominator. Unit $\frac{1}{4}''$.

The common denominator may be any number. Draw two lines O m, O n at any angle. On O m set off O a, O b equal to the numerators of the fractions, 3 and 5 units. On O n set off O A, O B equal to the denominators of the fractions, 4 and 8 units. On O n set off O D the required common denominator (say 10 units in length). Join A a, and from D draw D a' parallel to A a. Join B b, and from D draw D b' parallel to B b. Then O a' and O b' are the numerators of the required fractions. $\frac{\text{O}\,a'}{\text{O D}} = \frac{3}{4}$, and $\frac{\text{O}\,b'}{\text{O D}} = \frac{5}{8}$.

Note.—The triangles a' O D and a O A are similar; therefore O a : O A :: O a' : O D, that is, $\frac{\text{O}\,a}{\text{O A}} = \frac{\text{O}\,a'}{\text{O D}}$ or $\frac{3}{4} = \frac{\text{O}\,a'}{\text{O D}}$. The other result may be proved similarly.

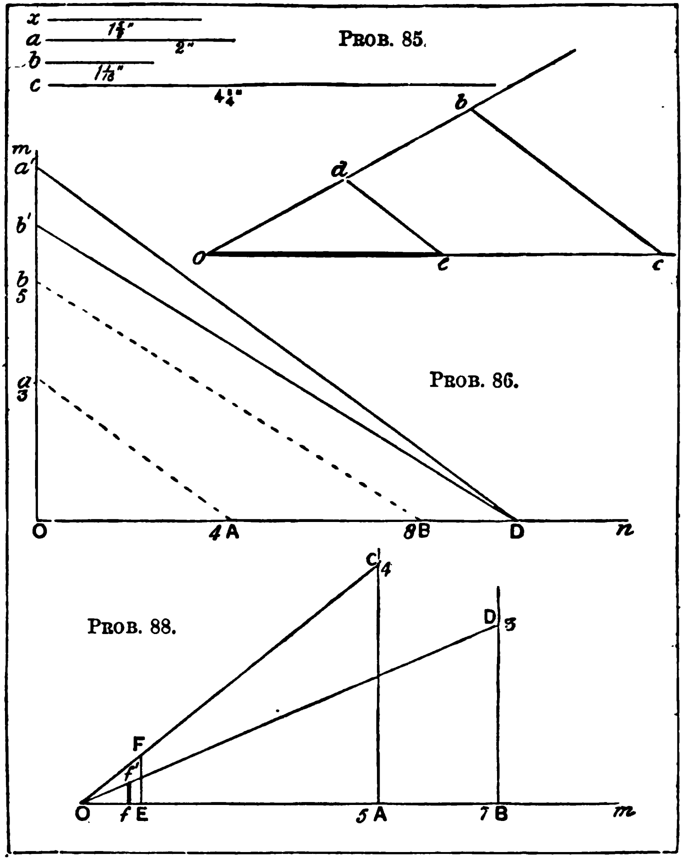
PROBLEM 87.—To add or subtract fractions.

From the previous problem we know that $\frac{3}{4} + \frac{5}{8} = \frac{\text{O}\,a'}{\text{O D}} + \frac{\text{O}\,b'}{\text{O D}} = \frac{\text{O}\,a' + \text{O}\,b'}{\text{O D}}$. Hence, to add fractions, reduce them to equal fractions having a common denominator by *Prob.* 86; add the numerators together by *Prob.* 78; and then divide the result by the common denominator O D by *Prob.* 83. Or a simpler plan would be to find lines representing $\frac{3}{4}$ and $\frac{5}{8}$ by *Prob.* 84, and then add the results together by *Prob.* 78.

PROBLEM 88.—To find the product of $\frac{4}{5} \times \frac{3}{7}$. Unit $\frac{1}{2}''$.

Draw O m of indefinite length. Set off O A, O B equal to the denominators of the fractions, 5 and 7 units respectively. At A draw A C perpendicular to O m and equal to 4, and at B draw B D perpendicular to O m and equal to 3 units. Join O C, O D. Set off O E equal to the unit, and draw E F perpendicular to O m. Make Of = E F, and draw $f\,f'$, the required product, perpendicular to O m.

Note.—$\frac{\text{A C}}{\text{A O}} = \frac{\text{E F}}{\text{E O}} = \frac{4}{5}$. Also $\frac{\text{D B}}{\text{B O}} = \frac{ff'}{f\,\text{O}} = \frac{3}{7}$. Hence $\frac{4}{5} \times \frac{3}{7} = \frac{\text{E F}}{\text{E O}} \times \frac{ff'}{f\,\text{O}}$. But E F and f O will cancel, being equal, leaving $\frac{ff'}{\text{E O}}$, that is $\frac{ff'}{1}$, which gives the result ff'.



PROBLEM 89.—To divide $\frac{4}{5}$ by $\frac{3}{7}$. Unit $\frac{1}{2}''$.

As $\frac{4}{5} \div \frac{3}{7} = \frac{4}{5} \times \frac{7}{3}$, the method of working becomes the same as that in the previous problem.

INVOLUTION

If a number be multiplied by itself any number of times the results obtained are called **powers** of the number. Thus 3×3, or 3^2 as it is usually written, is called the second power of 3; $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or $(\frac{1}{2})^4$, is the fourth power of $\frac{1}{2}$; and the small numbers $(^2)$ and $(^4)$ over the 3 and $\frac{1}{2}$ are called the **indices** of 3 and $\frac{1}{2}$ respectively. To raise or involve a number to its power is simply multiplication, and we may use the method employed in Problems 81 and 82, or the arrangement shown in the following problem. The student is advised to try both methods as a test of accuracy.

PROBLEM 90.—To obtain the fourth power of the line A. Unit = B.

Draw two axes, *x x* and *y y*, at right angles to each other and intersecting in O. Set off O B = the unit, B, and O A = to the given line, A. Join A B and draw A C at right angles to it; then O C will be the *second power* of A, or A^2. Draw C D at right angles to A C, giving O D as the *third power*. Draw D E at right angles to C D, giving O E the required *fourth power*. If the process be continued, the given line may be raised to any power.

Notes.—1. A O B and A O C are similar triangles; therefore B O : O A :: O A : O C, and $BO \times OC = OA \times OA = OA^2$; but B O = unity; therefore $OC = OA^2$. Again, A O : O C :: O C : O D; and $AO \times OD = OC \times OC = OC^2$; but $OC = OA^2$; therefore $OC^2 = OA^4$, and $AO \times OD = OA^4$; therefore $OD = \frac{OA^4}{OA} = OA^3$. In a similar manner OE may be shown to be equal to OA^4.

2. If the unit be greater than the given line, the lines A C, C D, &c., will get less and less.

PROBLEM 91.—To obtain any power of a fraction.

Let $(\frac{5}{6})^3$ be the given expression, and let the unit be $1\frac{3}{4}''$.

We may find a line to represent the fraction by *Problem* 84, and then raise it to the third power by *Problem* 90. A more convenient method, however, is as follows:—Draw two lines, O *m* and O *n*, at any angle. Set off O A and O B equal to 5 and 6 parts respectively *from any scale*. Describe arcs A *a*, B *b* and draw the lines *a b*, A B. On O *m* set off O *c* equal to the unit, $1\frac{3}{4}''$. From *c* draw *c*1 parallel to *a b*, from 1 draw 1 2 parallel to A B, and from 2 draw 2 3 parallel to *a b*. Then O 3 will be the *third power*. O 2 is the *second power*, and a parallel to A B from 3 will give the *fourth power*, &c.

Note.—If the numerator had been larger than the denominator, the parallels would have been drawn *from* the angle O instead of towards it.

EVOLUTION

If the power of a number be given, and it is required to find the root, or number from which the power was obtained, the process is called **evolution.** Thus $8 \times 8 = 64$, and the square root of 64 (written thus, $\sqrt{64}$) would be 8; $4 \times 4 \times 4 = 64$, and 4 would be the third or cube root of 64 (written $\sqrt[3]{64}$). The expression, $\sqrt[5]{32}$ means that the fifth root has to be extracted, which would be 2, because $32 = 2 \times 2 \times 2 \times 2 \times 2$.

PROBLEM 92.—To find the square root of the line A B. Unit = C.

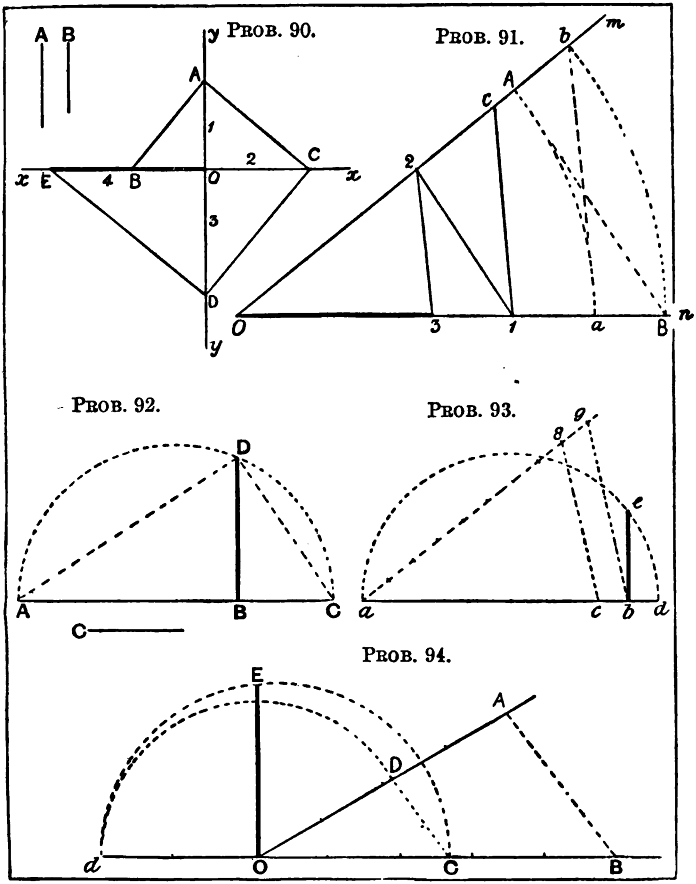
Find a mean proportional between the given line and the unit. Set off A B and C in the same straight line; then the mean proportional, B D, is the required square root.

Note.—The triangles A B D and D B C are similar; ∴ A B : B D :: B D : B C; and $AB \times BC = BD^2$; but B C = unity; ∴ $AB = BD^2$; or $BD = \sqrt{AB}$.

PROBLEM 93.—The line *a b* represents 9; obtain by construction a line representing $\sqrt{ab}$. (*Sc.*, 1884.)

First find the unit *b c* (*Prob.* 75). Make *b d* equal to the unit *b c*. The mean proportional (*b e*) between *a b* and *b d* is the required line.

Note.—When the number represented is large, it is advisable to modify the construction by dividing the number into two factors and using one of them as the unit. Thus, in the problem just worked we could have divided 9 into $4\frac{1}{2} \times 2$. The mean proportional between $4\frac{1}{2}$ units and 2 units will equal *b e*, as $2 \times 4\frac{1}{2} = 9 \times 1$, and the construction would have been better, because *b e* would come more to the middle of the line.



PROBLEM 94.—To find the square root of a fraction, the unit being given.

Let the $\sqrt{\frac{4}{5}}$ be the expression, and 1″ be the unit. The $\sqrt{\frac{4}{5}}$ will be the mean proportional between $\frac{4}{5}$ and the unit. Draw two lines meeting at any angle, and set off O A, O B equal to 4 and 5 equal parts, to any scale. Join A B, and on O B mark off O C equal to the unit, 1″. Draw C D parallel to A B. Then O D represents $\frac{4}{5}$ (*Prob.* 84). Produce O B and make O d = O D. The mean proportional, O E, between O D and O C will represent $\sqrt{\frac{4}{5}}$.

Note.—The fourth root is the square root of the square root. In the same way the eighth and sixteenth roots may be determined from the square root. The other roots are determined by using the logarithmic spiral.

PROBLEM 95.—The line A B represents $\sqrt{10}$; what is the unit?

Take a point, C, in any line and set off C D, any unit of length, and to the right of C set off 10 of these units. Find C F, the square root of C E (*Prob.* 92). Produce C F, and make C G = A B. From G draw G H parallel to F D. Then C H will be the unit required.

Note.—As C D is the unit when C F represents $\sqrt{10}$, then C H must be the unit when C G represents $\sqrt{10}$.

PROBLEM 96.—Supposing the area of a circle is $\frac{22}{7} r^2$, where r is the radius; obtain by construction a line representing the area of a circle 3″ in diameter. Unit 1″. (*Sc.*, 1887.)

Find two lines representing $\frac{22}{7}$ and r^2, and multiply them together; or, set off O B = the unit, 1″, draw O A = r ($1\frac{1}{2}$″), and obtain O C equal to r^2 (*Prob.* 90). $3\frac{1}{7}$ times O C will be the required line.

PROBLEM 97.—Draw a curve which will give the squares of all the quantities from 0 to 4, taking $\frac{1}{8}$″ as the unit.

Draw two lines perpendicular to each other and meeting at O. Set off the unit 4 times to the left of O. Make O X = the unit. Set off four equal distances to the right of O (any unit), and from each of the points 1, 2, &c., draw perpendiculars. Join d X and draw $d\,d'$ at right angles to it. From d' draw a line parallel to O 4 to meet the perpendicular from 4 in D. Join c with X, and draw $c\,c'$ at right angles to c X. From c' draw a line parallel to O 4 to meet the perpendicular from 3. Proceed similarly for the other points.

PROBLEM 98.—Taking one-third of a as unit; determine a line representing $\frac{\sqrt{c}}{b}$. (*Sc.*, 1886.)

Make $o\,a = \frac{1}{3}\,a$, and $o\,c = c$. Then the mean proportional $o\,d$ will equal $\sqrt{c}$ (*Prob.* 92). Make $o\,b = b$ and join $b\,d$. Draw $a\,e$ parallel to $b\,d$. Then $o\,e = \frac{\sqrt{c}}{b}$.

Note.—$oe : o\,d :: o\,a : o\,b$; that is, $o\,e : \sqrt{c} :: 1 : b$. Then $o\,e \times b = \sqrt{c} \times 1$. Therefore $o\,e = \frac{\sqrt{c}}{b}$.

PROBLEM 99.—A line 3″ long represents $3\frac{1}{2}$ units. Obtain a line representing $2\sqrt{5}$.

Draw $a\,b = 3''$ and produce it. From a draw a line and set off $3\frac{1}{2}$ and 5 equal parts. Join $3\frac{1}{2}$ and b, and from 5 and 1 draw parallels to this line. Then $a\,d$ = the unit, and $a\,c = 5$. Make $a\,d' = a\,d$, and find a mean proportional, $a\,f$. Then $a\,f = \sqrt{5}$, and $f\,g = 2\sqrt{5}$.

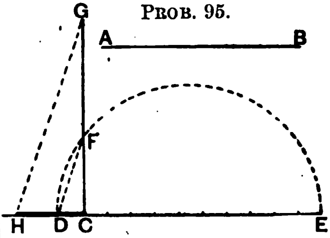
To represent areas by straight lines.

Reduce the given figure to an equivalent triangle as shown in Chapter XIII., and then reduce the triangle to another triangle of equal area, but having an altitude of 2″. The area of a triangle equals $\frac{\text{base} \times \text{height}}{2}$. But if the height = 2″, then the area will be represented by the number of inches in the base.

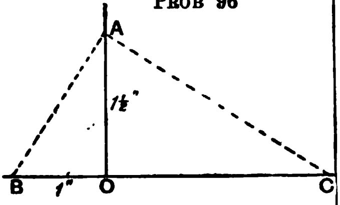
PROBLEM 100.—Given the triangle A B C. Determine another triangle equal to it in area, but having an altitude of 2″.

With centre B and radius 2″ describe an arc. From A draw A D tangent to the arc, and from C draw C D parallel to A B. Join B and D. Then the triangle A B D = triangle A B C, and A D will represent the area of the given triangle.

PROB. 95.



PROB 96



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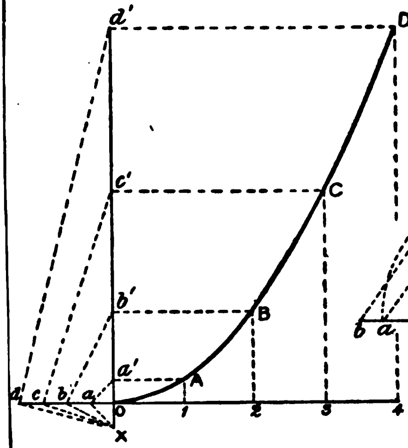

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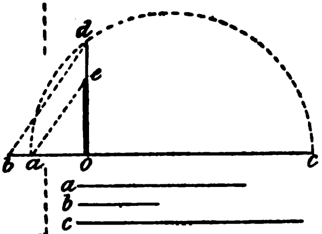

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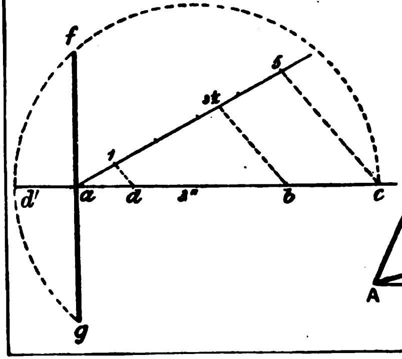
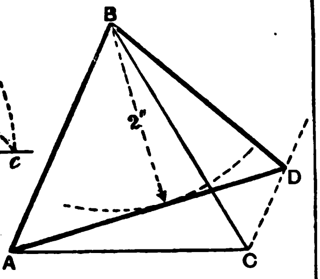


PROB. 100.



FIRST STAGE, OR ELEMENTARY EXAMINATION.

1890.

Only eight *questions are to be attempted. Questions marked* (*) *have accompanying diagrams.*

PLANE GEOMETRY.

11. Draw a triangle the sides of which are respectively twice the length of those of the given triangle *a b c* (Fig. to Question 3) and produce the sides indefinitely. Draw four circles each touching the three lines. (10.)

12. Construct a regular pentagon *a b c d e* on the side of 1·5″. Join the centre points of the sides *a b* and *d e*, and circumscribe the four-sided figure thus obtained by a circle. What is this figure called? (8.)

13. In a circle of 2⅝″ diameter place seven equi-distant radii. From the outer extremity of any radius, draw a line to the outer extremity of the next radius but one, and continue the process, always missing out a radius. (8.)

SOLID GEOMETRY.

14. A point P is ¾″ in front of the vertical and 1″ above the horizontal plane of projection. Show it by its plan and elevation, and through it draw a plane inclined at 50° to the horizontal plane and perpendicular to the vertical plane. (10.)

***15.** The traces of a plane are given. Determine the plan and elevation of a point which is in the given plane, ½″ in front of the vertical plane, and the plan of which is 1¼″ from the horizontal trace of the given plane. (10.)

16. The horizontal and vertical traces of a plane make respecively angles of 55° and 80° with the ground line. Determine the plane which bisects the obtuse dihedral angle between the horizontal plane and the given plane. (10.)

***17.** The traces of a plane are given and the vertical trace of a second plane. This second plane is inclined at 65° to the horizontal plane. Determine the inclination of the first plane to the horizontal plane, and the plan and elevation of the line in which the planes intersect. (12.)

***18.** The letter L as given rests on a plane inclined at 35°, with the points *a* and *b* on the horizontal trace of the plane. Draw its plan. (10.)

19. An equilateral triangle *a′ b′ c′* of 1½″ side is the end elevation of a prism 2″ long. Draw its plan when the long edge through **A** is in the horizontal plane, and the face containing the edge **A B** is inclined at an angle of 40° to the horizontal plane. (12.)

***20.** The plan and elevation of a solid, made up of two square prisms, are given. Draw its plan when, instead of standing vertically as in the figure, the edges through the points marked **A** and **B** are both in the horizontal plane. (16.)

***21.** The plan and side elevation of a wooden coal-box are given. Draw the elevation on the given ground line *x y*. (14.)

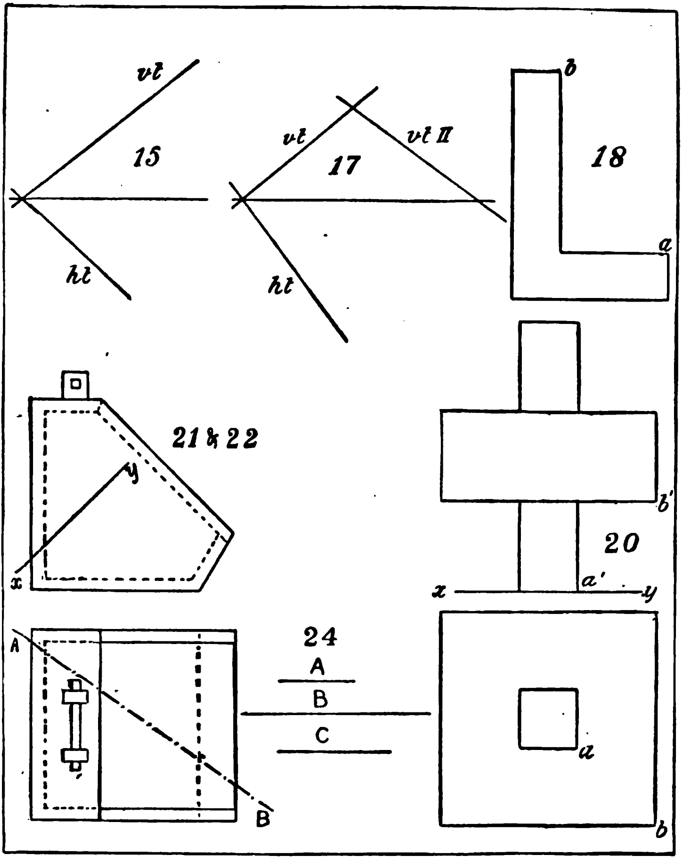
***22.** Draw the section of the coal-box made by the vertical plane, the horizontal trace of which is the line **A B**. (14.)

GRAPHIC ARITHMETIC.

23. Taking ¼″ as unit, obtain lines representing—

$$15,\ \sqrt{15},\ \text{and}\ \frac{5}{\sqrt{15}}. \qquad (10.)$$

***24.** Taking the given line **A** as unit of length, determine and write down the number of units of area in the rectangle contained by the given lines **B** and **C**. (12.)



EXERCISES

CHAPTER XXII

1. A line 3" long represents 9. Find the unit and draw a line to represent 15 to the same unit.
2. Find graphically the value of 2·5 − 1·7 + 3·1 − 2·4. Unit 1".
3. Obtain a line to represent 3 × 12 × 2. Unit $\frac{1}{8}$".
4. Find the value of $\frac{1}{3}+\frac{3}{4}-\frac{1}{5}$. Unit 2".
5. Determine the $\sqrt{29}$. Unit $\frac{1}{4}$".
6. A line 3" long represents 5, find the value of $\sqrt{5}$.
7. A line 2" long represents $\sqrt{7}$. Find the unit.
8. Find the value of $\frac{2}{3}+2\sqrt{5}$. Unit $\frac{1}{2}$".

9. ab ($4\frac{1}{2}''$) represents 9. Obtain by construction a line which will represent the square root of ab. (*Sc.*, 1884.)

10. Determine the product of the lines a and b, $2\frac{3}{4}''$ and $1\frac{1}{4}''$ respectively, and write down the number of units it contains. (*Sc.*, 1885.)

11. If a ($2\frac{3}{4}''$) represents $\sqrt{23}$, what is the unit? (*Sc.*, 1885.)

12. The given line a ($2\frac{1}{2}''$) represents the product of the lines b ($1\frac{1}{4}''$) and c ($3\frac{1}{2}''$). Determine and write down the length of the unit. (*Sc.*, 1886.)

13. Determine by construction a line representing the sum of the given lines A ($1\frac{3}{16}''$), B ($2\frac{5}{16}''$), divided by C ($1\frac{5}{8}''$). Unit $1\frac{1}{8}''$. (*Sc.*, 1887.)

14. A line $2{\cdot}6''$ long represents $4\frac{1}{3}$ units. Obtain lines representing 13, $\frac{10}{3}$, and $2\sqrt{6}$. (*Sc.*, 1889.)

15. If the given line A ($\frac{3}{4}''$) represents the unit, what numbers do the lines B ($3\frac{5}{16}''$), C ($1\frac{1}{8}''$), and D ($1\frac{9}{16}''$) severally represent? Determine also a line to represent $\frac{\mathrm{B}}{\mathrm{C} \times \mathrm{D}}$. (*Sc.*, 1889.

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